Elektromagnetische Feldtheorie I (EFT I) / Electromagnetic Field Theory I (EFT I)

2nd Lecture / 2. Vorlesung

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Governing Equations of Electromagnetic Fields and Waves / Grundgleichungen elektromagnetischer Felder und Wellen

Governing Equations in Integral Form / Grundgleichungen in Integralform

$$\oint_{C=\partial S} \underline{\mathbf{E}}(\underline{\mathbf{R}},t)\cdot \underline{\mathbf{dR}} = -\iint_S \frac{\partial}{\partial t}\underline{\mathbf{B}}(\underline{\mathbf{R}},t)\cdot\underline{\mathbf{dS}} - \iint_S \underline{\mathbf{J}}_\mathrm{m}(\underline{\mathbf{R}},t)\cdot\underline{\mathbf{dS}}$$

$$\oint_{C=\partial S} \underline{\mathbf{H}}(\underline{\mathbf{R}},t)\cdot \underline{\mathbf{dR}} = \iint_S \frac{\partial}{\partial t}\underline{\mathbf{D}}(\underline{\mathbf{R}},t)\cdot\underline{\mathbf{dS}} + \iint_S \underline{\mathbf{J}}_\mathrm{e}(\underline{\mathbf{R}},t)\cdot\underline{\mathbf{dS}}$$

$$\oiint_{S=\partial V} \underline{\mathbf{D}}(\underline{\mathbf{R}},t)\cdot\underline{\mathbf{dS}} = \iiint_V \rho_\mathrm{e}(\underline{\mathbf{R}},t)\,\mathrm{d}V$$

$$\oiint_{S=\partial V} \underline{\mathbf{B}}(\underline{\mathbf{R}},t)\cdot\underline{\mathbf{dS}} = \iiint_V \rho_\mathrm{m}(\underline{\mathbf{R}},t)\,\mathrm{d}V$$

$$\oiint_{S=\partial V} \underline{\mathbf{J}}_\mathrm{e}(\underline{\mathbf{R}},t)\cdot\underline{\mathbf{dS}} = -\iiint_V \frac{\partial}{\partial t}\rho_\mathrm{e}(\underline{\mathbf{R}},t)\,\mathrm{d}V$$

$$\oiint_{S=\partial V} \underline{\mathbf{J}}_\mathrm{m}(\underline{\mathbf{R}},t)\cdot\underline{\mathbf{dS}} = -\iiint_V \frac{\partial}{\partial t}\rho_\mathrm{m}(\underline{\mathbf{R}},t)\,\mathrm{d}V$$

Governing Equations in Differential Form / Grundgleichungen in Differentialform

$$\nabla\times\underline{\mathbf{E}}(\underline{\mathbf{R}},t) = -\frac{\partial}{\partial t}\underline{\mathbf{B}}(\underline{\mathbf{R}},t) - \underline{\mathbf{J}}_\mathrm{m}(\underline{\mathbf{R}},t)$$

$$\nabla\times\underline{\mathbf{H}}(\underline{\mathbf{R}},t) = \frac{\partial}{\partial t}\underline{\mathbf{D}}(\underline{\mathbf{R}},t) + \underline{\mathbf{J}}_\mathrm{e}(\underline{\mathbf{R}},t)$$

$$\nabla\cdot\underline{\mathbf{D}}(\underline{\mathbf{R}},t) = \rho_\mathrm{e}(\underline{\mathbf{R}},t)$$

$$\nabla\cdot\underline{\mathbf{B}}(\underline{\mathbf{R}},t) = \rho_\mathrm{m}(\underline{\mathbf{R}},t)$$

$$\nabla\cdot\underline{\mathbf{J}}_\mathrm{e}(\underline{\mathbf{R}},t) = -\frac{\partial}{\partial t}\rho_\mathrm{e}(\underline{\mathbf{R}},t)$$

