Elektromagnetische Feldtheorie I (EFT I) / Electromagnetic Field Theory I (EFT I)

12th Lecture / 12. Vorlesung

Dr.-Ing. René Marklein

marklein@uni-kassel.de

http://www.tet.e-technik.uni-kassel.de

http://www.uni-kassel.de/fb16/tet/marklein/index.html

Universität Kassel
Fachbereich Elektrotechnik / Informatik
(FB 16)
Fachgebiet Theoretische Elektrotechnik
(FG TET)
Wilhelmshöher Allee 71
Büro: Raum 2113 / 2115
D-34121 Kassel

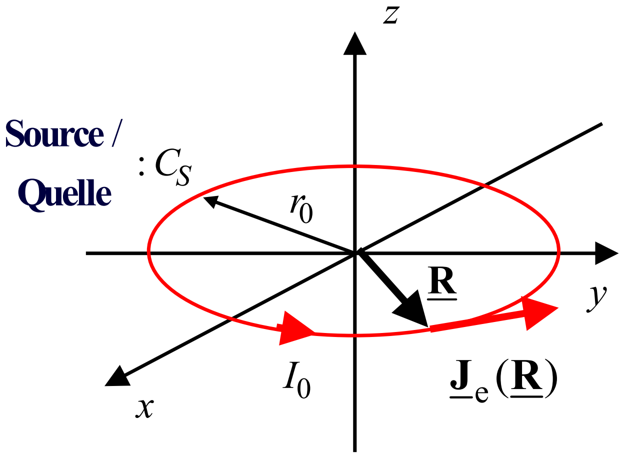
University of Kassel
Dept. Electrical Engineering / Computer Science (FB 16)
Electromagnetic Field Theory
(FG TET)
Wilhelmshöher Allee 71
Office: Room 2113 / 2115
D-34121 Kassel

MS Fields – Magnetic Dipole Moment / MS-Felder – Magnetisches Dipolmoment

Magnetic Dipole Moment / Magnetisches Dipolmoment

$$\underline{\mathbf{p}}_{\mathrm{m}} = \frac{\mu_0}{2} \iiint_{V_s} \underline{\mathbf{R}} \times \underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}}) \, \mathrm{d}^3 \underline{\mathbf{R}}$$

Example: Magnetic Dipole Moment of a Current Loop /
Beispiel: Magnetisches Dipolmoment einer elektrischen Stromschleife



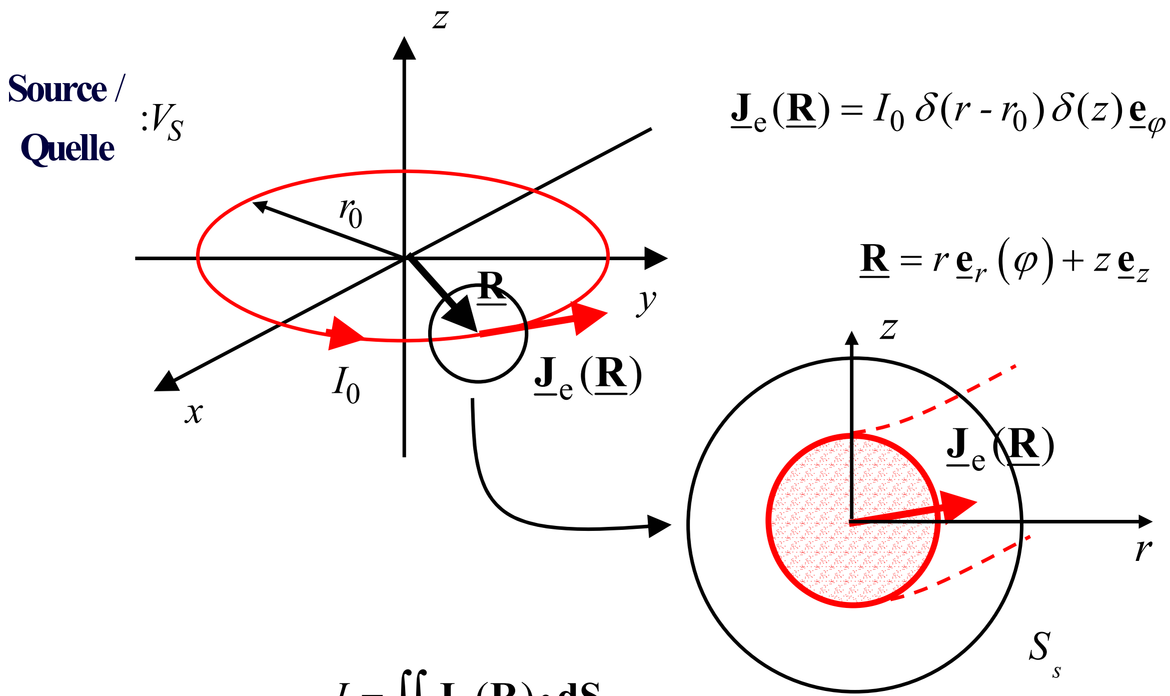
$$\underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}}) = I_0 \, \delta(r - r_0) \, \delta(z) \, \underline{\mathbf{e}}_{\varphi}(\varphi), \qquad r_0 > 0$$

$$\underline{\mathbf{R}} = r \, \underline{\mathbf{e}}_r(\varphi) + z \, \underline{\mathbf{e}}_z$$

$$\underline{\mathbf{p}}_{\mathrm{m}} = \frac{\mu_0}{2} \iiint_{V_s} \underline{\mathbf{R}} \times \underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}}) \, \mathrm{d}^3 \underline{\mathbf{R}}$$

$$\underline{\mathbf{R}} \times \underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}}) = \left[r \, \underline{\mathbf{e}}_r(\varphi) + z \, \underline{\mathbf{e}}_z \right] \times \left[I_0 \, \delta(r - r_0) \, \delta(z) \, \underline{\mathbf{e}}_{\varphi}(\varphi) \right]$$

MS Fields – Magnetic Dipole Moment of an Electric Current Loop / MS-Felder – Magnetisches Dipolmoment einer elektrischen Stromschleife



$$\underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}}) = I_0\,\delta(r - r_0)\,\delta(z)\,\underline{\mathbf{e}}_{\varphi}(\varphi), \qquad r_0 > 0$$

$$\underline{\mathbf{R}} = r\,\underline{\mathbf{e}}_r(\varphi) + z\,\underline{\mathbf{e}}_z$$

$$I = \iint\limits_{S_s} \underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}}) \cdot \underline{\mathbf{dS}}$$

$$= \int\limits_{r=0}^{\infty} \int\limits_{z=-\infty}^{\infty} I_0\,\delta(r - r_0)\,\delta(z)\,\underbrace{\underline{\mathbf{e}}_{\varphi}(\varphi) \cdot \underline{\mathbf{e}}_{\varphi}(\varphi)}_{=1}\,\mathrm{d}z\,\mathrm{d}r$$

$$= I_0$$

MS Fields – Magnetic Dipole Moment of an Electric Current Loop / MS-Felder – Magnetisches Dipolmoment einer elektrischen Stromschleife

$$\underline{\mathbf{R}} \times \underline{\mathbf{J}}_\mathrm{e}(\underline{\mathbf{R}}) = \left[r\,\underline{\mathbf{e}}_r(\varphi) + z\,\underline{\mathbf{e}}_z \right] \times \left[I_0\,\delta(r - r_0)\,\delta(z)\,\underline{\mathbf{e}}_\varphi(\varphi) \right]$$

$$= I_0\,\delta(r - r_0)\,\delta(z) \left[r\,\underline{\mathbf{e}}_r(\varphi) + z\,\underline{\mathbf{e}}_z \right] \times \underline{\mathbf{e}}_\varphi(\varphi)$$

$$= I_0\,\delta(r - r_0)\,\delta(z) \begin{vmatrix} \underline{\mathbf{e}}_r(\varphi) & \underline{\mathbf{e}}_\varphi(\varphi) & \underline{\mathbf{e}}_z \\ r & 0 & z \\ 0 & 1 & 0 \end{vmatrix}$$

$$= I_0\,\delta(r - r_0)\,\delta(z) \left[r\underline{\mathbf{e}}_z - z\underline{\mathbf{e}}_r(\varphi) \right]$$

$$\underline{\mathbf{p}}_\mathrm{m} = \frac{\mu_0}{2} \iiint_{V_s} \underline{\mathbf{R}} \times \underline{\mathbf{J}}_\mathrm{e}(\underline{\mathbf{R}})\,\mathrm{d}^3\underline{\mathbf{R}}$$

$$= \frac{\mu_0}{2} \int_{r=-\infty}^{\infty} \int_{\varphi=0}^{2\pi} \int_{z=-\infty}^{\infty} I_0\,\delta(r - r_0)\,\delta(z) \left[r\underline{\mathbf{e}}_z - z\underline{\mathbf{e}}_r(\varphi) \right] r\,\mathrm{d}r\,\mathrm{d}\varphi\,\mathrm{d}z$$

$$= \frac{\mu_0 I_0}{2} \left[\int_{r=-\infty}^{\infty} \int_{\varphi=0}^{2\pi} \int_{z=-\infty}^{\infty} \delta(r - r_0)\,\delta(z)\,r\underline{\mathbf{e}}_z\,r\,\mathrm{d}r\,\mathrm{d}\varphi\,\mathrm{d}z \right.$$

$$\left. - \int_{r=-\infty}^{\infty} \int_{\varphi=0}^{2\pi} \int_{z=-\infty}^{\infty} \delta(r - r_0)\,\delta(z)\,z\underline{\mathbf{e}}_r(\varphi)\,r\,\mathrm{d}r\,\mathrm{d}\varphi\,\mathrm{d}z \right]$$

MS Fields – Magnetic Dipole Moment of an Electric Current Loop / MS-Felder – Magnetisches Dipolmoment einer elektrischen Stromschleife

$$\underline{\mathbf{p}}_{\mathrm{m}} = \frac{\mu_0 I_0}{2}\left[\int\limits_{r=-\infty}^{\infty}\int\limits_{\varphi=0}^{2\pi}\int\limits_{z=-\infty}^{\infty}\delta(r-r_0)\,\delta(z)\,r^2\,\mathrm{d}r\,\mathrm{d}\varphi\,\mathrm{d}z\,\underline{\mathbf{e}}_z - \underbrace{\int\limits_{r=-\infty}^{\infty}\int\limits_{\varphi=0}^{2\pi}\int\limits_{z=-\infty}^{\infty}\delta(r-r_0)\,\delta(z)\,z\,\underline{\mathbf{e}}_r(\varphi)\,r\,\mathrm{d}r\,\mathrm{d}\varphi\,\mathrm{d}z}_{=0}\right]$$

$$= \frac{\mu_0 I_0}{2}\int\limits_{r=-\infty}^{\infty}\int\limits_{\varphi=0}^{2\pi}\int\limits_{z=-\infty}^{\infty}\delta(r-r_0)\,\delta(z)\,r^2\,\mathrm{d}r\,\mathrm{d}\varphi\,\mathrm{d}z\,\underline{\mathbf{e}}_z$$

because / weil $\int\limits_{z=-\infty}^{\infty}\delta(z)\,z\,\mathrm{d}z = 0$

$$= \frac{\mu_0 I_0}{2}\underbrace{\left[\int\limits_{r=-\infty}^{\infty}\delta(r-r_0)\,r^2\underbrace{\left[\int\limits_{\varphi=0}^{2\pi}\left[\underbrace{\int\limits_{z=-\infty}^{\infty}\delta(z)\,\mathrm{d}z}_{=1}\right]\mathrm{d}\varphi\right]}_{=2\pi}\mathrm{d}r\right]}_{=r_0^2}\underline{\mathbf{e}}_z$$

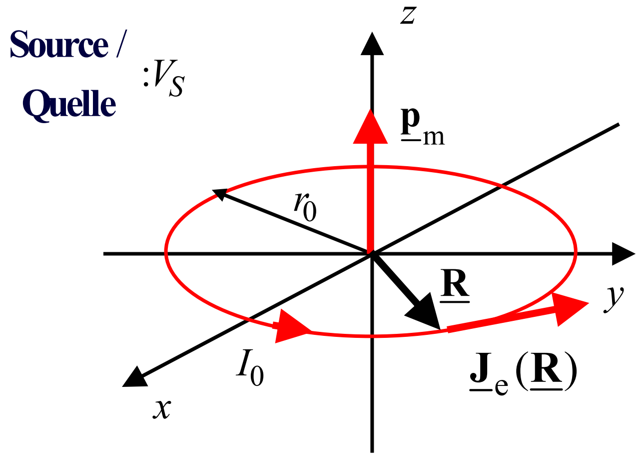
$$= \mu_0 \pi I_0\underbrace{\left[\int\limits_{r=-\infty}^{\infty}\delta(r-r_0)\,r^2\,\mathrm{d}r\right]}_{=r_0^2}\underline{\mathbf{e}}_z$$

$$= \mu_0\,\pi\,I_0\,r_0^2\,\underline{\mathbf{e}}_z$$

Example: Magnetic Dipole Moment of a Current Loop / Beispiel: Magnetisches Dipolmoment einer elektrischen Stromschleife

$$\underline{\mathbf{p}}_{\mathrm{m}} = \mu_0\,\pi\,I_0\,r_0^2\,\underline{\mathbf{e}}_z$$

MS Fields – Magnetic Dipole Moment of an Electric Current Loop / MS-Felder – Magnetisches Dipolmoment einer elektrischen Stromschleife

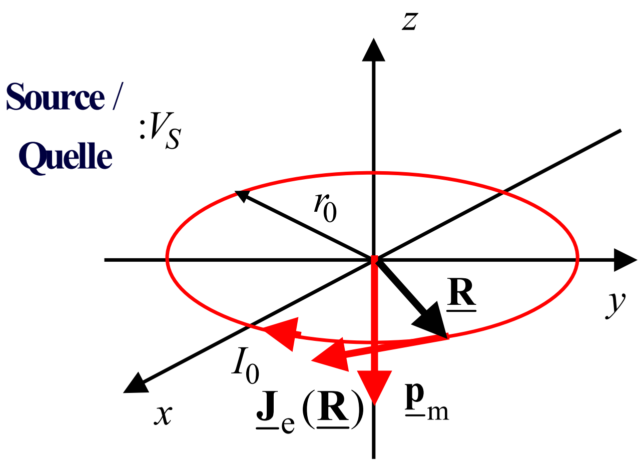


Example: Magnetic Dipole Moment of a Current Loop / Beispiel: Magnetisches Dipolmoment einer elektrischen Stromschleife

$$\underline{\mathbf{p}}_\mathrm{m} = \mu_0 \, \pi \, I_0 \, r_0^2 \, \underline{\mathbf{e}}_z$$

$$\underline{\mathbf{p}}_\mathrm{m} \sim I_0$$

$$\underline{\mathbf{p}}_\mathrm{m} \sim \underline{\mathbf{e}}_z \quad \rightarrow \quad \perp \quad xy \text{ Plane / } xy\text{ - Ebene}$$



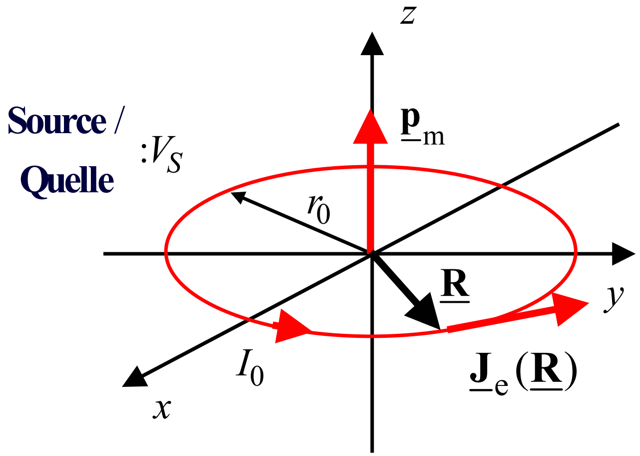
$$\underline{\mathbf{p}}_\mathrm{m} = -\mu_0 \, \pi \, I_0 \, r_0^2 \, \underline{\mathbf{e}}_z$$

$$\underline{\mathbf{p}}_\mathrm{m} \sim I_0$$

$$\underline{\mathbf{p}}_\mathrm{m} \sim -\underline{\mathbf{e}}_z \quad \rightarrow \quad \perp \quad xy \text{ Plane / } xy\text{ - Ebene}$$

MS Fields – Magnetic Dipole Moment of an Electric Current Loop / MS-Felder – Magnetisches Dipolmoment einer elektrischen Stromschleife

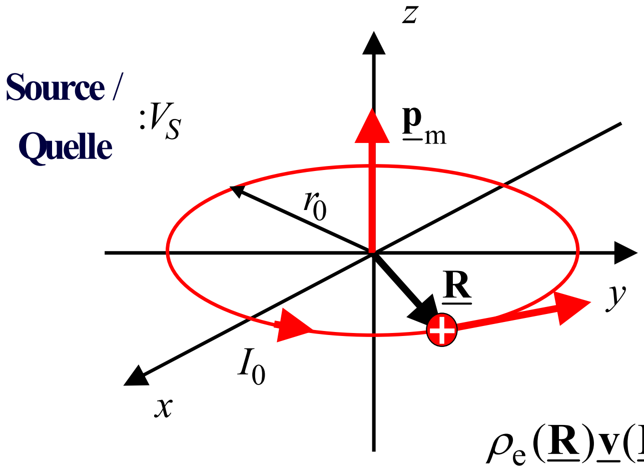
Example: Magnetic Dipole Moment of a Current Loop / Beispiel: Magnetisches Dipolmoment einer elektrischen Stromschleife



$$\underline{\mathbf{p}}_{\mathrm{m}} = \mu_0 \, \pi \, I_0 \, r_0^2 \, \underline{\mathbf{e}}_z$$

$$\underline{\mathbf{p}}_{\mathrm{m}} \sim I_0$$

$$\underline{\mathbf{p}}_{\mathrm{m}} \sim \underline{\mathbf{e}}_z \quad \rightarrow \quad \perp \quad xy \text{ Plane / } xy\text{ - Ebene}$$



$$\underline{\mathbf{p}}_{\mathrm{m}} = \mu_0 \, \pi \, Q_{\mathrm{e}} \, v_0 \, r_0^2 \, \underline{\mathbf{e}}_z$$

$$\underline{\mathbf{p}}_{\mathrm{m}} \sim I_0$$

$$\underline{\mathbf{p}}_{\mathrm{m}} \sim \underline{\mathbf{e}}_z \quad \rightarrow \quad \perp \quad xy \text{ Plane / } xy\text{ - Ebene}$$

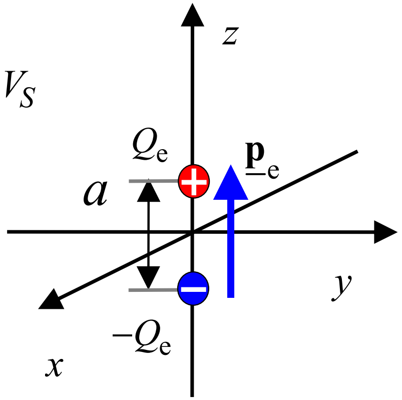
$$\rho_{\mathrm{e}}(\underline{\mathbf{R}})\underline{\mathbf{v}}(\underline{\mathbf{R}}) = \underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}})$$

ES and MS Fields – Electric and Magnetic Dipole Moment / ES- und MS-Felder – Elektrisches und Magnetisches Dipolmoment

Electric Dipole Moment / Elektrisches Dipolmoment

$$\underbrace{\underline{\mathbf{p}}_{\mathrm{e}}}_{=[\mathrm{As\,m}]} = \underbrace{\iiint_{V_s} \underbrace{\rho_{\mathrm{e}}(\underline{\mathbf{R}})}_{=[\mathrm{As/m^3}]} \underbrace{\underline{\mathbf{R}}}_{=[\mathrm{m}]} \underbrace{\mathrm{d}^3\underline{\mathbf{R}}}_{=[\mathrm{m^3}]}}_{=[\mathrm{As\,m}]}$$

Source / Quelle : V_S

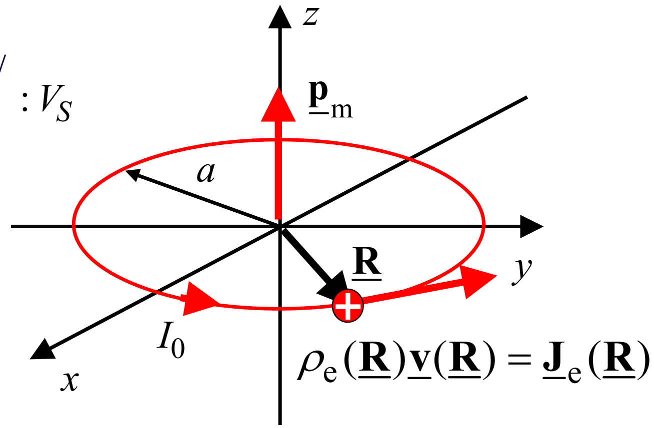


$$\underline{\mathbf{p}}_{\mathrm{e}} = Q_{\mathrm{e}} a \underline{\mathbf{e}}_z \quad [\mathrm{As\,m}]$$

Magnetic Dipole Moment / Magnetisches Dipolmoment

$$\underbrace{\underline{\mathbf{p}}_{\mathrm{m}}}_{=[\mathrm{Vs\,m}]} = \underbrace{\underbrace{\frac{\mu_0}{2}}_{=[\mathrm{Vs/Am}]} \iiint_{V_s} \underbrace{\underbrace{\underline{\mathbf{R}}}_{=[\mathrm{m}]} \times \underbrace{\underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}})}_{=[\mathrm{A/m^2}]} \underbrace{\mathrm{d}^3\underline{\mathbf{R}}}_{=[\mathrm{m^3}]}}_{=[\mathrm{Am^2}]}}_{=[\mathrm{Vs\,m}]}$$

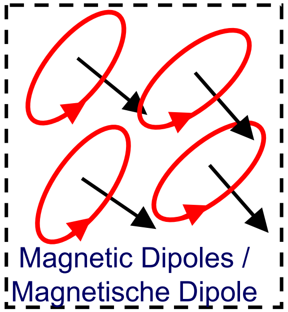
Source / Quelle : V_S



$$\underline{\mathbf{p}}_{\mathrm{m}} = \mu_0 \pi Q_{\mathrm{e}} |\underline{\mathbf{v}}| a^2 \underline{\mathbf{e}}_z$$

MS Fields – Magnetic Polarization of Materials / MS-Felder – Magnetische Polarisation von Materialien

$$\iiint_{V_M} \underline{\mathbf{P}}_\mathrm{m}(\underline{\mathbf{R}})\,\mathrm{d}V = \sum_{i=1}^{N} \underline{\mathbf{p}}_\mathrm{m}^{(i)}$$



$\underline{\mathbf{p}}_\mathrm{m}^{(i)}, i = 1,2,3,4$

$$\underline{\mathbf{B}}(\underline{\mathbf{R}}) = \begin{cases} \mu_0 \underline{\mathbf{H}}(\underline{\mathbf{R}}) + \underline{\mathbf{P}}_\mathrm{m}(\underline{\mathbf{R}}) & \underline{\mathbf{R}} \in V_\mathrm{mat} \\ \mu_0 \underline{\mathbf{H}}(\underline{\mathbf{R}}) & \underline{\mathbf{R}} \in V_\mathrm{vac} \end{cases}$$

$$\underline{\mathbf{B}}(\underline{\mathbf{R}}) = \begin{cases} \mu_0 \underline{\mathbf{H}}(\underline{\mathbf{R}}) + \mu_0 \underline{\mathbf{M}}(\underline{\mathbf{R}}) & \underline{\mathbf{R}} \in V_\mathrm{mat} \\ \mu_0 \underline{\mathbf{H}}(\underline{\mathbf{R}}) & \underline{\mathbf{R}} \in V_\mathrm{vac} \end{cases}$$

$$\begin{aligned} \underline{\mathbf{B}}(\underline{\mathbf{R}}) &= \mu_0 \underline{\mathbf{H}}(\underline{\mathbf{R}}) + \underline{\mathbf{P}}_\mathrm{m}(\underline{\mathbf{R}}) \\ &= \mu_0 \mu_\mathrm{r}(\underline{\mathbf{R}}) \underline{\mathbf{H}}(\underline{\mathbf{R}}) \\ &= \mu_0 \underline{\mathbf{H}}(\underline{\mathbf{R}}) + \underbrace{\mu_0 \left[\mu_\mathrm{r}(\underline{\mathbf{R}}) - 1\right] \underline{\mathbf{H}}(\underline{\mathbf{R}})}_{=\underline{\mathbf{P}}_\mathrm{m}(\underline{\mathbf{R}})} \end{aligned}$$

$\mu_\mathrm{r}(\underline{\mathbf{R}})$ Relative Permeability / Relative Permeabilität

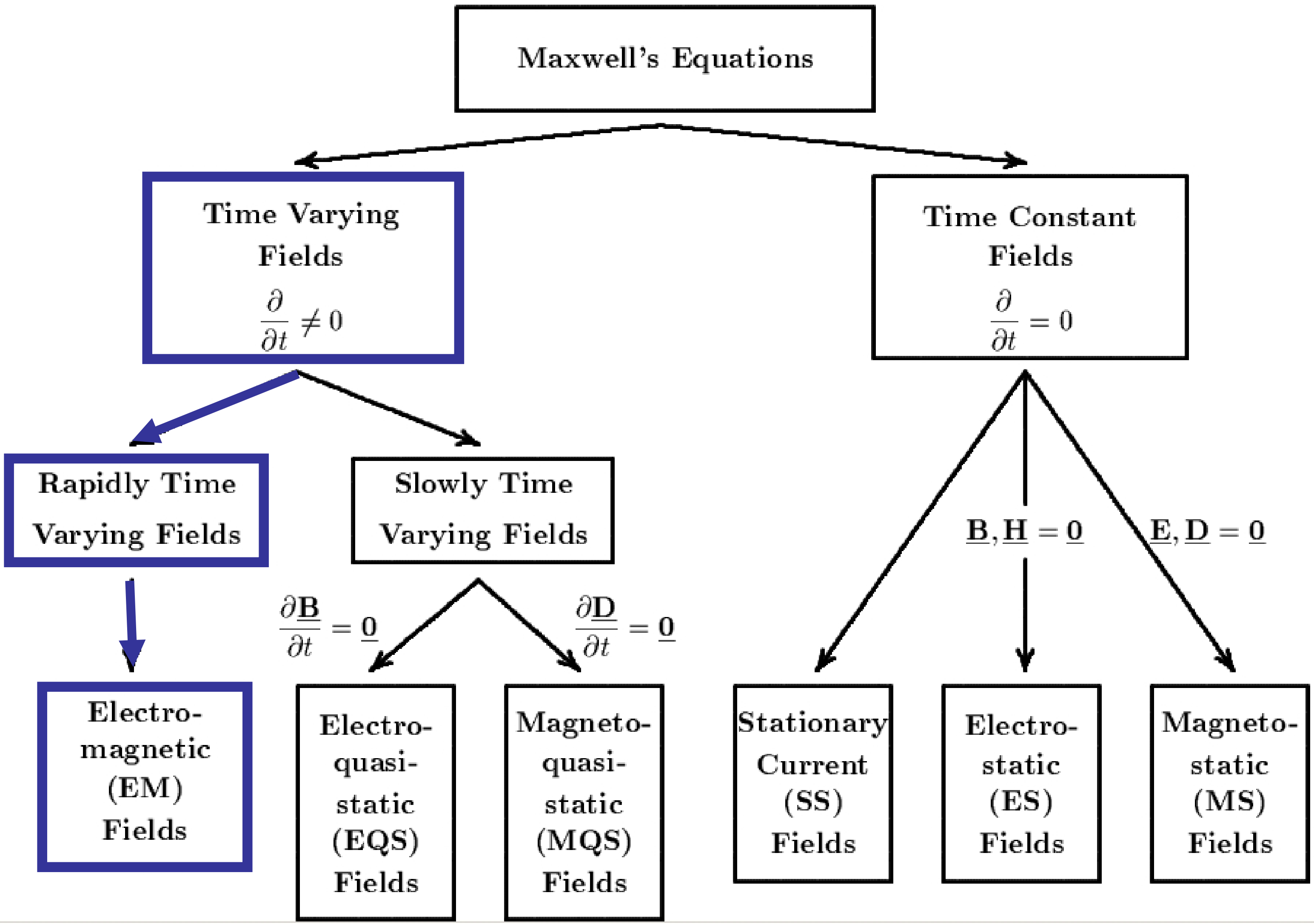
$$\begin{aligned} \underline{\mathbf{P}}_\mathrm{m}(\underline{\mathbf{R}}) &= \mu_0 \underbrace{\left[\mu_\mathrm{r}(\underline{\mathbf{R}}) - 1\right]}_{=\chi_\mathrm{m}(\underline{\mathbf{R}})} \underline{\mathbf{H}}(\underline{\mathbf{R}}) \\ &= \mu_0 \chi_\mathrm{m}(\underline{\mathbf{R}}) \underline{\mathbf{H}}(\underline{\mathbf{R}}) \end{aligned}$$

$\chi_\mathrm{m}(\underline{\mathbf{R}})$ Magnetic Susceptibility / Magnetische Suszeptibilität

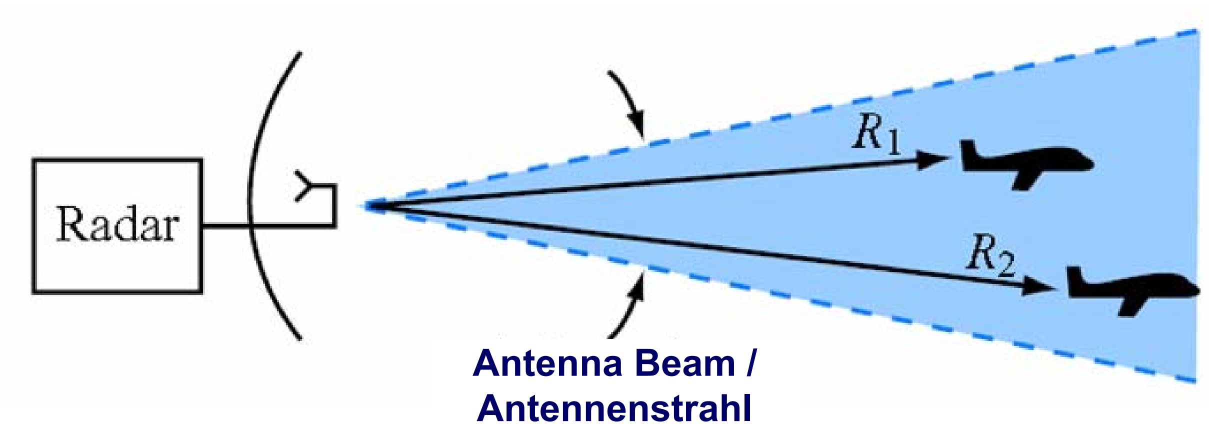
MS Fields – Relative Permeability / MS-Felder – Relative Permeabilität

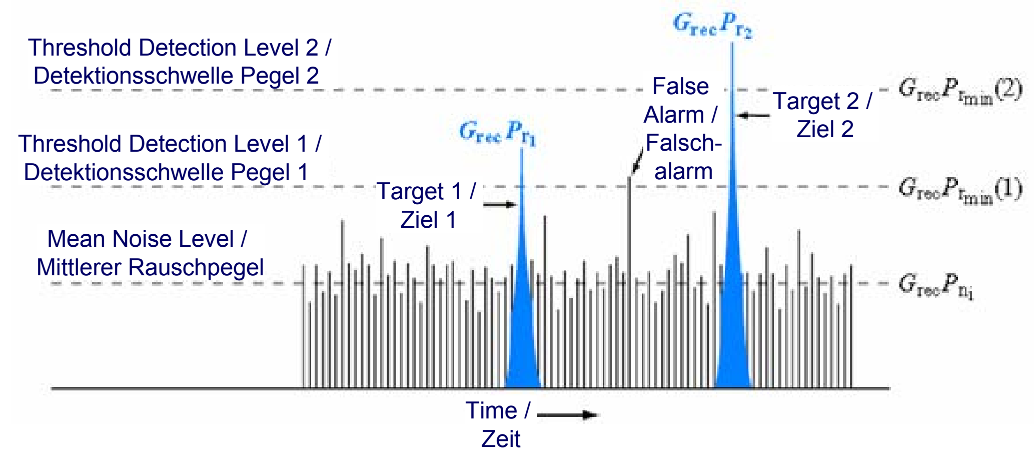
Material / Material	μ_r
Diamagnetic / Diamagnetisch	$\mu_r < 1$
Silver / Silber	0,99998
Lead / Blei	
Copper / Kupfer	0,99999
Paramagnetic / Paramagnetisch	$\mu_r > 1$
Vacuum / Vakuum	1
Air / Luft	1,00000035
Aluminum / Aluminium	1,000024
Tungsten / Wolfram	1,000067
Platinum / Platin	1,000256
Ferromagnetic / Ferromagnetisch	$\mu_r >> 1$
Nickel / Nickel	600
Iron / Eisen	5.000 (>> 1000)
Permalloy	150.000
Superalloy	1.000.000

Electromagnetic (EM) Fields – Classification / Elektromagnetische (EM) Felder - Klassifikation

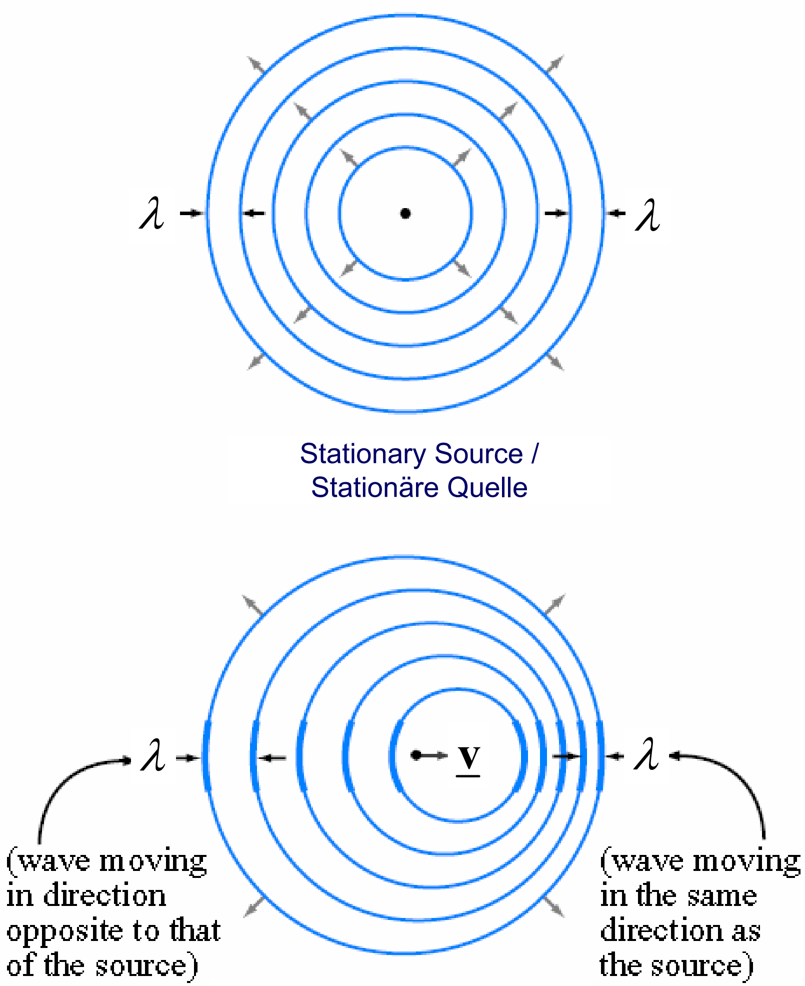


EM Fields – EM Wave – Radar Systems / EM Felder – EM-Wellen – Radar-Systeme



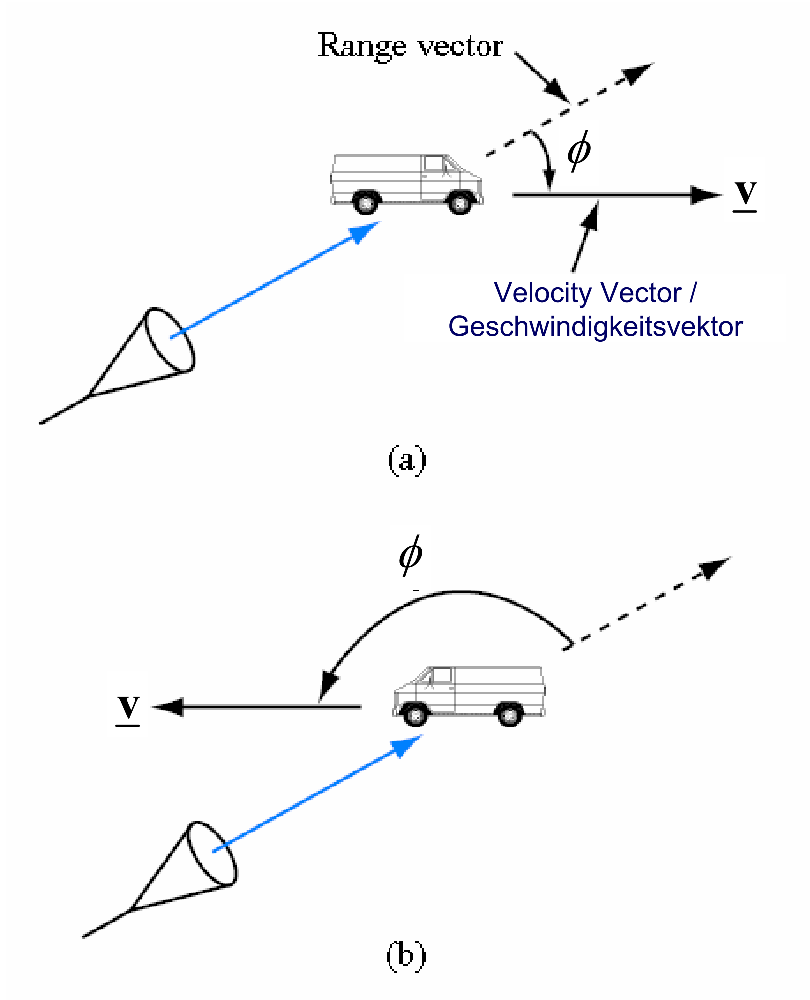


EM Fields – EM Waves – Doppler Radar Systems / EM Felder – EM-Wellen – Doppler Radar-Systeme

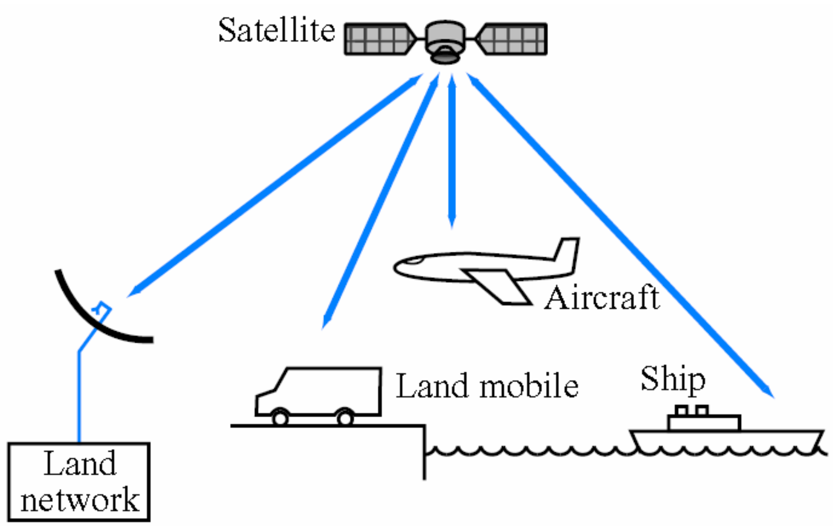


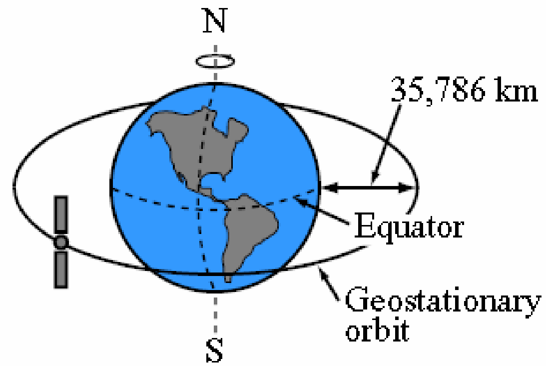
Stationary Source / Stationäre Quelle

Moving Source / Bewegte Quelle

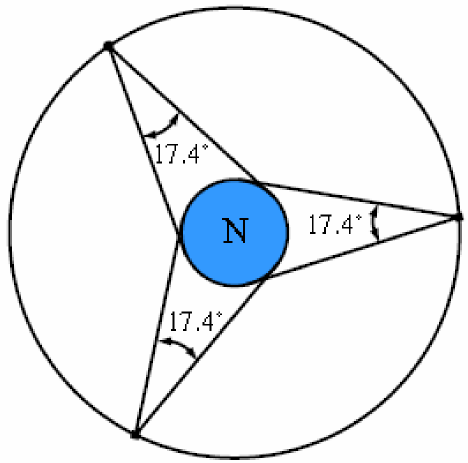


EM Fields – EM Waves – Satellite Communication / EM Felder – EM-Wellen– Satellitenkommunikation



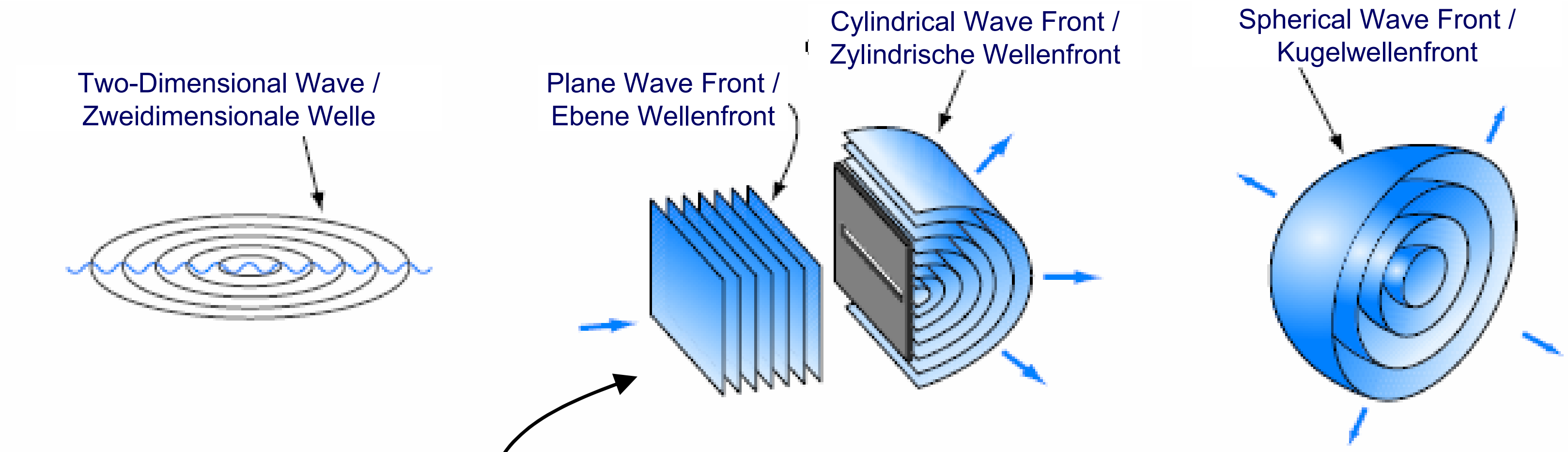


(a) Geostationary satellite orbit



(b) Worldwide coverage by three satellites spaced 120° apart

EM Fields – EM Waves – Elementary EM Waves / EM Felder – EM-Wellen – Elementare EM-Wellen



(a) Circular Wave / Zylinderwelle

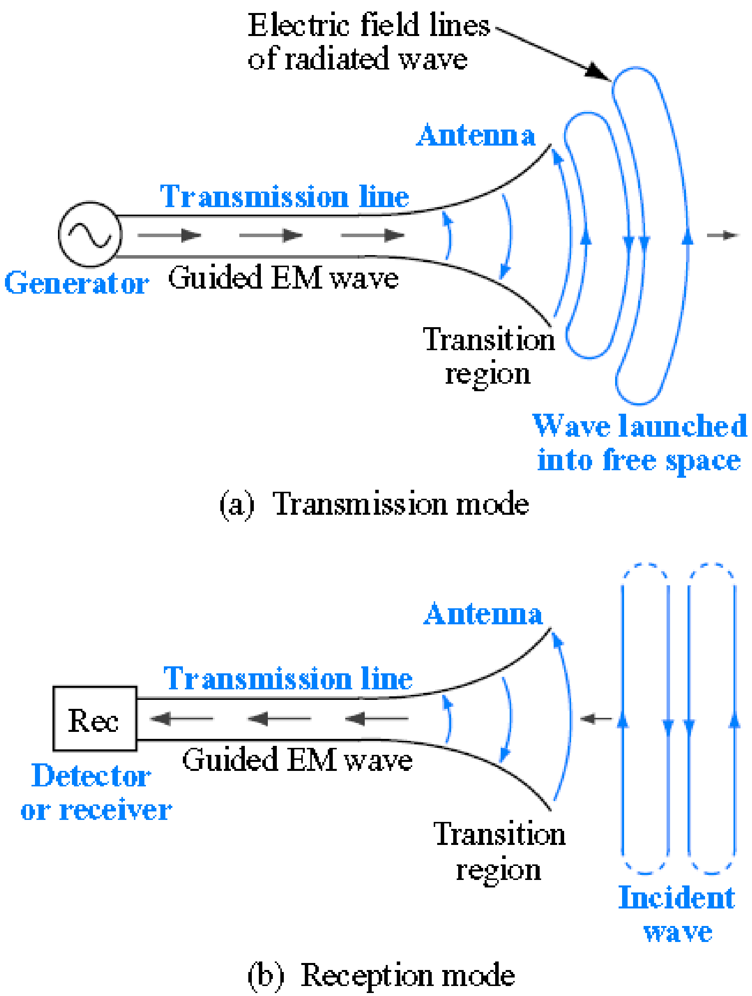
(b) Plane and Cylindrical Waves / Ebene und zylindrische Wellen

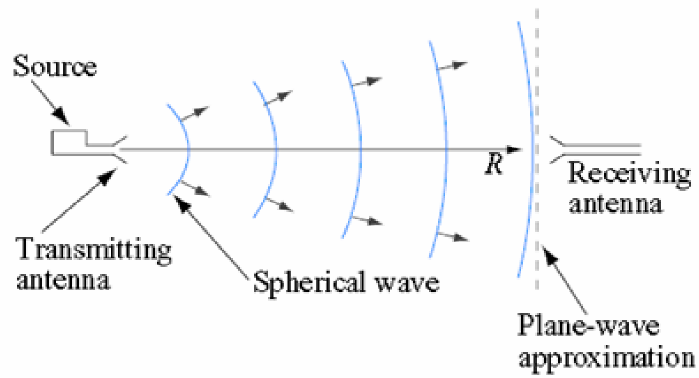
(c) Spherical Wave / Kugelwelle

$$\overbrace{\underline{\mathbf{E}}(\underline{\mathbf{R}},\omega) = \underline{\mathbf{E}}_0(\underline{\mathbf{k}},\omega)\,\mathrm{e}^{\mathrm{j}\underline{\mathbf{k}}\bullet\underline{\mathbf{R}}}} \\ = \underline{\mathbf{E}}_0(\hat{\underline{\mathbf{k}}},\omega)\,\mathrm{e}^{\mathrm{j}k\hat{\underline{\mathbf{k}}}\bullet\underline{\mathbf{R}}}$$

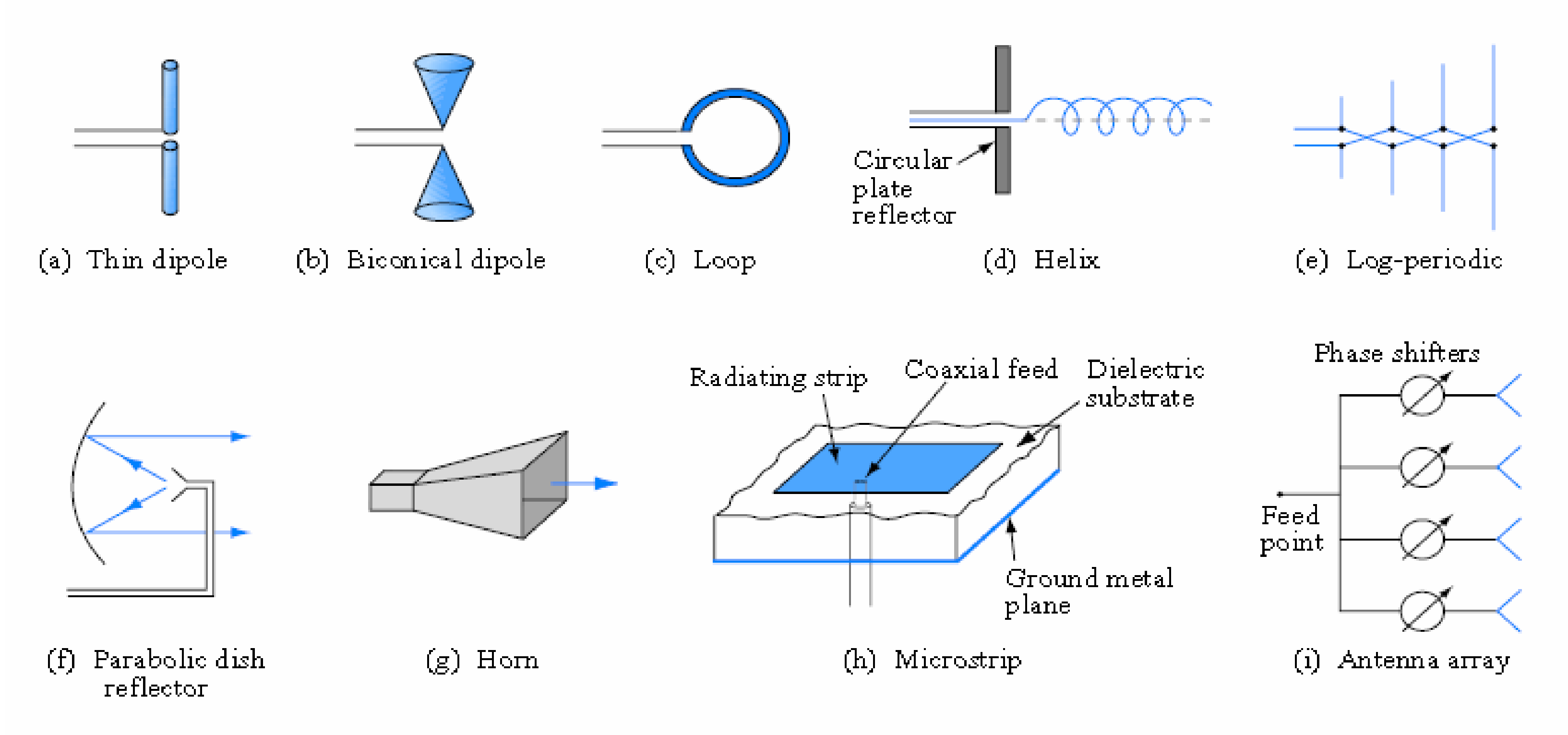
Plane Wave in the Frequency Domain Propagating in **k** Direction / Ebene Welle im Frequenzbereich, die sich in **k** Richtung ausbreitet

EM Fields – EM Waves – Antenna Systems / EM Felder – EM-Wellen - Antennensysteme

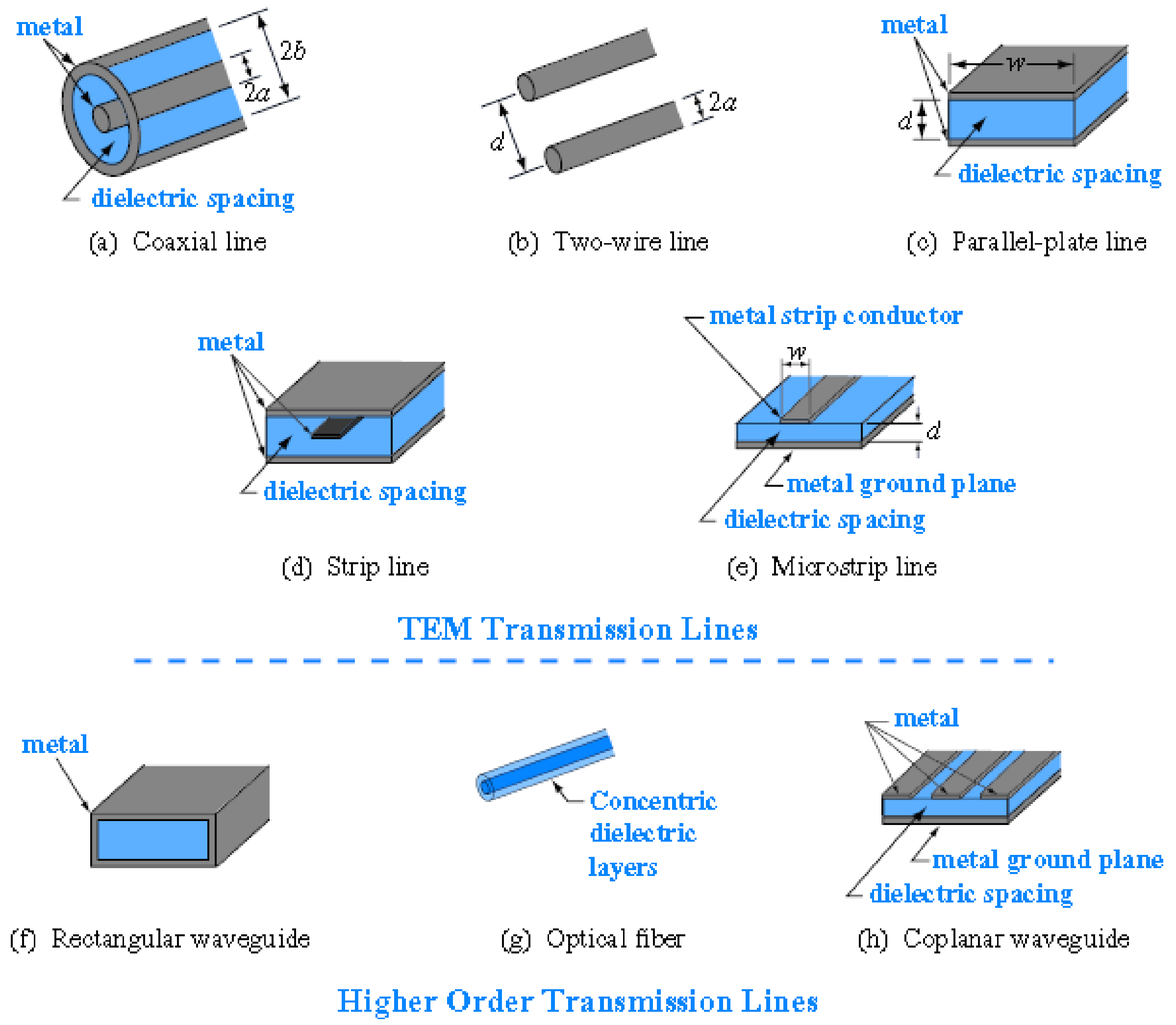




EM Fields – EM Waves – Antennas / EM Felder – EM Wellen - Antennen



EM Fields – EM Waves – Guided EM Waves / EM Felder – EM-Wellen – Geführte EM-Wellen



EM Fields – Maxwell's Equations – Vector Wave Equation / EM Felder – Maxwellsche Gleichungen – Vektorielle Wellengleichung

Maxwell's equations / Maxwellsche Gleichungen

$$\left\{ \begin{aligned} \frac{\partial}{\partial t}\underline{\mathbf{B}}(\underline{\mathbf{R}},t) &= -\nabla\times\underline{\mathbf{E}}(\underline{\mathbf{R}},t) - \underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},t) \\ \frac{\partial}{\partial t}\underline{\mathbf{D}}(\underline{\mathbf{R}},t) &= \nabla\times\underline{\mathbf{H}}(\underline{\mathbf{R}},t) - \underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}},t) \end{aligned} \right.$$

$$\nabla\bullet\underline{\mathbf{B}}(\underline{\mathbf{R}},t) = \rho_{\mathrm{m}}(\underline{\mathbf{R}},t)$$

$$\nabla\bullet\underline{\mathbf{D}}(\underline{\mathbf{R}},t) = \rho_{\mathrm{e}}(\underline{\mathbf{R}},t)$$

Constitutive Equations for Vacuum / Konstituierende Gleichungen (Materialgleichungen) für Vakuum

$$\underline{\mathbf{B}}(\underline{\mathbf{R}},t) = \mu_0\,\underline{\mathbf{H}}(\underline{\mathbf{R}},t)$$

$$\underline{\mathbf{D}}(\underline{\mathbf{R}},t) = \varepsilon_0\,\underline{\mathbf{E}}(\underline{\mathbf{R}},t)$$

Continuity equations / Kontinuitätsgleichungen

$$\nabla\bullet\underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},t) = -\frac{\partial}{\partial t}\rho_{\mathrm{m}}(\underline{\mathbf{R}},t)$$

$$\nabla\bullet\underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}},t) = -\frac{\partial}{\partial t}\rho_{\mathrm{e}}(\underline{\mathbf{R}},t)$$

$$\frac{\partial}{\partial t}\underline{\mathbf{H}}(\underline{\mathbf{R}},t) = -\frac{1}{\mu_0}\nabla\times\underline{\mathbf{E}}(\underline{\mathbf{R}},t) - \frac{1}{\mu_0}\underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},t)$$

$$\frac{\partial}{\partial t}\underline{\mathbf{E}}(\underline{\mathbf{R}},t) = \frac{1}{\varepsilon_0}\nabla\times\underline{\mathbf{H}}(\underline{\mathbf{R}},t) - \frac{1}{\varepsilon_0}\underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}},t)$$

EM Fields – EM Waves – Vector Wave Equation / EM Felder – EM-Wellen – Vektorielle Wellengleichung

$$\frac{\partial}{\partial t}\underline{\mathbf{H}}(\underline{\mathbf{R}},t) = -\frac{1}{\mu_0}\nabla\times\underline{\mathbf{E}}(\underline{\mathbf{R}},t) - \frac{1}{\mu_0}\underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},t) \qquad (1)$$

$$\frac{\partial}{\partial t}\underline{\mathbf{E}}(\underline{\mathbf{R}},t) = \frac{1}{\varepsilon_0}\nabla\times\underline{\mathbf{H}}(\underline{\mathbf{R}},t) - \frac{1}{\varepsilon_0}\underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}},t) \qquad (2)$$

$$\frac{\partial^2}{\partial t^2}\underline{\mathbf{H}}(\underline{\mathbf{R}},t) = -\frac{1}{\mu_0}\nabla\times\frac{\partial}{\partial t}\underline{\mathbf{E}}(\underline{\mathbf{R}},t) - \frac{1}{\mu_0}\frac{\partial}{\partial t}\underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},t) \qquad (3)$$

$$\frac{\partial^2}{\partial t^2}\underline{\mathbf{E}}(\underline{\mathbf{R}},t) = \frac{1}{\varepsilon_0}\nabla\times\frac{\partial}{\partial t}\underline{\mathbf{H}}(\underline{\mathbf{R}},t) - \frac{1}{\varepsilon_0}\frac{\partial}{\partial t}\underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}},t) \qquad (4)$$

$$\frac{\partial^2}{\partial t^2}\underline{\mathbf{H}}(\underline{\mathbf{R}},t) = -\frac{1}{\mu_0}\nabla\times\left[\frac{1}{\varepsilon_0}\nabla\times\underline{\mathbf{H}}(\underline{\mathbf{R}},t) - \frac{1}{\varepsilon_0}\underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}},t)\right] - \frac{1}{\mu_0}\frac{\partial}{\partial t}\underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},t) \qquad (5)$$

$$\frac{\partial^2}{\partial t^2}\underline{\mathbf{E}}(\underline{\mathbf{R}},t) = \frac{1}{\varepsilon_0}\nabla\times\left[-\frac{1}{\mu_0}\nabla\times\underline{\mathbf{E}}(\underline{\mathbf{R}},t) - \frac{1}{\mu_0}\underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},t)\right] - \frac{1}{\varepsilon_0}\frac{\partial}{\partial t}\underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}},t) \qquad (6)$$

$$\frac{\partial^2}{\partial t^2}\underline{\mathbf{H}}(\underline{\mathbf{R}},t) = -\frac{1}{\varepsilon_0\mu_0}\nabla\times\nabla\times\underline{\mathbf{H}}(\underline{\mathbf{R}},t) + \frac{1}{\varepsilon_0\mu_0}\nabla\times\underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}},t) - \frac{1}{\mu_0}\frac{\partial}{\partial t}\underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},t) \qquad (7)$$

$$\frac{\partial^2}{\partial t^2}\underline{\mathbf{E}}(\underline{\mathbf{R}},t) = -\frac{1}{\varepsilon_0\mu_0}\nabla\times\nabla\times\underline{\mathbf{E}}(\underline{\mathbf{R}},t) - \frac{1}{\varepsilon_0\mu_0}\nabla\times\underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},t) - \frac{1}{\varepsilon_0}\frac{\partial}{\partial t}\underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}},t) \qquad (8)$$

EM Fields – EM Waves – Vector Wave Equation / EM Felder – EM-Wellen – Vektorielle Wellengleichung

$$\frac{1}{\varepsilon_0\mu_0}\nabla\times\nabla\times\underline{\mathbf{H}}(\underline{\mathbf{R}},t)+\frac{\partial^2}{\partial t^2}\underline{\mathbf{H}}(\underline{\mathbf{R}},t)=+\frac{1}{\varepsilon_0\mu_0}\nabla\times\underline{\mathbf{J}}_\mathrm{e}(\underline{\mathbf{R}},t)-\frac{1}{\mu_0}\frac{\partial}{\partial t}\underline{\mathbf{J}}_\mathrm{m}(\underline{\mathbf{R}},t) \quad (1)$$

$$\frac{1}{\varepsilon_0\mu_0}\nabla\times\nabla\times\underline{\mathbf{E}}(\underline{\mathbf{R}},t)+\frac{\partial^2}{\partial t^2}\underline{\mathbf{E}}(\underline{\mathbf{R}},t)=-\frac{1}{\varepsilon_0\mu_0}\nabla\times\underline{\mathbf{J}}_\mathrm{m}(\underline{\mathbf{R}},t)-\frac{1}{\varepsilon_0}\frac{\partial}{\partial t}\underline{\mathbf{J}}_\mathrm{e}(\underline{\mathbf{R}},t) \quad (2)$$

Velocity of Light in Vacuum / Lichtgeschwindigkeit in Vakuum $\quad c_0=\frac{1}{\sqrt{\varepsilon_0\mu_0}}$

$$-\nabla\times\nabla\times\underline{\mathbf{H}}(\underline{\mathbf{R}},t)-\frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}\underline{\mathbf{H}}(\underline{\mathbf{R}},t)=-\nabla\times\underline{\mathbf{J}}_\mathrm{e}(\underline{\mathbf{R}},t)+\varepsilon_0\frac{\partial}{\partial t}\underline{\mathbf{J}}_\mathrm{m}(\underline{\mathbf{R}},t) \quad (3)$$

$$-\nabla\times\nabla\times\underline{\mathbf{E}}(\underline{\mathbf{R}},t)-\frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}\underline{\mathbf{E}}(\underline{\mathbf{R}},t)=\nabla\times\underline{\mathbf{J}}_\mathrm{m}(\underline{\mathbf{R}},t)+\mu_0\frac{\partial}{\partial t}\underline{\mathbf{J}}_\mathrm{e}(\underline{\mathbf{R}},t) \quad (4)$$

Vector identity / Vektoridentität $\quad \nabla\times\nabla\times=\nabla\nabla\bullet-\underbrace{\nabla\bullet\nabla}_{=\nabla^2=\Delta}=\nabla\nabla\bullet-\Delta$

Short-hand notation / Abkürzende Schreibweise $\quad \nabla\bullet\nabla=\nabla^2=\Delta$

$$-\left[\nabla\nabla\bullet-\Delta\right]\underline{\mathbf{H}}(\underline{\mathbf{R}},t)-\frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}\underline{\mathbf{H}}(\underline{\mathbf{R}},t)=-\nabla\times\underline{\mathbf{J}}_\mathrm{e}(\underline{\mathbf{R}},t)+\varepsilon_0\frac{\partial}{\partial t}\underline{\mathbf{J}}_\mathrm{m}(\underline{\mathbf{R}},t) \quad (5)$$

$$-\left[\nabla\nabla\bullet-\Delta\right]\underline{\mathbf{E}}(\underline{\mathbf{R}},t)-\frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}\underline{\mathbf{E}}(\underline{\mathbf{R}},t)=\nabla\times\underline{\mathbf{J}}_\mathrm{m}(\underline{\mathbf{R}},t)+\mu_0\frac{\partial}{\partial t}\underline{\mathbf{J}}_\mathrm{e}(\underline{\mathbf{R}},t) \quad (6)$$

EM Fields – EM Waves – Vector Wave Equation / EM Felder – EM-Wellen – Vektorielle Wellengleichung

$$-\left[\nabla\nabla\bullet-\Delta\right]\underline{\mathbf{H}}(\underline{\mathbf{R}},t)-\frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}\underline{\mathbf{H}}(\underline{\mathbf{R}},t)=-\nabla\times\underline{\mathbf{J}}_\mathrm{e}(\underline{\mathbf{R}},t)+\varepsilon_0\frac{\partial}{\partial t}\underline{\mathbf{J}}_\mathrm{m}(\underline{\mathbf{R}},t)$$

$$-\left[\nabla\nabla\bullet-\Delta\right]\underline{\mathbf{E}}(\underline{\mathbf{R}},t)-\frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}\underline{\mathbf{E}}(\underline{\mathbf{R}},t)=\nabla\times\underline{\mathbf{J}}_\mathrm{m}(\underline{\mathbf{R}},t)+\mu_0\frac{\partial}{\partial t}\underline{\mathbf{J}}_\mathrm{e}(\underline{\mathbf{R}},t)$$

3rd and 4th Maxwell's equations /
3. und 4. Maxwellsche Gleichung

$$\nabla\bullet\underline{\mathbf{D}}(\underline{\mathbf{R}},t)=\rho_\mathrm{e}(\underline{\mathbf{R}},t)$$
$$\nabla\bullet\underline{\mathbf{B}}(\underline{\mathbf{R}},t)=\rho_\mathrm{m}(\underline{\mathbf{R}},t)$$

$+$

Constitutive equations /
Materialgleichungen

$$\underline{\mathbf{B}}(\underline{\mathbf{R}},t)=\mu_0\underline{\mathbf{H}}(\underline{\mathbf{R}},t)$$
$$\underline{\mathbf{D}}(\underline{\mathbf{R}},t)=\varepsilon_0\underline{\mathbf{E}}(\underline{\mathbf{R}},t)$$

$\Rightarrow$

$$\nabla\bullet\underline{\mathbf{H}}(\underline{\mathbf{R}},t)=\frac{1}{\mu_0}\rho_\mathrm{m}(\underline{\mathbf{R}},t)$$
$$\nabla\bullet\underline{\mathbf{E}}(\underline{\mathbf{R}},t)=\frac{1}{\varepsilon_0}\rho_\mathrm{e}(\underline{\mathbf{R}},t)$$

$$\Delta\underline{\mathbf{H}}(\underline{\mathbf{R}},t)-\nabla\underbrace{\nabla\bullet\underline{\mathbf{H}}(\underline{\mathbf{R}},t)}_{=\frac{1}{\mu_0}\rho_\mathrm{m}(\underline{\mathbf{R}},t)}-\frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}\underline{\mathbf{H}}(\underline{\mathbf{R}},t)=-\nabla\times\underline{\mathbf{J}}_\mathrm{e}(\underline{\mathbf{R}},t)+\varepsilon_0\frac{\partial}{\partial t}\underline{\mathbf{J}}_\mathrm{m}(\underline{\mathbf{R}},t)$$

$$\Delta\underline{\mathbf{E}}(\underline{\mathbf{R}},t)-\nabla\underbrace{\nabla\bullet\underline{\mathbf{E}}(\underline{\mathbf{R}},t)}_{=\frac{1}{\varepsilon_0}\rho_\mathrm{e}(\underline{\mathbf{R}},t)}-\frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}\underline{\mathbf{E}}(\underline{\mathbf{R}},t)=\nabla\times\underline{\mathbf{J}}_\mathrm{m}(\underline{\mathbf{R}},t)+\mu_0\frac{\partial}{\partial t}\underline{\mathbf{J}}_\mathrm{e}(\underline{\mathbf{R}},t)$$

EM Fields – EM Waves – Vector Wave Equation / EM Felder – EM-Wellen – Vektorielle Wellengleichung

$$\Delta\underline{\mathbf{H}}(\underline{\mathbf{R}},t)-\nabla\left[\frac{1}{\mu_0}\rho_\mathrm{m}(\underline{\mathbf{R}},t)\right]-\frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}\underline{\mathbf{H}}(\underline{\mathbf{R}},t)=-\nabla\times\underline{\mathbf{J}}_\mathrm{e}(\underline{\mathbf{R}},t)+\varepsilon_0\frac{\partial}{\partial t}\underline{\mathbf{J}}_\mathrm{m}(\underline{\mathbf{R}},t)$$

$$\Delta\underline{\mathbf{E}}(\underline{\mathbf{R}},t)-\nabla\left[\frac{1}{\varepsilon_0}\rho_\mathrm{e}(\underline{\mathbf{R}},t)\right]-\frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}\underline{\mathbf{E}}(\underline{\mathbf{R}},t)=\nabla\times\underline{\mathbf{J}}_\mathrm{m}(\underline{\mathbf{R}},t)+\mu_0\frac{\partial}{\partial t}\underline{\mathbf{J}}_\mathrm{e}(\underline{\mathbf{R}},t)$$

$$\Delta\underline{\mathbf{H}}(\underline{\mathbf{R}},t)-\frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}\underline{\mathbf{H}}(\underline{\mathbf{R}},t)=-\nabla\times\underline{\mathbf{J}}_\mathrm{e}(\underline{\mathbf{R}},t)+\varepsilon_0\frac{\partial}{\partial t}\underline{\mathbf{J}}_\mathrm{m}(\underline{\mathbf{R}},t)+\frac{1}{\mu_0}\nabla\rho_\mathrm{m}(\underline{\mathbf{R}},t)$$

$$\Delta\underline{\mathbf{E}}(\underline{\mathbf{R}},t)-\frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}\underline{\mathbf{E}}(\underline{\mathbf{R}},t)=\nabla\times\underline{\mathbf{J}}_\mathrm{m}(\underline{\mathbf{R}},t)+\mu_0\frac{\partial}{\partial t}\underline{\mathbf{J}}_\mathrm{e}(\underline{\mathbf{R}},t)+\frac{1}{\varepsilon_0}\nabla\rho_\mathrm{e}(\underline{\mathbf{R}},t)$$

Laplace operator in Cartesian coordinates /
Laplace-Operator in Kartesischen Koordinaten

$$\begin{aligned}\Delta&=\nabla\bullet\nabla\\&=\left(\underline{\mathrm{e}}_x\frac{\partial}{\partial x}+\underline{\mathrm{e}}_y\frac{\partial}{\partial y}+\underline{\mathrm{e}}_z\frac{\partial}{\partial z}\right)\bullet\left(\underline{\mathrm{e}}_x\frac{\partial}{\partial x}+\underline{\mathrm{e}}_y\frac{\partial}{\partial y}+\underline{\mathrm{e}}_z\frac{\partial}{\partial z}\right)\\&=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\end{aligned}$$

EM Fields – EM Waves – Vector Wave Equation / EM Felder – EM-Wellen – Vektorielle Wellengleichung

$$\Delta\underline{\mathbf{E}}(\underline{\mathbf{R}},t) = \nabla \bullet \left[\nabla\underline{\mathbf{E}}(\underline{\mathbf{R}},t)\right]$$

$$= \left(\underline{e}_x \frac{\partial}{\partial x} + \underline{e}_y \frac{\partial}{\partial y} + \underline{e}_z \frac{\partial}{\partial z}\right) \bullet \left[\left(\underline{e}_x \frac{\partial}{\partial x} + \underline{e}_y \frac{\partial}{\partial y} + \underline{e}_z \frac{\partial}{\partial z}\right)\underline{\mathbf{E}}(\underline{\mathbf{R}},t)\right]$$

Short-hand notation / Abkürzende Schreibweise $\quad \frac{\partial}{\partial x} = \partial_x \quad \frac{\partial}{\partial y} = \partial_y \quad \frac{\partial}{\partial z} = \partial_z \quad \frac{\partial}{\partial t} = \partial_t$

$$\Delta\underline{\mathbf{E}}(\underline{\mathbf{R}},t) = \nabla \bullet \left[\nabla\underline{\mathbf{E}}(\underline{\mathbf{R}},t)\right]$$

$$= \left(\underline{e}_x\partial_x + \underline{e}_y\partial_y + \underline{e}_z\partial_z\right) \bullet \left[\left(\underline{e}_x\partial_x + \underline{e}_y\partial_y + \underline{e}_z\partial_z\right)\underline{\mathbf{E}}(\underline{\mathbf{R}},t)\right]$$

$$\left(\underline{e}_x\partial_x + \underline{e}_y\partial_y + \underline{e}_z\partial_z\right)\left[E_x(\underline{\mathbf{R}},t)\underline{e}_x + E_y(\underline{\mathbf{R}},t)\underline{e}_y + E_z(\underline{\mathbf{R}},t)\underline{e}_z\right]$$

$$= \underline{e}_x\underline{e}_x\partial_x E_x(\underline{\mathbf{R}},t) + \underline{e}_x\underline{e}_y\partial_x E_y(\underline{\mathbf{R}},t) + \underline{e}_x\underline{e}_z\partial_x E_z(\underline{\mathbf{R}},t)$$

$$+ \underline{e}_y\underline{e}_x\partial_y E_y(\underline{\mathbf{R}},t) + \underline{e}_y\underline{e}_y\partial_y E_y(\underline{\mathbf{R}},t) + \underline{e}_y\underline{e}_z\partial_y E_z(\underline{\mathbf{R}},t)$$

$$+ \underline{e}_z\underline{e}_x\partial_x E_z(\underline{\mathbf{R}},t) + \underline{e}_z\underline{e}_y\partial_z E_y(\underline{\mathbf{R}},t) + \underline{e}_z\underline{e}_z\partial_z E_z(\underline{\mathbf{R}},t)$$

EM Fields – EM Waves – Vector Wave Equation / EM Felder – EM-Wellen – Vektorielle Wellengleichung

$$
\begin{aligned}
\Delta\underline{\mathbf{E}}(\underline{\mathbf{R}},t) &= \nabla\bullet\left[\nabla\underline{\mathbf{E}}(\underline{\mathbf{R}},t)\right]\\
&= \left(\underline{\mathrm{e}}_x\partial_x+\underline{\mathrm{e}}_y\partial_y+\underline{\mathrm{e}}_z\partial_z\right)\bullet\left[\left(\underline{\mathrm{e}}_x\partial_x+\underline{\mathrm{e}}_y\partial_y+\underline{\mathrm{e}}_z\partial_z\right)\underline{\mathbf{E}}(\underline{\mathbf{R}},t)\right]\\
&= \left(\underline{\mathrm{e}}_x\partial_x+\underline{\mathrm{e}}_y\partial_y+\underline{\mathrm{e}}_z\partial_z\right)\bullet\Big[\ \underline{\mathrm{e}}_x\underline{\mathrm{e}}_x\partial_x E_x(\underline{\mathbf{R}},t)+\underline{\mathrm{e}}_x\underline{\mathrm{e}}_y\partial_x E_y(\underline{\mathbf{R}},t)+\underline{\mathrm{e}}_x\underline{\mathrm{e}}_z\partial_x E_z(\underline{\mathbf{R}},t)\\
&\qquad +\underline{\mathrm{e}}_y\underline{\mathrm{e}}_x\partial_x E_y(\underline{\mathbf{R}},t)+\underline{\mathrm{e}}_y\underline{\mathrm{e}}_y\partial_y E_y(\underline{\mathbf{R}},t)+\underline{\mathrm{e}}_y\underline{\mathrm{e}}_z\partial_y E_z(\underline{\mathbf{R}},t)\\
&\qquad +\underline{\mathrm{e}}_z\underline{\mathrm{e}}_x\partial_x E_z(\underline{\mathbf{R}},t)+\underline{\mathrm{e}}_z\underline{\mathrm{e}}_y\partial_y E_z(\underline{\mathbf{R}},t)+\underline{\mathrm{e}}_z\underline{\mathrm{e}}_z\partial_z E_z(\underline{\mathbf{R}},t)\Big]
\end{aligned}
$$

$$
\begin{aligned}
\Delta\underline{\mathbf{E}}(\underline{\mathbf{R}},t) &= \nabla\bullet\left[\nabla\underline{\mathbf{E}}(\underline{\mathbf{R}},t)\right]\\
&= \underline{\mathrm{e}}_x\partial_x\bullet\Big[\ \underline{\mathrm{e}}_x\underline{\mathrm{e}}_x\partial_x E_x(\underline{\mathbf{R}},t)+\underline{\mathrm{e}}_x\underline{\mathrm{e}}_y\partial_x E_y(\underline{\mathbf{R}},t)+\underline{\mathrm{e}}_x\underline{\mathrm{e}}_z\partial_x E_z(\underline{\mathbf{R}},t)\\
&\qquad +\underline{\mathrm{e}}_y\underline{\mathrm{e}}_x\partial_y E_x(\underline{\mathbf{R}},t)+\underline{\mathrm{e}}_y\underline{\mathrm{e}}_y\partial_y E_y(\underline{\mathbf{R}},t)+\underline{\mathrm{e}}_y\underline{\mathrm{e}}_z\partial_y E_z(\underline{\mathbf{R}},t)\\
&\qquad +\underline{\mathrm{e}}_z\underline{\mathrm{e}}_x\partial_z E_x(\underline{\mathbf{R}},t)+\underline{\mathrm{e}}_z\underline{\mathrm{e}}_y\partial_z E_y(\underline{\mathbf{R}},t)+\underline{\mathrm{e}}_z\underline{\mathrm{e}}_z\partial_z E_z(\underline{\mathbf{R}},t)\Big]\\
&+\underline{\mathrm{e}}_y\partial_y\bullet\Big[\ \underline{\mathrm{e}}_x\underline{\mathrm{e}}_x\partial_x E_x(\underline{\mathbf{R}},t)+\underline{\mathrm{e}}_x\underline{\mathrm{e}}_y\partial_x E_y(\underline{\mathbf{R}},t)+\underline{\mathrm{e}}_x\underline{\mathrm{e}}_z\partial_z E_z(\underline{\mathbf{R}},t)\\
&\qquad +\underline{\mathrm{e}}_y\underline{\mathrm{e}}_x\partial_y E_x(\underline{\mathbf{R}},t)+\underline{\mathrm{e}}_y\underline{\mathrm{e}}_y\partial_y E_y(\underline{\mathbf{R}},t)+\underline{\mathrm{e}}_y\underline{\mathrm{e}}_z\partial_y E_z(\underline{\mathbf{R}},t)\\
&\qquad +\underline{\mathrm{e}}_z\underline{\mathrm{e}}_x\partial_z E_x(\underline{\mathbf{R}},t)+\underline{\mathrm{e}}_z\underline{\mathrm{e}}_y\partial_z E_y(\underline{\mathbf{R}},t)+\underline{\mathrm{e}}_z\underline{\mathrm{e}}_z\partial_z E_z(\underline{\mathbf{R}},t)\Big]\\
&+\underline{\mathrm{e}}_z\partial_z\bullet\Big[\ \underline{\mathrm{e}}_x\underline{\mathrm{e}}_x\partial_x E_x(\underline{\mathbf{R}},t)+\underline{\mathrm{e}}_x\underline{\mathrm{e}}_y\partial_x E_y(\underline{\mathbf{R}},t)+\underline{\mathrm{e}}_x\underline{\mathrm{e}}_z\partial_x E_z(\underline{\mathbf{R}},t)\\
&\qquad +\underline{\mathrm{e}}_y\underline{\mathrm{e}}_x\partial_y E_x(\underline{\mathbf{R}},t)+\underline{\mathrm{e}}_y\underline{\mathrm{e}}_y\partial_y E_y(\underline{\mathbf{R}},t)+\underline{\mathrm{e}}_y\underline{\mathrm{e}}_z\partial_y E_z(\underline{\mathbf{R}},t)\\
&\qquad +\underline{\mathrm{e}}_z\underline{\mathrm{e}}_x\partial_z E_x(\underline{\mathbf{R}},t)+\underline{\mathrm{e}}_z\underline{\mathrm{e}}_y\partial_z E_y(\underline{\mathbf{R}},t)+\underline{\mathrm{e}}_z\underline{\mathrm{e}}_z\partial_z E_z(\underline{\mathbf{R}},t)\Big]
\end{aligned}
$$

EM Fields – EM Waves – Vector Wave Equation / EM Felder – EM-Wellen – Vektorielle Wellengleichung

$$\Delta\underline{\mathbf{E}}(\underline{\mathbf{R}},t) = \nabla \bullet \left[\nabla\underline{\mathbf{E}}(\underline{\mathbf{R}},t)\right]$$

$$= \left[\ \underline{\mathbf{e}}_x\partial_x^2 E_x(\underline{\mathbf{R}},t) + \underline{\mathbf{e}}_y\partial_x^2 E_y(\underline{\mathbf{R}},t) + \underline{\mathbf{e}}_z\partial_x^2 E_z(\underline{\mathbf{R}},t)\right.$$

$$+\left[\ \underline{\mathbf{e}}_x\partial_y^2 E_x(\underline{\mathbf{R}},t) + \underline{\mathbf{e}}_y\partial_y^2 E_y(\underline{\mathbf{R}},t) + \underline{\mathbf{e}}_z\partial_y^2 E_z(\underline{\mathbf{R}},t)\right.$$

$$+\left[\ \underline{\mathbf{e}}_x\partial_z^2 E_x(\underline{\mathbf{R}},t) + \underline{\mathbf{e}}_y\partial_z^2 E_y(\underline{\mathbf{R}},t) + \underline{\mathbf{e}}_z\partial_z^2 E_z(\underline{\mathbf{R}},t)\right.$$

$$= \left(\partial_x^2 + \partial_y^2 + \partial_z^2\right)\underbrace{\left[E_x(\underline{\mathbf{R}},t)\underline{\mathbf{e}}_x + E_y(\underline{\mathbf{R}},t)\underline{\mathbf{e}}_y + E_z(\underline{\mathbf{R}},t)\underline{\mathbf{e}}_z\right]}_{=\underline{\mathbf{E}}(\underline{\mathbf{R}},t)}$$

$$= \left(\partial_x^2 + \partial_y^2 + \partial_z^2\right)\underline{\mathbf{E}}(\underline{\mathbf{R}},t)$$

$$\Delta\underline{\mathbf{E}}(\underline{\mathbf{R}},t) = \left(\partial_x^2 + \partial_y^2 + \partial_z^2\right)\underline{\mathbf{E}}(\underline{\mathbf{R}},t)$$

$$= \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)\underline{\mathbf{E}}(\underline{\mathbf{R}},t)$$

EM Fields – EM Waves – Vector Wave Equation / EM Felder – EM-Wellen – Vektorielle Wellengleichung

$$\Delta \underline{\mathbf{H}}(\underline{\mathbf{R}},t) - \frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}\underline{\mathbf{H}}(\underline{\mathbf{R}},t) = -\nabla \times \underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}},t) + \varepsilon_0 \frac{\partial}{\partial t}\underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},t) + \frac{1}{\mu_0}\nabla \rho_{\mathrm{m}}(\underline{\mathbf{R}},t)$$

$$\Delta \underline{\mathbf{E}}(\underline{\mathbf{R}},t) - \frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}\underline{\mathbf{E}}(\underline{\mathbf{R}},t) = \nabla \times \underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},t) + \mu_0 \frac{\partial}{\partial t}\underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}},t) + \frac{1}{\varepsilon_0}\nabla \rho_{\mathrm{e}}(\underline{\mathbf{R}},t)$$

$$\Delta \underline{\mathbf{E}}(\underline{\mathbf{R}},t) = \left(\partial_x^2 + \partial_y^2 + \partial_z^2\right)\underline{\mathbf{E}}(\underline{\mathbf{R}},t)$$

$$\Delta \underline{\mathbf{H}}(\underline{\mathbf{R}},t) = \left(\partial_x^2 + \partial_y^2 + \partial_z^2\right)\underline{\mathbf{H}}(\underline{\mathbf{R}},t)$$

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)\underline{\mathbf{H}}(\underline{\mathbf{R}},t) - \frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}\underline{\mathbf{H}}(\underline{\mathbf{R}},t) = -\nabla \times \underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}},t) + \varepsilon_0 \frac{\partial}{\partial t}\underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},t) + \frac{1}{\mu_0}\nabla \rho_{\mathrm{m}}(\underline{\mathbf{R}},t)$$

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)\underline{\mathbf{E}}(\underline{\mathbf{R}},t) - \frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}\underline{\mathbf{E}}(\underline{\mathbf{R}},t) = \nabla \times \underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},t) + \mu_0 \frac{\partial}{\partial t}\underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}},t) + \frac{1}{\varepsilon_0}\nabla \rho_{\mathrm{e}}(\underline{\mathbf{R}},t)$$

$$\nabla \bullet \underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},t) = -\frac{\partial}{\partial t}\rho_{\mathrm{m}}(\underline{\mathbf{R}},t)$$

$$\nabla \bullet \underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}},t) = -\frac{\partial}{\partial t}\rho_{\mathrm{e}}(\underline{\mathbf{R}},t)$$

EM Fields – EM Waves – Homogeneous Vector Wave Equation / EM Felder – EM-Wellen – Homogene vektorielle Wellengleichung

Source-free Case / Quellenfreier Fall

$$\underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}},t) = \underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},t) = \underline{\mathbf{0}}$$

$$\rho_{\mathrm{e}}(\underline{\mathbf{R}},t) = \rho_{\mathrm{m}}(\underline{\mathbf{R}},t) = 0$$

$$\Delta\underline{\mathbf{H}}(\underline{\mathbf{R}},t) - \frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}\underline{\mathbf{H}}(\underline{\mathbf{R}},t) = \underbrace{-\nabla\times\underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}},t) + \varepsilon_0\frac{\partial}{\partial t}\underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},t) + \frac{1}{\mu_0}\nabla\rho_{\mathrm{m}}(\underline{\mathbf{R}},t)}_{=\underline{\mathbf{0}}}$$

$$\Delta\underline{\mathbf{E}}(\underline{\mathbf{R}},t) - \frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}\underline{\mathbf{E}}(\underline{\mathbf{R}},t) = \underbrace{\nabla\times\underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},t) + \mu_0\frac{\partial}{\partial t}\underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}},t) + \frac{1}{\varepsilon_0}\nabla\rho_{\mathrm{e}}(\underline{\mathbf{R}},t)}_{=\underline{\mathbf{0}}}$$

Homogeneous Vector Wave Equations / Homogene vektorielle Wellengleichungen

$$\Delta\underline{\mathbf{H}}(\underline{\mathbf{R}},t) - \frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}\underline{\mathbf{H}}(\underline{\mathbf{R}},t) = \underline{\mathbf{0}}$$

$$\Delta\underline{\mathbf{E}}(\underline{\mathbf{R}},t) - \frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}\underline{\mathbf{E}}(\underline{\mathbf{R}},t) = \underline{\mathbf{0}}$$

D'Alembert Operator / D'Alembert-Operator

$$\Box = \Delta - \frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}$$

$$\Box\,\underline{\mathbf{H}}(\underline{\mathbf{R}},t) = \underline{\mathbf{0}}$$

$$\Box\,\underline{\mathbf{E}}(\underline{\mathbf{R}},t) = \underline{\mathbf{0}}$$

EM Fields – Homogeneous Vector Wave Equation – Fourier Transform / EM-Felder – Homogene vektorielle Wellengleichung – Fourier-Transformation

$$\left(\Delta - \frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}\right)\underline{\mathbf{H}}(\underline{\mathbf{R}},t) = \underline{\mathbf{0}}$$

$$\left(\Delta - \frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}\right)\underline{\mathbf{E}}(\underline{\mathbf{R}},t) = \underline{\mathbf{0}}$$

Fourier Transform with Regard to Time / Fourier-Transformation bzgl. der Zeit

$$\underline{\mathbf{E}}(\underline{\mathbf{R}},\omega) = \int_{t=-\infty}^{\infty} \underline{\mathbf{E}}(\underline{\mathbf{R}},t)\, \mathrm{e}^{+\mathrm{j}\omega t}\, \mathrm{d}t = \mathcal{FT}_{t\to\omega}\left\{\underline{\mathbf{E}}(\underline{\mathbf{R}},t)\right\} \qquad \underline{\mathbf{E}}(\underline{\mathbf{R}},\omega) \overset{\omega\leftarrow t}{\bullet\!\!-\!\!\circ} \underline{\mathbf{E}}(\underline{\mathbf{R}},t)$$

$$\underline{\mathbf{E}}(\underline{\mathbf{R}},t) = \frac{1}{2\pi}\int_{\omega=-\infty}^{\infty} \underline{\mathbf{E}}(\underline{\mathbf{R}},\omega)\, \mathrm{e}^{-\mathrm{j}\omega t}\, \mathrm{d}\omega = \mathcal{FT}_{\omega\to t}^{-1}\left\{\underline{\mathbf{E}}(\underline{\mathbf{R}},\omega)\right\} \qquad \underline{\mathbf{E}}(\underline{\mathbf{R}},t) \overset{t\leftarrow \omega}{\circ\!\!-\!\!\bullet} \underline{\mathbf{E}}(\underline{\mathbf{R}},\omega)$$

Inverse Fourier Transform with Regard to Circular Frequency /
Inverse Fourier-Transformation bzgl. der Kreisfrequenz

$$\frac{\partial}{\partial t} \overset{t\leftrightarrow\omega}{\circ\!\!-\!\!\bullet} -\mathrm{j}\omega$$

$$\frac{\partial^2}{\partial t^2} \overset{t\leftrightarrow\omega}{\circ\!\!-\!\!\bullet} -\omega^2$$

$$\left(\frac{\partial}{\partial t}\right)^n \overset{t\leftrightarrow\omega}{\circ\!\!-\!\!\bullet} (-\mathrm{j}\omega)^n$$

EM Fields – Fourier Transform / EM-Felder – Fourier-Transformation

Direct and Inverse Fourier Transform with Regard to Time and Circular Frequency / Direkte und Inverse Fourier-Transformation bzgl. der Zeit und Kreisfrequenz

$$\underline{\mathbf{E}}(\underline{\mathbf{R}},\omega) = \int\limits_{t=-\infty}^{\infty} \underline{\mathbf{E}}(\underline{\mathbf{R}},t)\ \mathrm{e}^{\mathrm{j}\omega t}\ \mathrm{d}t = \mathcal{FT}_{t\to\omega}\left\{\underline{\mathbf{E}}(\underline{\mathbf{R}},t)\right\} \qquad \underline{\mathbf{E}}(\underline{\mathbf{R}},\omega) \overset{\omega\leftarrow t}{\bullet\!\!-\!\!\circ} \underline{\mathbf{E}}(\underline{\mathbf{R}},t)$$

$$\underline{\mathbf{E}}(\underline{\mathbf{R}},t) = \frac{1}{2\pi} \int\limits_{\omega=-\infty}^{\infty} \underline{\mathbf{E}}(\underline{\mathbf{R}},\omega)\,\mathrm{e}^{-\mathrm{j}\omega t}\,\mathrm{d}\omega = \mathcal{FT}^{-1}_{\omega\to t}\left\{\underline{\mathbf{E}}(\underline{\mathbf{R}},\omega)\right\} \qquad \underline{\mathbf{E}}(\underline{\mathbf{R}},t) \overset{t\leftarrow\omega}{\circ\!\!-\!\!\bullet} \underline{\mathbf{E}}(\underline{\mathbf{R}},\omega)$$

$$\frac{\partial}{\partial t} \overset{t\leftrightarrow\omega}{\circ\!\!-\!\!\bullet} -\mathrm{j}\omega$$

Derive the Following Identity / Leite die folgende Identität ab

$$\frac{\partial}{\partial t} \overset{t\leftrightarrow\omega}{\circ\!\!-\!\!\bullet} -\mathrm{j}\omega$$

$$\frac{\partial}{\partial t}\underline{\mathbf{E}}(\underline{\mathbf{R}},t) = \frac{\partial}{\partial t}\left[\frac{1}{2\pi} \int\limits_{\omega=-\infty}^{\infty} \underline{\mathbf{E}}(\underline{\mathbf{R}},\omega)\,\mathrm{e}^{-\mathrm{j}\omega t}\,\mathrm{d}\omega\right]$$

$$= \frac{1}{2\pi} \int\limits_{\omega=-\infty}^{\infty} \underline{\mathbf{E}}(\underline{\mathbf{R}},\omega)\left[\frac{\partial}{\partial t}\mathrm{e}^{-\mathrm{j}\omega t}\right]\mathrm{d}\omega$$

$$\frac{\partial}{\partial t}\mathrm{e}^{-\mathrm{j}\omega t} = -\mathrm{j}\omega\,\mathrm{e}^{-\mathrm{j}\omega t}$$

$$\left[\frac{\partial}{\partial t}\underline{\mathbf{E}}(\underline{\mathbf{R}},t)\right] = \frac{1}{2\pi} \int\limits_{\omega=-\infty}^{\infty} \left[-\mathrm{j}\omega\underline{\mathbf{E}}(\underline{\mathbf{R}},\omega)\right]\mathrm{e}^{-\mathrm{j}\omega t}\,\mathrm{d}\omega$$

$$\frac{\partial}{\partial t} \overset{t\leftrightarrow\omega}{\circ\!\!-\!\!\bullet} -\mathrm{j}\omega$$

EM Fields – Homogeneous Vector Helmholtz Equation / EM-Felder – Homogene vektorielle Helmholtz-Gleichung

Vector Wave Equation / Vektorielle Wellengleichung

$$\Delta \underline{\mathbf{H}}(\underline{\mathbf{R}},t) - \frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}\underline{\mathbf{H}}(\underline{\mathbf{R}},t) = \underline{\mathbf{0}}$$

$$\Delta \underline{\mathbf{E}}(\underline{\mathbf{R}},t) - \frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}\underline{\mathbf{E}}(\underline{\mathbf{R}},t) = \underline{\mathbf{0}}$$

$$-\omega^2 \bullet\!\!-\!\!\circ \frac{\partial^2}{\partial t^2}$$

Homogeneous Vector Helmholtz Equation / Homogene vektorielle Helmholtz-Gleichung

$$\Delta \underline{\mathbf{H}}(\underline{\mathbf{R}},\omega) + \frac{\omega^2}{c_0^2}\underline{\mathbf{H}}(\underline{\mathbf{R}},\omega) = \underline{\mathbf{0}}$$

$$\Delta \underline{\mathbf{E}}(\underline{\mathbf{R}},\omega) + \frac{\omega^2}{c_0^2}\underline{\mathbf{E}}(\underline{\mathbf{R}},\omega) = \underline{\mathbf{0}}$$

with the wave number / mit der Wellenzahl

$$\Delta \underline{\mathbf{H}}(\underline{\mathbf{R}},\omega) + k^2\underline{\mathbf{H}}(\underline{\mathbf{R}},\omega) = \underline{\mathbf{0}}$$

$$\Delta \underline{\mathbf{E}}(\underline{\mathbf{R}},\omega) + k^2\underline{\mathbf{E}}(\underline{\mathbf{R}},\omega) = \underline{\mathbf{0}}$$

in Operator Notation / in Operatorschreibweise

$$\left(\Delta + k^2\right)\underline{\mathbf{H}}(\underline{\mathbf{R}},\omega) = \underline{\mathbf{0}}$$

$$\left(\Delta + k^2\right)\underline{\mathbf{E}}(\underline{\mathbf{R}},\omega) = \underline{\mathbf{0}}$$

Wave Number / Wellenzahl

$$k = \frac{\omega}{c_0} \quad \left[\frac{1}{\mathrm{m}} = \frac{1/\mathrm{s}}{\mathrm{m/s}}\right]$$

$$k = \frac{2\pi f}{c_0} = \frac{2\pi}{\lambda}$$

Wave Length / Wellenlänge

$$\lambda = \frac{c_0}{f} \quad \left[\mathrm{m} = \frac{\mathrm{m/s}}{1/\mathrm{s}}\right]$$

EM Fields – Homogeneous Vector Helmholtz Equation – Plane Wave / EM-Felder – Homogene vektorielle Helmholtz-Gleichung – Ebene Welle

Homogeneous Vector Helmholtz Equation /
Homogene vektorielle Helmholtz-Gleichung

$$\left(\Delta + k^2\right)\underline{\mathbf{H}}(\underline{\mathbf{R}},\omega) = \underline{\mathbf{0}}$$

$$\left(\Delta + k^2\right)\underline{\mathbf{E}}(\underline{\mathbf{R}},\omega) = \underline{\mathbf{0}}$$

Elementary Solution of the Homogeneous Vector Helmholtz Equation: Plane Wave /
Elementare Lösung der homogenen vektoriellen Helmholtz-Gleichung: Ebene Welle

$$\underline{\mathbf{H}}(\underline{\mathbf{R}},\omega) = \underline{\mathbf{H}}_0(\hat{\underline{\mathbf{k}}},\omega)\, e^{j\underline{\mathbf{k}}\cdot\underline{\mathbf{R}}}$$

$$\underline{\mathbf{E}}(\underline{\mathbf{R}},\omega) = \underbrace{\underline{\mathbf{E}}_0(\hat{\underline{\mathbf{k}}},\omega)}_{\text{Amplitude / Amplitude}}\, \underbrace{e^{j\underline{\mathbf{k}}\cdot\underline{\mathbf{R}}}}_{\text{Phase Function / Phasenterm}}$$

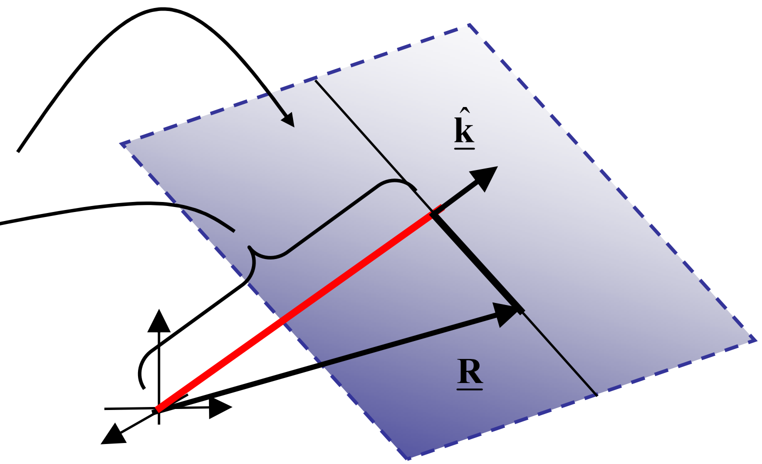
Plane Wave: Because the Phase is Constant on a Plane! /
Ebene Welle: Weil die Phase auf einer Ebene konstant ist.

$$\underline{\mathbf{H}}(\underline{\mathbf{R}},\omega) = \underline{\mathbf{H}}_0(\hat{\underline{\mathbf{k}}},\omega)\, e^{jk\hat{\underline{\mathbf{k}}}\cdot\underline{\mathbf{R}}}$$

$$\underline{\mathbf{E}}(\underline{\mathbf{R}},\omega) = \underline{\mathbf{E}}_0(\hat{\underline{\mathbf{k}}},\omega)\, e^{jk\hat{\underline{\mathbf{k}}}\cdot\underline{\mathbf{R}}}$$

Plane of constant phase /
Ebene konstanter Phase

$$\hat{\underline{\mathbf{k}}}\cdot\underline{\mathbf{R}} = \text{const.}$$



EM Fields – Plane Wave / EM-Felder – Ebene Welle

Example: Plane Wave Propagating in Positive z Direction /
Beispiel: ebene Welle, die sich in positive z Richtung ausbreitet

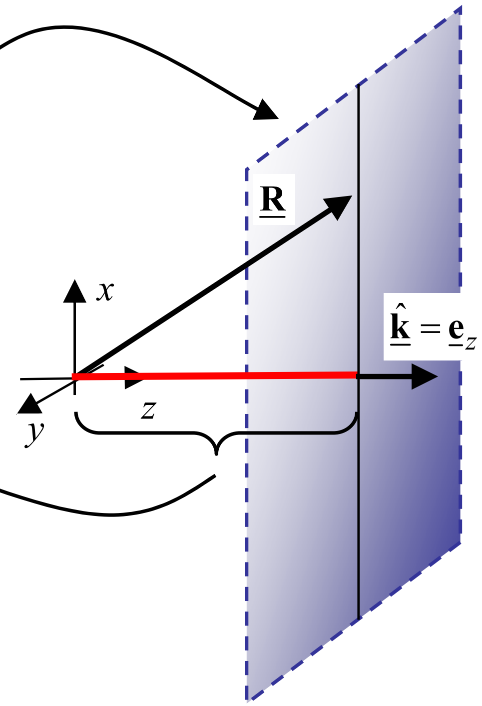
$$\underline{\mathbf{k}} = k\hat{\underline{\mathbf{k}}} = k\underline{\mathbf{e}}_z$$

$$\begin{aligned}\underline{\mathbf{E}}(\underline{\mathbf{R}},\omega) &= \underline{\mathbf{E}}_0(\hat{\underline{\mathbf{k}}},\omega)\,\mathrm{e}^{\mathrm{j}\underline{\mathbf{k}}\cdot\underline{\mathbf{R}}} \\ &= \underline{\mathbf{E}}_0(\hat{\underline{\mathbf{k}}},\omega)\,\mathrm{e}^{\mathrm{j}k\hat{\underline{\mathbf{k}}}\cdot\underline{\mathbf{R}}} \\ &= \underline{\mathbf{E}}_0(\hat{\underline{\mathbf{k}}},\omega)\,\mathrm{e}^{\mathrm{j}k\underline{\mathbf{e}}_z\cdot\underline{\mathbf{R}}} \\ &= \underline{\mathbf{E}}_0(\hat{\underline{\mathbf{k}}},\omega)\,\mathrm{e}^{\mathrm{j}kz}\end{aligned}$$

Plane Wave: Because the Phase is Constant on a Plane! /
Ebene Welle: Weil die Phase auf einer Ebene konstant ist.

Plane of constant phase /
Ebene konstanter Phase

$$\underline{\mathbf{E}}(\underline{\mathbf{R}},\omega) = \underline{\mathbf{E}}_0(\hat{\underline{\mathbf{k}}},\omega)\,\mathrm{e}^{\mathrm{j}kz} \longrightarrow \hat{\underline{\mathbf{k}}}\cdot\underline{\mathbf{R}} = z = \text{const.}$$

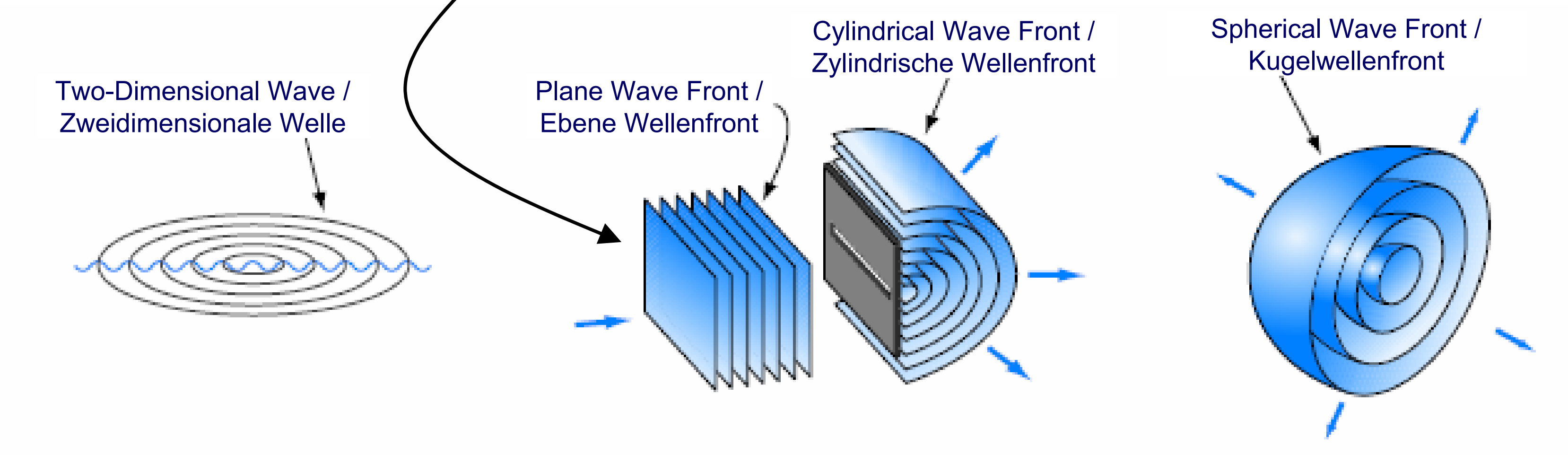


EM Fields – Plane Wave / EM-Felder – Ebene Welle

Wave Vector of a Plane Wave in the Frequency Domain Propagating in Positive z Direction / Wellenvektor für eine ebene Welle im Frequenzbereich, die sich in positive z-Richtung ausbreitet

$$\underline{\mathbf{k}} = k\hat{\underline{\mathbf{k}}} = k\underline{\mathbf{e}}_z$$

$$\begin{aligned}\underline{\mathbf{E}}(\underline{\mathbf{R}},\omega) &= \underline{\mathbf{E}}_0(\hat{\underline{\mathbf{k}}},\omega)\,\mathrm{e}^{\mathrm{j}\underline{\mathbf{k}}\bullet\underline{\mathbf{R}}} \\ &= \underline{\mathbf{E}}_0(\hat{\underline{\mathbf{k}}},\omega)\,\mathrm{e}^{\mathrm{j}k\hat{\underline{\mathbf{k}}}\bullet\underline{\mathbf{R}}} \\ &= \underline{\mathbf{E}}_0(\hat{\underline{\mathbf{k}}},\omega)\,\mathrm{e}^{\mathrm{j}k\underline{\mathbf{e}}_z\bullet\underline{\mathbf{R}}} \\ &= \underline{\mathbf{E}}_0(\hat{\underline{\mathbf{k}}},\omega)\,\mathrm{e}^{\mathrm{j}kz}\end{aligned}$$



(a) Circular Wave / Zylinderwelle

(b) Plane and Cylindrical Waves / Ebene und zylindrische Wellen

(c) Spherical Wave / Kugelwelle

EM Fields – 3-D and 1-D Plane EM Wave – Frequency and Time Domain / EM-Felder – 3D und 1D ebene EM-Welle – Frequenz- und Zeitbereich

$$\underline{\mathbf{E}}(\underline{\mathbf{R}},\omega) = E_0\left(\hat{\underline{\mathbf{k}}},\omega\right)\mathrm{e}^{\mathrm{j}k\hat{\underline{\mathbf{k}}}\bullet\underline{\mathbf{R}}}\,\underline{\mathbf{e}}_x$$

$$\underline{\mathbf{E}}(\underline{\mathbf{R}},\omega) = E_0\left(\hat{\underline{\mathbf{k}}},\omega\right)\mathrm{e}^{\mathrm{j}kz}\,\underline{\mathbf{e}}_x$$

$$\begin{aligned}\underline{\mathbf{E}}(\underline{\mathbf{R}},t) &= \frac{1}{2\pi}\int\limits_{\omega=-\infty}^{\infty}\underline{\mathbf{E}}(\underline{\mathbf{R}},\omega)\,\mathrm{e}^{-\mathrm{j}\omega\mathrm{t}}\,\mathrm{d}\omega\\ &= \frac{1}{2\pi}\int\limits_{\omega=-\infty}^{\infty}E_0\left(\hat{\underline{\mathbf{k}}},\omega\right)\mathrm{e}^{\mathrm{j}k\hat{\underline{\mathbf{k}}}\bullet\underline{\mathbf{R}}}\,\underline{\mathbf{e}}_x\,\mathrm{e}^{-\mathrm{j}\omega\mathrm{t}}\,\mathrm{d}\omega\\ &= \frac{1}{2\pi}\int\limits_{\omega=-\infty}^{\infty}E_0\left(\hat{\underline{\mathbf{k}}},\omega\right)\mathrm{e}^{\mathrm{j}\omega\frac{\hat{\underline{\mathbf{k}}}\bullet\underline{\mathbf{R}}}{c_0}}\,\mathrm{e}^{-\mathrm{j}\omega t}\,\mathrm{d}\omega\,\underline{\mathbf{e}}_x\\ &= E_0\left(t-\frac{\hat{\underline{\mathbf{k}}}\bullet\underline{\mathbf{R}}}{c_0}\right)\underline{\mathbf{e}}_x\end{aligned}$$

$$\begin{aligned}\underline{\mathbf{E}}(\underline{\mathbf{R}},t) &= \frac{1}{2\pi}\int\limits_{\omega=-\infty}^{\infty}\underline{\mathbf{E}}(\underline{\mathbf{R}},\omega)\,\mathrm{e}^{-\mathrm{j}\omega\mathrm{t}}\,\mathrm{d}\omega\\ &= \frac{1}{2\pi}\int\limits_{\omega=-\infty}^{\infty}E_0\left(\hat{\underline{\mathbf{k}}},\omega\right)\mathrm{e}^{\mathrm{j}kz}\,\underline{\mathbf{e}}_x\,\mathrm{e}^{-\mathrm{j}\omega\mathrm{t}}\,\mathrm{d}\omega\\ &= \frac{1}{2\pi}\int\limits_{\omega=-\infty}^{\infty}E_0\left(\hat{\underline{\mathbf{k}}},\omega\right)\mathrm{e}^{\mathrm{j}\omega\frac{z}{c_0}}\,\underline{\mathbf{e}}_x\,\mathrm{e}^{-\mathrm{j}\omega\mathrm{t}}\,\mathrm{d}\omega\\ &= E_0\left(t-\frac{z}{c_0}\right)\underline{\mathbf{e}}_x\end{aligned}$$

$$E_0\left(\hat{\underline{\mathbf{k}}},t\right) = \frac{1}{2\pi}\int\limits_{\omega=-\infty}^{\infty}E_0\left(\hat{\underline{\mathbf{k}}},\omega\right)\mathrm{e}^{-\mathrm{j}\omega\mathrm{t}}\,\mathrm{d}\omega$$

$$E_0\left(\hat{\underline{\mathbf{k}}},t\right) = \frac{1}{2\pi}\int\limits_{\omega=-\infty}^{\infty}E_0\left(\hat{\underline{\mathbf{k}}},\omega\right)\mathrm{e}^{-\mathrm{j}\omega\mathrm{t}}\,\mathrm{d}\omega$$

EM Fields – 1-D Plane EM Wave / EM-Felder – 1D ebene EM-Welle

Plane Wave Propagating in Positive z Direction /
Ebene Welle, die sich in positive z Richtung ausbreitet

$$\underline{\mathbf{k}} = k\hat{\underline{\mathbf{k}}} = k\underline{\mathbf{e}}_z$$

$$\underline{\mathbf{E}}(\underline{\mathbf{R}},\omega) = \underline{\mathbf{E}}_0(\underline{\mathbf{k}},\omega)\mathrm{e}^{\mathrm{j}kz}$$

Maxwell's Equations in the Time Domain for the Source-Free Case /
Maxwellsche Gleichungen im Zeitbereich für den quellenfreien Fall

$$\partial/\partial t\,\underline{\mathbf{B}}(\underline{\mathbf{R}},t) = -\nabla\times\underline{\mathbf{E}}(\underline{\mathbf{R}},t)$$

$$\partial/\partial t\,\underline{\mathbf{D}}(\underline{\mathbf{R}},t) = \nabla\times\underline{\mathbf{H}}(\underline{\mathbf{R}},t)$$

$$\nabla\bullet\underline{\mathbf{B}}(\underline{\mathbf{R}},t) = 0$$

$$\nabla\bullet\underline{\mathbf{D}}(\underline{\mathbf{R}},t) = 0$$

$$\frac{\partial}{\partial t} \overset{t\leftrightarrow\omega}{\circ\!\!-\!\!\bullet} -\mathrm{j}\omega$$

Maxwell's Equations in the Frequency Domain for the Source-Free Case /
Maxwellsche Gleichungen im Frequenzbereich für den quellenfreien Fall

$$\nabla\times\underline{\mathbf{E}}(\underline{\mathbf{R}},\omega) = \mathrm{j}\omega\underline{\mathbf{B}}(\underline{\mathbf{R}},\omega)$$

$$\nabla\times\underline{\mathbf{H}}(\underline{\mathbf{R}},\omega) = -\mathrm{j}\omega\underline{\mathbf{D}}(\underline{\mathbf{R}},\omega)$$

$$\nabla\bullet\underline{\mathbf{D}}(\underline{\mathbf{R}},\omega) = 0$$

$$\nabla\bullet\underline{\mathbf{B}}(\underline{\mathbf{R}},\omega) = 0$$

$$\begin{aligned}
\underline{\mathbf{B}}(\underline{\mathbf{R}},\omega) &= \frac{1}{\mathrm{j}\omega}\nabla\times\underline{\mathbf{E}}(\underline{\mathbf{R}},\omega)\\
&= \frac{1}{\mathrm{j}\omega}\nabla\times\left[\underline{\mathbf{E}}_0(\underline{\mathbf{k}},\omega)\mathrm{e}^{\mathrm{j}kz}\right]\\
&= \frac{1}{\mathrm{j}\omega}\left[\mathrm{e}^{\mathrm{j}kz}\underbrace{\nabla\times\underline{\mathbf{E}}_0(\underline{\mathbf{k}},\omega)}_{=\underline{\mathbf{0}}} + \left(\nabla\mathrm{e}^{\mathrm{j}kz}\right)\underline{\mathbf{E}}_0(\underline{\mathbf{k}})\right]
\end{aligned}$$

EM Fields – 1-D Plane EM Wave / EM-Felder – 1D ebene EM-Welle

Plane Wave Propagating in Positive z Direction /
Ebene Welle, die sich in positive z Richtung ausbreitet

$$\underline{\mathbf{k}} = k\hat{\underline{\mathbf{k}}} = k\underline{\mathbf{e}}_z$$

$$\underline{\mathbf{E}}(\underline{\mathbf{R}},\omega) = \underline{\mathbf{E}}_0(\hat{\underline{\mathbf{k}}},\omega)\mathrm{e}^{\mathrm{j}kz}$$

$$\begin{aligned}
\underline{\mathbf{B}}(\underline{\mathbf{R}},\omega) &= \frac{1}{\mathrm{j}\omega}\left[\mathrm{e}^{\mathrm{j}kz}\underbrace{\nabla\times\underline{\mathbf{E}}_0(\hat{\underline{\mathbf{k}}},\omega)}_{=\underline{\mathbf{0}}} + \left(\nabla\mathrm{e}^{\mathrm{j}kz}\right)\times\underline{\mathbf{E}}_0(\hat{\underline{\mathbf{k}}},\omega)\right] \\
&= \frac{1}{\mathrm{j}\omega}\underbrace{\left(\nabla\mathrm{e}^{\mathrm{j}kz}\right)}\times\underline{\mathbf{E}}_0(\hat{\underline{\mathbf{k}}},\omega) \\
&= \frac{1}{\mathrm{j}\omega}\mathrm{j}k\underbrace{\underline{\mathbf{e}}_z}_{=\hat{\underline{\mathbf{k}}}}\mathrm{e}^{\mathrm{j}kz}\times\underline{\mathbf{E}}_0(\hat{\underline{\mathbf{k}}},\omega) \\
&= \frac{k}{\omega}\hat{\underline{\mathbf{k}}}\times\underline{\mathbf{E}}_0(\hat{\underline{\mathbf{k}}},\omega)\mathrm{e}^{\mathrm{j}kz} \\
&= \frac{k}{\omega}\hat{\underline{\mathbf{k}}}\times\underline{\mathbf{E}}(\underline{\mathbf{R}},\omega)
\end{aligned}$$

$$\begin{aligned}
\nabla\mathrm{e}^{\mathrm{j}kz} &= \left(\underline{\mathbf{e}}_x\frac{\partial}{\partial x} + \underline{\mathbf{e}}_y\frac{\partial}{\partial y} + \underline{\mathbf{e}}_z\frac{\partial}{\partial z}\right)\mathrm{e}^{\mathrm{j}kz} \\
&= \left(\underline{\mathbf{e}}_x\underbrace{\frac{\partial}{\partial x}\mathrm{e}^{\mathrm{j}kz}}_{=0} + \underline{\mathbf{e}}_y\underbrace{\frac{\partial}{\partial y}\mathrm{e}^{\mathrm{j}kz}}_{=0} + \underline{\mathbf{e}}_z\underbrace{\frac{\partial}{\partial z}\mathrm{e}^{\mathrm{j}kz}}_{\mathrm{j}k\mathrm{e}^{\mathrm{j}kz}}\right) \\
&= \mathrm{j}k\underbrace{\underline{\mathbf{e}}_z}_{=\hat{\underline{\mathbf{k}}}}\mathrm{e}^{\mathrm{j}kz} \\
&= \mathrm{j}k\hat{\underline{\mathbf{k}}}\mathrm{e}^{\mathrm{j}kz}
\end{aligned}$$

$$\underline{\mathbf{B}}(\underline{\mathbf{R}},\omega) = \frac{k}{\omega}\hat{\underline{\mathbf{k}}}\times\underline{\mathbf{E}}(\underline{\mathbf{R}},\omega)$$

EM Fields – 1-D Plane EM-Wave / EM-Felder – 1D ebene EM-Welle

$$\underline{\mathbf{H}}(\underline{\mathbf{R}},\omega)=\frac{1}{\mu_0}\underline{\mathbf{B}}(\underline{\mathbf{R}},\omega) \qquad \underline{\mathbf{B}}(\underline{\mathbf{R}},\omega)=\frac{k}{\omega}\hat{\underline{\mathbf{k}}}\times\underline{\mathbf{E}}(\underline{\mathbf{R}},\omega)$$

$$\begin{aligned}
\underline{\mathbf{H}}(\underline{\mathbf{R}},\omega) &= \frac{1}{\mu_0}\underline{\mathbf{B}}(\underline{\mathbf{R}},\omega)\\
&= \frac{1}{\mu_0}\frac{k}{\omega}\mathrm{e}^{\mathrm{j}kz}\underline{\mathbf{e}}_z\times\underline{\mathbf{E}}_0(\hat{\underline{\mathbf{k}}},\omega)\\
&= \frac{1}{\mu_0}\frac{\omega/c_0}{\omega}\mathrm{e}^{\mathrm{j}kz}\underline{\mathbf{e}}_z\times\underline{\mathbf{E}}_0(\hat{\underline{\mathbf{k}}},\omega)\\
&= \frac{1}{\mu_0}\frac{1}{c_0}\mathrm{e}^{\mathrm{j}kz}\underline{\mathbf{e}}_z\times\underline{\mathbf{E}}_0(\hat{\underline{\mathbf{k}}},\omega)\\
&= \frac{1}{\mu_0}\sqrt{\varepsilon_0\mu_0}\,\mathrm{e}^{\mathrm{j}kz}\underline{\mathbf{e}}_z\times\underline{\mathbf{E}}_0(\hat{\underline{\mathbf{k}}},\omega)\\
&= \sqrt{\frac{\varepsilon_0}{\mu_0}}\mathrm{e}^{\mathrm{j}kz}\underbrace{\underline{\mathbf{e}}_z}_{=\hat{\underline{\mathbf{k}}}}\times\underline{\mathbf{E}}_0(\hat{\underline{\mathbf{k}}},\omega)\\
&= \underbrace{Y_0\hat{\underline{\mathbf{k}}}\times\underline{\mathbf{E}}_0(\hat{\underline{\mathbf{k}}},\omega)}_{=\underline{\mathbf{H}}_0(\hat{\underline{\mathbf{k}}},\omega)}\mathrm{e}^{\mathrm{j}kz}\\
&= \underline{\mathbf{H}}_0(\hat{\underline{\mathbf{k}}},\omega)\mathrm{e}^{\mathrm{j}kz}
\end{aligned}$$

$$\underline{\mathbf{H}}(\underline{\mathbf{R}},\omega)=Y_0\hat{\underline{\mathbf{k}}}\times\underline{\mathbf{E}}(\underline{\mathbf{R}},\omega)$$

Wave Impedance of Free Space (Vacuum) / Wellenimpedanz des Freiraumes (Vakuum)

$$\sqrt{\frac{\mu_0}{\varepsilon_0}}=Z_0=376.7\ \Omega\approx 377\ \Omega\approx 120\pi\ \Omega$$

Wave Admittance of Free Space (Vacuum) / Wellenadmittanz des Freiraumes (Vakuum)

$$\frac{1}{Y_0}=Z_0$$

EM Fields – 1-D Plane EM Wave / EM-Felder – 1D ebene EM-Welle

Homogeneous Vector Wave Equation →
Proof: The Divergence of a Plane Wave must be Zero /

Homogene vektorielle Wellengleichung →
Beweis: Die Divergenz einer ebenen Wellen muss null sein

$$\nabla \bullet \underline{\mathbf{D}}(\underline{\mathbf{R}}, \omega) = 0$$

$$\begin{aligned}
\nabla \bullet \underline{\mathbf{D}}(\underline{\mathbf{R}}, \omega) &= \nabla \bullet \left[\varepsilon_0 \underline{\mathbf{E}}(\underline{\mathbf{R}}, \omega) \right] \\
&= \varepsilon_0 \nabla \bullet \underline{\mathbf{E}}(\underline{\mathbf{R}}, \omega) \\
&= \varepsilon_0 \nabla \bullet \left[\underline{\mathbf{E}}_0(\hat{\underline{\mathbf{k}}}, \omega) \mathrm{e}^{\mathrm{j}kz} \right] \\
&= \varepsilon_0 \left[\underbrace{\nabla \bullet \underline{\mathbf{E}}_0(\hat{\underline{\mathbf{k}}}, \omega)}_{=0} \mathrm{e}^{\mathrm{j}kz} + \underline{\mathbf{E}}_0(\hat{\underline{\mathbf{k}}}, \omega) \bullet \underbrace{\nabla \mathrm{e}^{\mathrm{j}kz}}_{= \mathrm{j}k \underline{\mathbf{e}}_z \mathrm{e}^{\mathrm{j}kz}} \right] \\
&= \mathrm{j}k \varepsilon_0 \mathrm{e}^{\mathrm{j}kz} \underline{\mathbf{E}}_0(\hat{\underline{\mathbf{k}}}, \omega) \bullet \underbrace{\underline{\mathbf{e}}_z}_{= \hat{\underline{\mathbf{k}}}}
\end{aligned}$$

$$\underline{\mathbf{E}}_0(\hat{\underline{\mathbf{k}}}, \omega) \bullet \hat{\underline{\mathbf{k}}} = 0 \quad \text{for / für} \quad \underline{\mathbf{E}}_0(\hat{\underline{\mathbf{k}}}, \omega) \perp \hat{\underline{\mathbf{k}}}$$

$$= 0$$

The Divergence of a Plane Wave is Zero, if the Field is Perpendicular to the Propagation Direction! /
Die Divergenz einer ebenen Wellen ist null, wenn das Feld senkrecht auf der Ausbreitungsrichtung steht!

EM Fields – 1-D Plane EM Wave – TEM Wave / EM-Felder – 1D ebene EM-Welle – TEM-Welle

Example: Linear Polarized in x Direction /
Beispiel: Linear polarisiert in x Richtung

$$\underline{\mathbf{E}}_0(\hat{\underline{\mathbf{k}}},\omega) = \underline{\mathbf{E}}_0(\omega) = E_0(\omega)\underline{\mathbf{e}}_x$$

$$\underline{\mathbf{E}}(\underline{\mathbf{R}},\omega) = E_0(\omega)\mathrm{e}^{\mathrm{j}kz}\underline{\mathbf{e}}_x$$

$$\begin{aligned}\underline{\mathbf{H}}(\underline{\mathbf{R}},\omega) &= \underline{\mathbf{H}}_0(\underline{\mathbf{k}})\mathrm{e}^{\mathrm{j}kz}\\ &= Y_0\underline{\mathbf{e}}_z \times \underline{\mathbf{E}}_0(\underline{\mathbf{k}})\mathrm{e}^{\mathrm{j}kz}\\ &= Y_0\underline{\mathbf{e}}_z \times E_0(\omega)\underline{\mathbf{e}}_x\mathrm{e}^{\mathrm{j}kz}\\ &= Y_0E_0(\omega)\mathrm{e}^{\mathrm{j}kz}\underbrace{\underline{\mathbf{e}}_z \times \underline{\mathbf{e}}_x}_{=\underline{\mathbf{e}}_y}\\ &= \underbrace{Y_0E_0(\omega)}_{=H_0(\omega)}\underline{\mathbf{e}}_y\mathrm{e}^{\mathrm{j}kz}\\ &= H_0(\omega)\mathrm{e}^{\mathrm{j}kz}\underline{\mathbf{e}}_y\end{aligned}$$

$$\underline{\mathbf{H}}(\underline{\mathbf{R}},\omega) = H_0(\omega)\mathrm{e}^{\mathrm{j}kz}\underline{\mathbf{e}}_y$$

TEM Wave (TEM: Transversal Electromagnetic Wave)
TEM-Welle (TEM: transversale elektromagnetische Welle

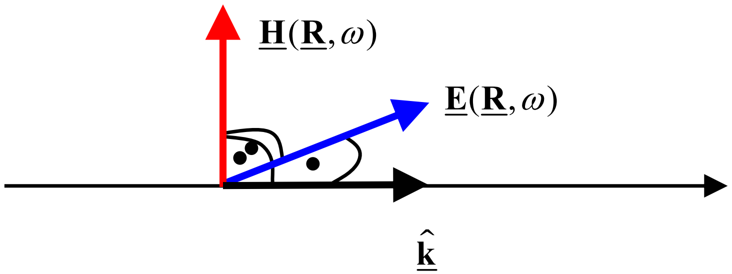
$$\underline{\mathbf{E}}(\underline{\mathbf{R}},\omega) = E_0(\omega)\mathrm{e}^{\mathrm{j}kz}\underline{\mathbf{e}}_x$$

$$\underline{\mathbf{H}}(\underline{\mathbf{R}},\omega) = H_0(\omega)\mathrm{e}^{\mathrm{j}kz}\underline{\mathbf{e}}_y$$

TEM Wave (TEM: Transversal Electromagnetic Wave)
TEM-Welle (TEM: transversale elektromagnetische Welle

$$\underline{\mathbf{E}}(\underline{\mathbf{R}},\omega) = -Z_0\hat{\underline{\mathbf{k}}} \times \underline{\mathbf{H}}(\underline{\mathbf{R}},\omega) \qquad \underline{\mathbf{E}}(\underline{\mathbf{R}},\omega) \perp \hat{\underline{\mathbf{k}}}$$

$$\underline{\mathbf{H}}(\underline{\mathbf{R}},\omega) = \frac{1}{Z_0}\hat{\underline{\mathbf{k}}} \times \underline{\mathbf{E}}(\underline{\mathbf{R}},\omega) \qquad \underline{\mathbf{H}}(\underline{\mathbf{R}},\omega) \perp \hat{\underline{\mathbf{k}}}$$



EM Fields – Plane Wave – Energy Flow – Poynting Vector / EM-Felder – Ebene Welle – Energiefluss – Poynting-Vektor

Plane Wave Traveling in z Direction /
Ebene Welle, die sich in z Richtung ausbreitet

$$\underline{\mathbf{E}}(\underline{\mathbf{R}},\omega) = E_0(\omega)\mathrm{e}^{\mathrm{j}kz}\underline{\mathbf{e}}_x$$

$$\underline{\mathbf{H}}(\underline{\mathbf{R}},\omega) = H_0(\omega)\mathrm{e}^{\mathrm{j}kz}\underline{\mathbf{e}}_y$$

Complex Poynting Vector /
Komplexer Poynting-Vektor

$$\begin{aligned}
\underline{\mathbf{S}}_\mathrm{C}(\underline{\mathbf{R}},\omega) &= \frac{1}{2}\underline{\mathbf{E}}(\underline{\mathbf{R}},\omega)\times\underline{\mathbf{H}}^*(\underline{\mathbf{R}},\omega) \\
&= \frac{1}{2}E_{x0}(\omega)\mathrm{e}^{\mathrm{j}kz}\underline{\mathbf{e}}_x \times H_{y0}^*(\omega)\mathrm{e}^{-\mathrm{j}kz}\underline{\mathbf{e}}_y \\
&= \frac{1}{2}E_{x0}(\omega)H_{y0}^*(\omega)\underbrace{\underline{\mathbf{e}}_x\times\underline{\mathbf{e}}_y}_{=\underline{\mathbf{e}}_z}\underbrace{\mathrm{e}^{\mathrm{j}kz}\mathrm{e}^{-\mathrm{j}kz}}_{=1} \\
&= \frac{1}{2}E_{x0}(\omega)\left[Y_0E_{x0}(\omega)\right]^*\underline{\mathbf{e}}_z \\
&= \frac{1}{2}Y_0E_{x0}(\omega)E_{x0}^*(\omega)\underline{\mathbf{e}}_z \\
&= \underbrace{\frac{1}{2}Y_0\left|E_{x0}(\omega)\right|^2}_{=S_z(\omega)}\underline{\mathbf{e}}_z \\
&= S_z(\omega)\underline{\mathbf{e}}_z
\end{aligned}$$

$$\frac{\mathrm{V}}{\mathrm{m}}\frac{\mathrm{A}}{\mathrm{m}} = \frac{\mathrm{W(att)}}{\mathrm{m}^2}$$

The Energy Propagates in Positive z Direction! /
Die Energie breitet sich in positive z Richtung aus!

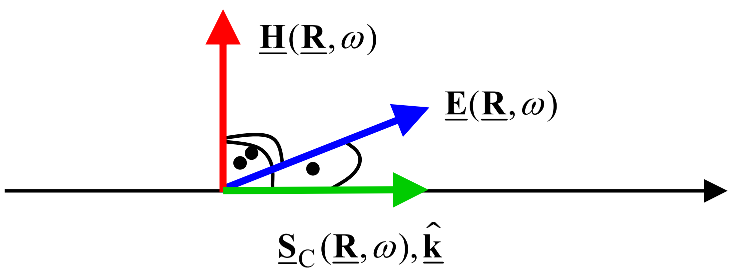
EM Fields – 1-D Plane EM Wave – TEM Wave / EM-Felder – 1D ebene EM-Welle – TEM-Welle

TEM Wave (TEM: Transversal Electromagnetic Wave)
TEM-Welle (TEM: transversal elektromagnetische Welle

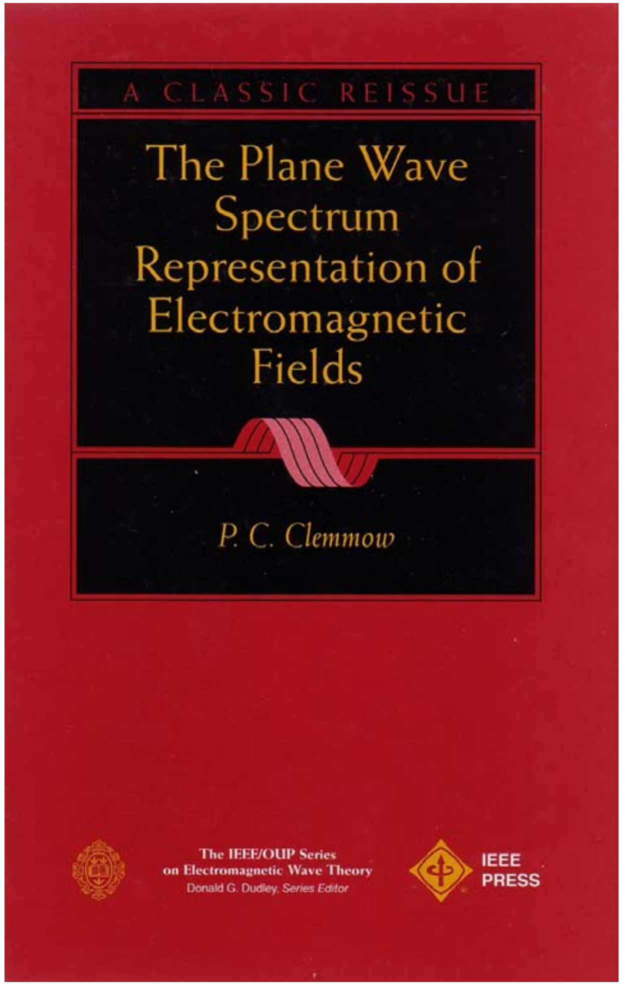
$$\underline{\mathbf{E}}(\underline{\mathbf{R}},\omega) = -Z_0 \hat{\underline{\mathbf{k}}} \times \underline{\mathbf{H}}(\underline{\mathbf{R}},\omega) \qquad \underline{\mathbf{E}}(\underline{\mathbf{R}},\omega) \perp \hat{\underline{\mathbf{k}}}$$

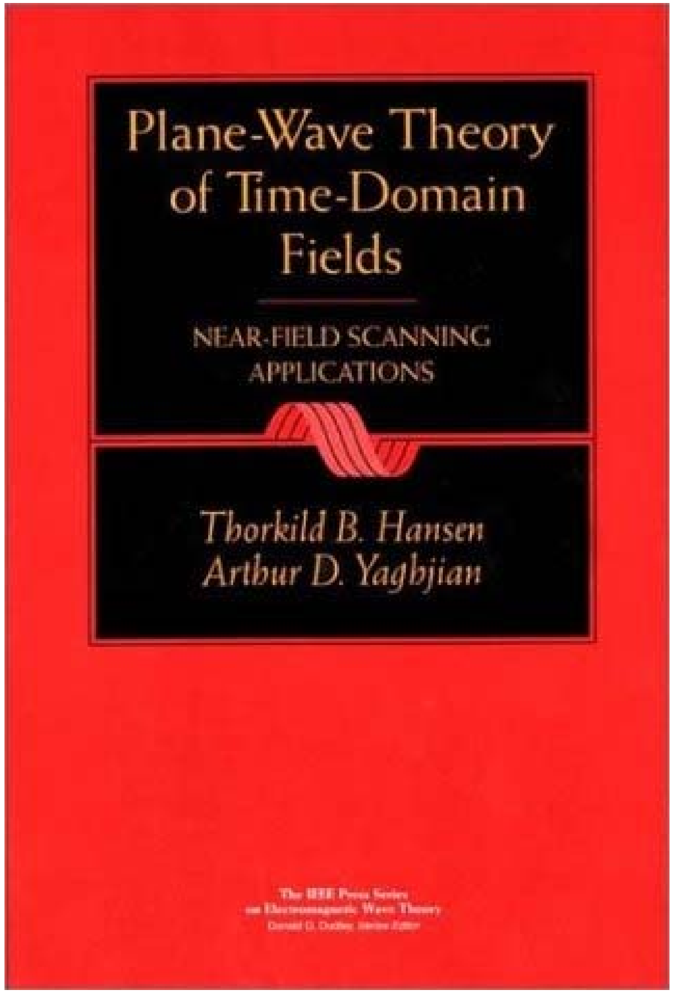
$$\underline{\mathbf{H}}(\underline{\mathbf{R}},\omega) = \frac{1}{Z_0} \hat{\underline{\mathbf{k}}} \times \underline{\mathbf{E}}(\underline{\mathbf{R}},\omega) \qquad \underline{\mathbf{H}}(\underline{\mathbf{R}},\omega) \perp \hat{\underline{\mathbf{k}}}$$

$$\begin{aligned}
\underline{\mathbf{S}}_C(\underline{\mathbf{R}},\omega) &= \frac{1}{2} \underline{\mathbf{E}}(\underline{\mathbf{R}},\omega) \times \underline{\mathbf{H}}^*(\underline{\mathbf{R}},\omega) \\
&= \underbrace{\frac{1}{2} Y_0 \left| E_{x0}(\omega) \right|^2}_{=S_z(\omega)} \underbrace{\underline{\mathbf{e}}_z}_{=\hat{\underline{\mathbf{k}}}} \\
&= S_z(\omega) \underline{\mathbf{e}}_z \\
&= S_z(\omega) \hat{\underline{\mathbf{k}}}
\end{aligned}$$



EM Fields – Plane Wave – Theory – Frequency and Time Domain / EM-Felder – Ebene Welle – Theorie – Frequenz- und Zeitbereich



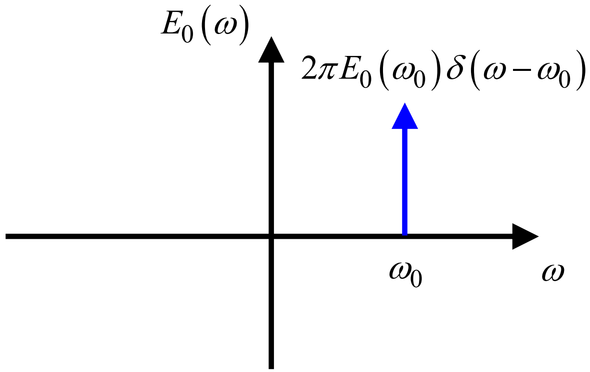


EM Fields – 1-D Plane EM Wave – Frequency and Time Domain / EM-Felder – 1D ebene EM-Welle – Frequenz- und Zeitbereich

$$\underline{\mathbf{E}}(\underline{\mathbf{R}},\omega) = E_0(\omega)\,\mathrm{e}^{\mathrm{j}kz}\,\underline{\mathbf{e}}_x$$

Assume the following Frequency Spectrum /
Nehme das folgende Frequenzspektrum an

$$E_0(\omega) = 2\pi E_0(\omega_0)\,\delta(\omega-\omega_0)$$



$$\underline{\mathbf{E}}(\underline{\mathbf{R}},t) = \frac{1}{2\pi}\int_{\omega=-\infty}^{\infty} \underline{\mathbf{E}}(\underline{\mathbf{R}},\omega)\,\mathrm{e}^{-\mathrm{j}\omega t}\,\mathrm{d}\omega$$

$$= \frac{1}{2\pi}\int_{\omega=-\infty}^{\infty} E_0(\omega)\,\mathrm{e}^{\mathrm{j}kz}\,\underline{\mathbf{e}}_x\,\mathrm{e}^{-\mathrm{j}\omega t}\,\mathrm{d}\omega$$

$$= \frac{1}{2\pi}\int_{\omega=-\infty}^{\infty} E_0(\omega)\,\mathrm{e}^{\mathrm{j}\omega\frac{z}{c_0}}\,\mathrm{e}^{-\mathrm{j}\omega t}\,\mathrm{d}\omega\,\underline{\mathbf{e}}_x$$

$$= \frac{1}{2\pi}\int_{\omega=-\infty}^{\infty} E_0(\omega)\,\mathrm{e}^{-\mathrm{j}\omega\left(t-\frac{z}{c_0}\right)}\,\mathrm{d}\omega\,\underline{\mathbf{e}}_x$$

$$= \frac{1}{2\pi}\int_{\omega=-\infty}^{\infty} \left\{2\pi E_0(\omega_0)\,\delta(\omega-\omega_0)\right\}\mathrm{e}^{-\mathrm{j}\omega\left(t-\frac{z}{c_0}\right)}\,\mathrm{d}\omega\,\underline{\mathbf{e}}_x$$

$$= E_0(\omega_0)\,\mathrm{e}^{-\mathrm{j}\omega_0\left(t-\frac{z}{c_0}\right)}\,\underline{\mathbf{e}}_x$$

Complex Monochromatic Plane Wave /
Komplex monochromatische ebene Welle

$$\underline{\mathbf{E}}(\underline{\mathbf{R}},t) = E_0(\omega_0)\,\mathrm{e}^{-\mathrm{j}\omega_0\left(t-\frac{z}{c_0}\right)}\,\underline{\mathbf{e}}_x$$

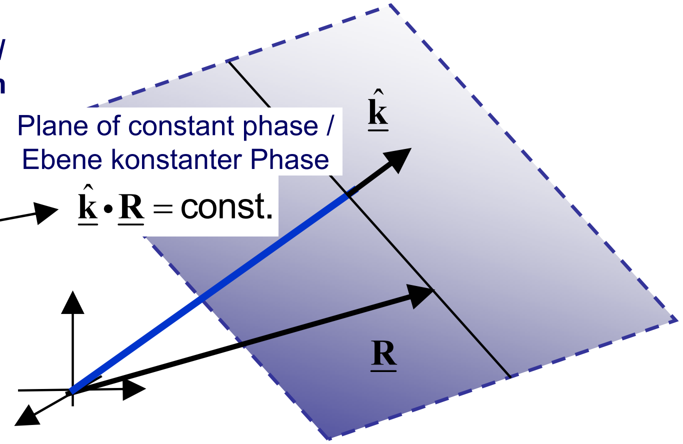
EM Fields – Complex Monochromatic Plane Wave / EM-Felder – Komplexe monochromatische Ebene Welle

Monofrequent (monochromatic) plane wave in the time domain / Monofrequente (monochromatische) ebene Welle im Zeitbereich

$$\underline{\mathbf{E}}(\underline{\mathbf{R}},t) = \underline{\mathbf{E}}_0(\hat{\underline{\mathbf{k}}},\omega_0)\, \mathrm{e}^{-\mathrm{j}\left(\omega_0 t - k\hat{\underline{\mathbf{k}}}\bullet\underline{\mathbf{R}}\right)}$$

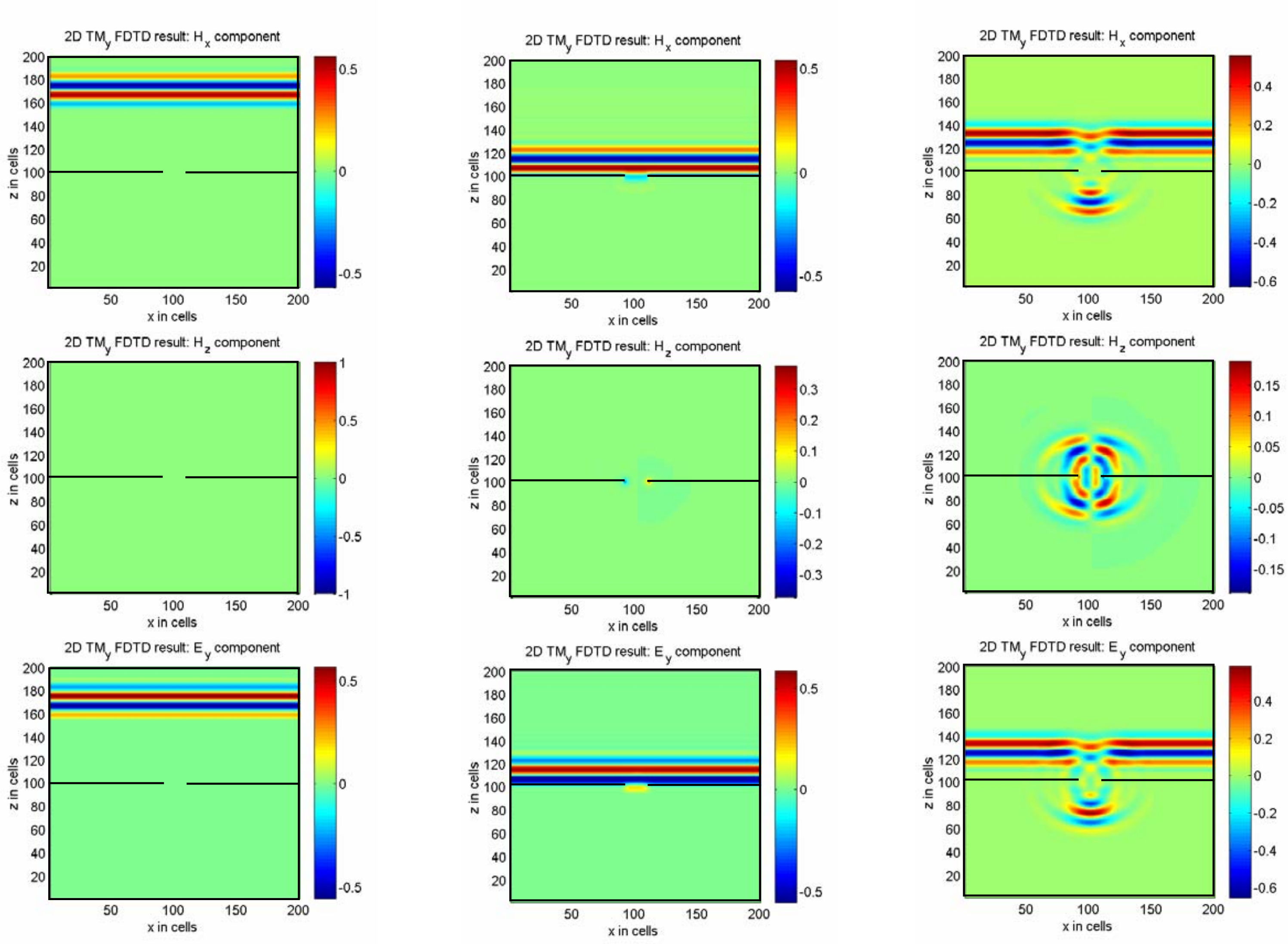
$$= \underline{\mathbf{E}}_0(\hat{\underline{\mathbf{k}}},\omega_0)\, \mathrm{e}^{-\mathrm{j}\omega_0 t}\; \mathrm{e}^{\mathrm{j}k\hat{\underline{\mathbf{k}}}\bullet\underline{\mathbf{R}}}$$

$$= \underline{\mathbf{E}}_0(\hat{\underline{\mathbf{k}}},\omega_0)\, \mathrm{e}^{-\mathrm{j}\omega_0 t}\; \mathrm{e}^{\mathrm{j}\underline{\mathbf{k}}\bullet\underline{\mathbf{R}}}$$



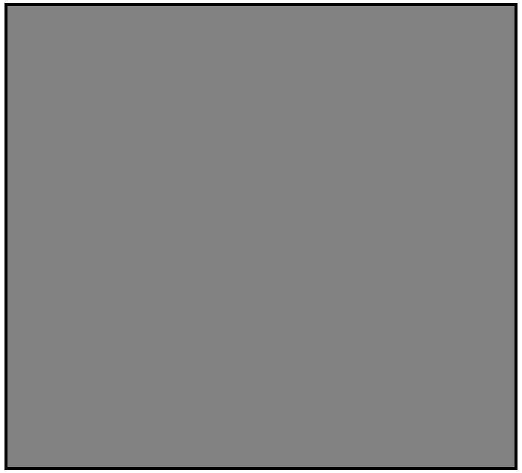
Wave vector / Wellenvektor	$\underline{\mathbf{k}} = \mathrm{k}_x\underline{\mathbf{e}}_x + \mathrm{k}_z\underline{\mathbf{e}}_z + \mathrm{k}_z\underline{\mathbf{e}}_z = \mathrm{k}_z\underline{\mathbf{e}}_z$
Magnitude of the wave vector / Betrag des Wellenvektors	$\lvert\underline{\mathbf{k}}\rvert = \sqrt{\underline{\mathbf{k}}\bullet\underline{\mathbf{k}}} = \sqrt{\mathrm{k}_x^2 + \mathrm{k}_y^2 + \mathrm{k}_z^2} = k$
Wavenumber / Wellenzahl	$k = \dfrac{\omega_0}{c}$
Circular frequency / Kreisfrequenz	$\omega_0 = 2\pi f_0$
Propagation direction / Ausbreitungsrichtung	$\hat{\underline{\mathbf{k}}} = \dfrac{\underline{\mathbf{k}}}{\lvert\underline{\mathbf{k}}\rvert}$
Phase of the plane wave / Phase der ebenen Welle	$\hat{\underline{\mathbf{k}}}\bullet\underline{\mathbf{R}} = \text{const.}$

2-D TM FDTD – Diffraction on a Single Slit / 2D-TM-FDTD – Beugung an einem Spalt

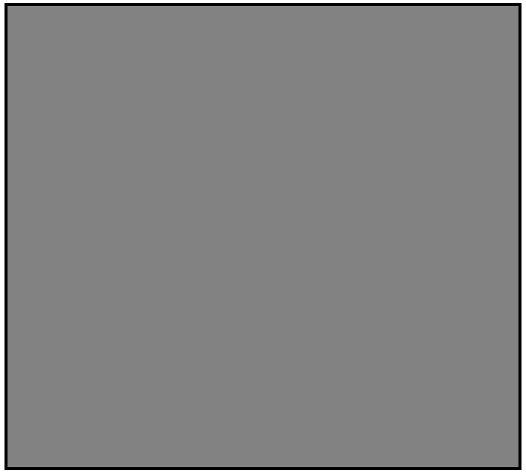


2-D TM FDTD – Diffraction on a Single Slit / 2D-TM-FDTD – Beugung am Spalt

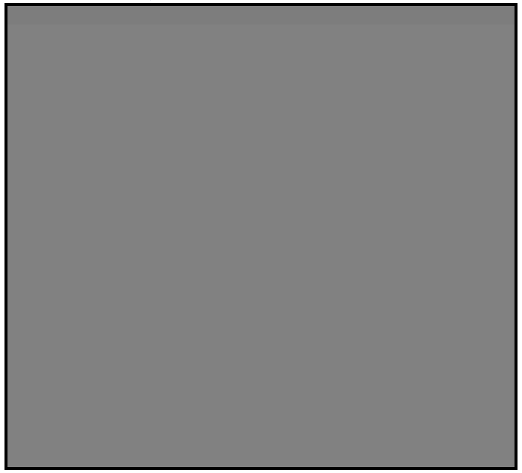
Wave field movie of the H_x field component / Wellenfeldfilm der H_x-Feldkomponente



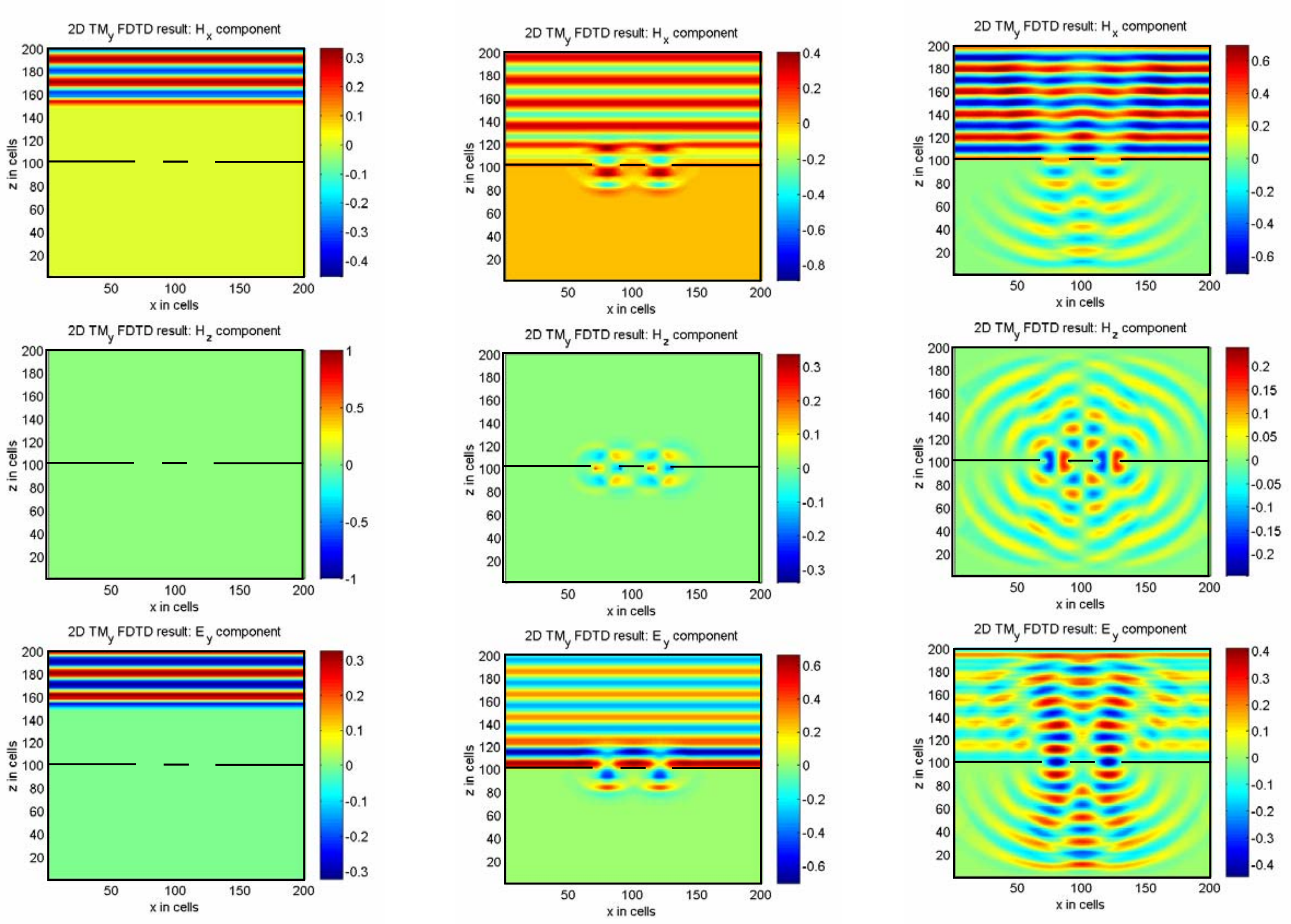
Wave field movie of the H_z field component / Wellenfeldfilm der H_z-Feldkomponente



Wave field movie of the E_y field component / Wellenfeldfilm der E_y-Feldkomponente

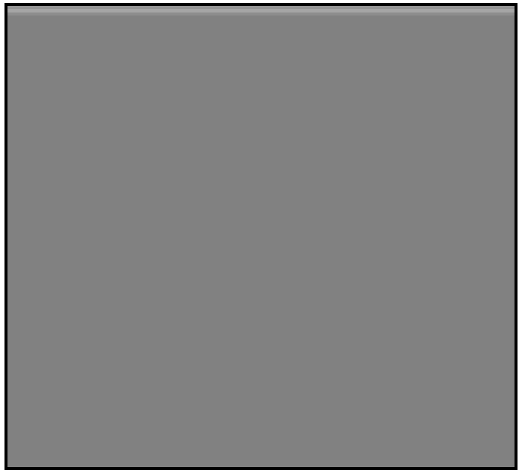


2-D TM FDTD – Diffraction on a Double Slit / 2D-TM-FDTD – Beugung am Doppelspalt

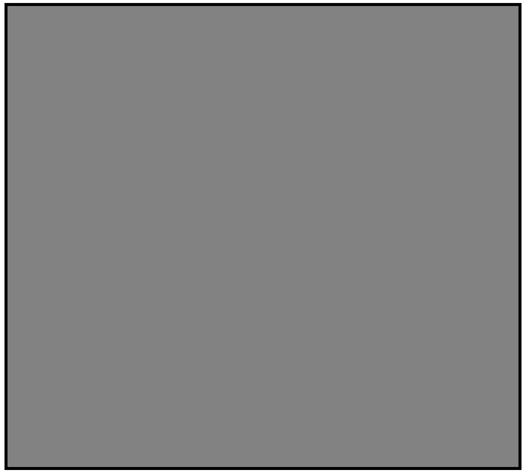


2-D TM FDTD – Diffraction on a Double Slit / 2D-TM-FDTD – Beugung am Doppelspalt

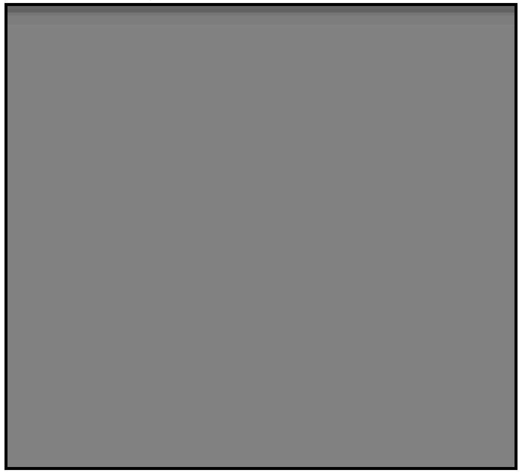
Wave field movie of the H_x field component / Wellenfeldfilm der H_x-Feldkomponente



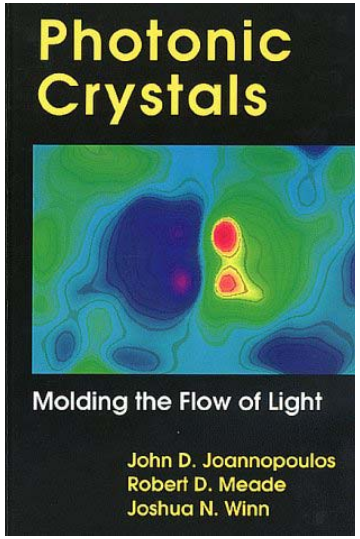
Wave field movie of the H_z field component / Wellenfeldfilm der H_z-Feldkomponente



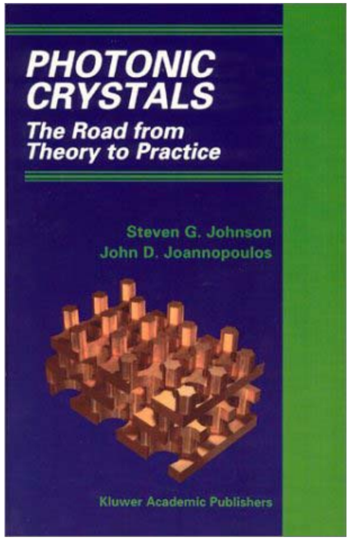
Wave field movie of the E_y field component / Wellenfeldfilm der E_y-Feldkomponente



Photonic Crystals / Photonische Kristalle



Joannopoulos, J. D., R. D. Meade, J. N. Winn: ***Photonic Crystals – Molding the Flow of Light. Princeton University Press, Princeton, 1995.***



Johnson, S. G.: ***Photonic Crystals: The Road from Theory to Practice*****. Kluwer Academic Press, 2001.**

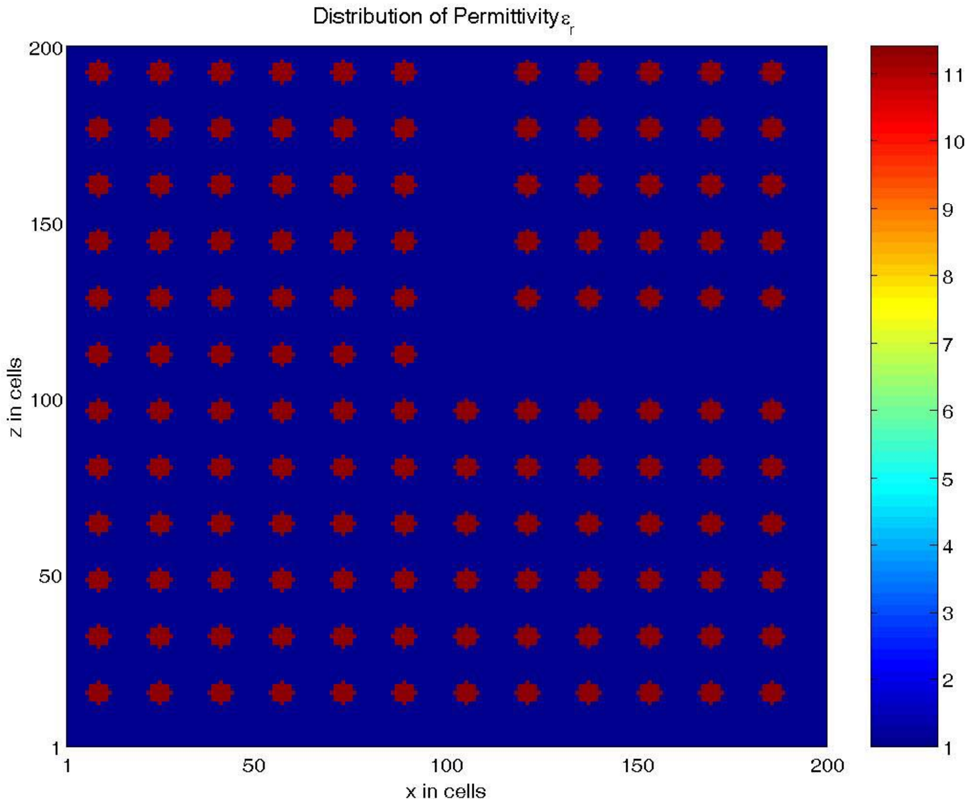
Links:

Photonic Crystals Research at MIT
Homepage of Prof. Sajeev John, University of Toronto, Canada

2-D TM FDTD – Photonic Crystals / 2D-TM-FDTD – Photonische Kristalle

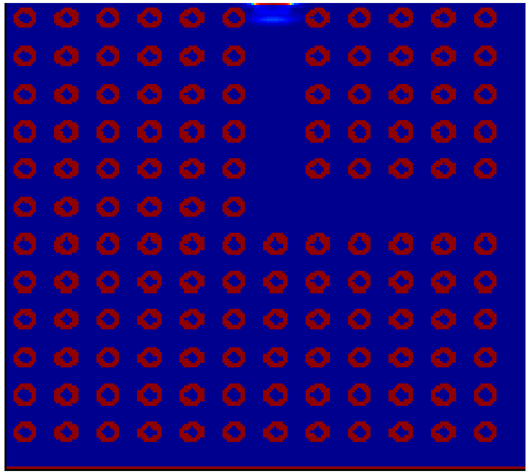
Relative permittivity of the background
Relative Permittivität des Hintergrundes $\varepsilon_{\mathrm{r}}^{(b)} = 1$

Relative permittivity of the rods
Relative Permittivität der Stäbe $\varepsilon_{\mathrm{r}}^{(r)} = 11.4$

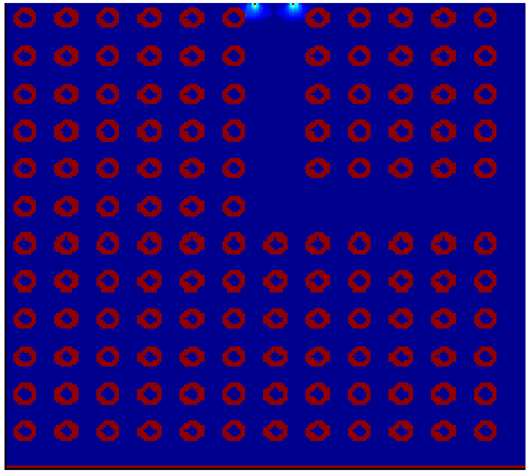


2-D TM FDTD – Photonic Crystals / 2D-TM-FDTD – Photonische Kristalle

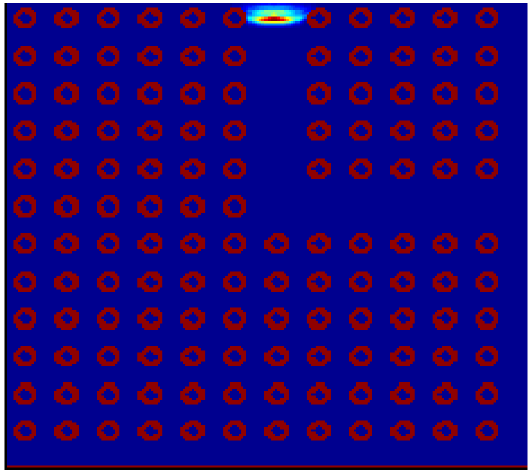
Wave field movie of the H_x field component / Wellenfeldfilm der H_x-Feldkomponente



Wave field movie of the H_z field component / Wellenfeldfilm der H_z-Feldkomponente

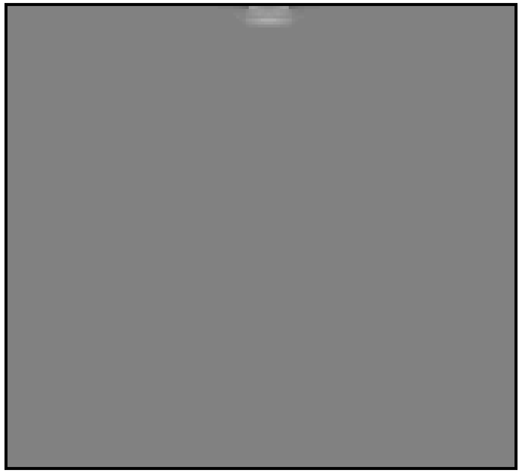


Wave field movie of the E_y field component / Wellenfeldfilm der E_y-Feldkomponente

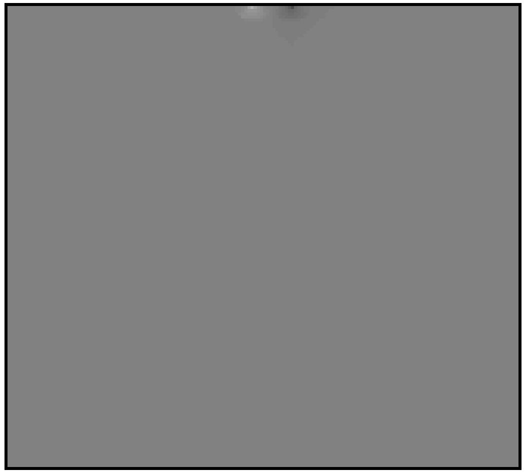


2-D TM FDTD – Photonic Crystals / 2D-TM-FDTD – Photonische Kristalle

Wave field movie of the H_x field component / Wellenfeldfilm der H_x-Feldkomponente



Wave field movie of the H_z field component / Wellenfeldfilm der H_z-Feldkomponente



Wave field movie of the E_y field component / Wellenfeldfilm der E_y-Feldkomponente

End of the 12th Lecture / Ende der 12. Vorlesung