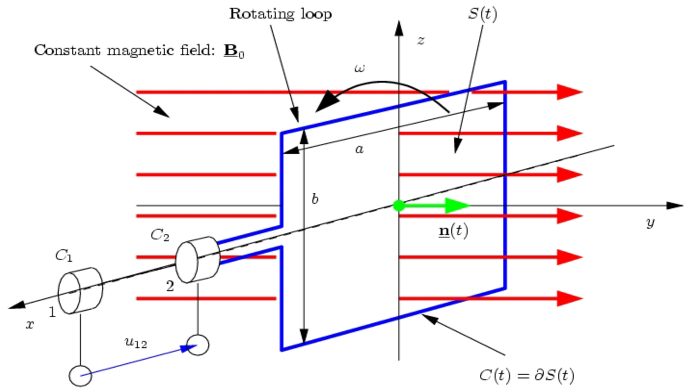
$$\nabla\cdot\underline{\mathbf{J}}_\mathrm{m}(\underline{\mathbf{R}},t) = -\frac{\partial}{\partial t}\rho_\mathrm{m}(\underline{\mathbf{R}},t)$$

Faraday's Induction Law in Integral Form / Faradaysches Induktionsgesetz in Integralform (1)

Faraday's Induction Law / Faradaysches Induktionsgesetz

$$\oint_{C(t)=\partial S(t)} \underline{\mathbf{E}}(\underline{\mathbf{R}},t)\bullet \underline{\mathbf{dR}} = -\frac{d}{dt}\iint_{S(t)} \underline{\mathbf{B}}(\underline{\mathbf{R}},t)\bullet \underline{\mathbf{dS}} - \iint_{S(t)} \underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},t)\bullet \underline{\mathbf{dS}}$$

Time Dependent Surface / Zeitabhängige Fläche $S(t) \quad C(t)=\partial S(t)$ **Time Dependent Contour / Zeitabhängige Kontur**



Faraday's Induction Law in Integral Form / Faradaysches Induktionsgesetz in Integralform (2)

Faraday's Induction Law / Faradaysches Induktionsgesetz

$$\oint_{C(t)=\partial S(t)} \underline{\mathbf{E}}(\underline{\mathbf{R}},t)\bullet \underline{\mathbf{dR}} = -\frac{d}{dt}\iint_{S(t)} \underline{\mathbf{B}}(\underline{\mathbf{R}},t)\bullet \underline{\mathbf{dS}} - \iint_{S(t)} \underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},t)\bullet \underline{\mathbf{dS}}$$

$\oint_{C(t)=\partial S(t)}[\circ]\bullet\underline{\mathbf{dR}}$	[m]	Closed Contour Integral / Geschlossenes Kurvenintegral
$\underline{\mathbf{E}}(\underline{\mathbf{R}},t)$	[V/m]	Electric Field Strength / Elektrische Feldstärke
$\underline{\mathbf{dR}}$	[m]	Vectorial Differential Line Element / Vektorielles differentielles Linienelement
$\underline{\mathbf{E}}(\underline{\mathbf{R}},t)\bullet\underline{\mathbf{dR}}$	[V]	Scalar Product of E and dR = tangential projection of E onto dR / Skalarprodukt von E auf dR = Tangentialprojektion von E auf dR

Vectorial Differential Line Element / Vektorielles differentielles Linienelement

$$\underline{\mathbf{dR}} = \underline{\mathbf{s}}\; dR$$

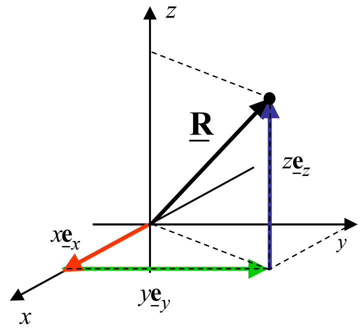
Tangential Unit Vector / Tangentialer Einheitsvektor

Scalar Differential Line Element / Skalares differentielles Linienelement

Position Vector / Ortsvektor

Cartesian Coordinate System / Kartesisches Koordinatensystem

$$\underline{\mathbf{R}} = \mathrm{R}_x(\underline{\mathbf{R}})\underline{\mathbf{e}}_x + \mathrm{R}_y(\underline{\mathbf{R}})\underline{\mathbf{e}}_y + \mathrm{R}_z(\underline{\mathbf{R}})\underline{\mathbf{e}}_z$$
$$= x\underline{\mathbf{e}}_x + y\underline{\mathbf{e}}_y + z\underline{\mathbf{e}}_z$$



Coordinates / Koordinaten x, y, z

Orthonormal Unit Vectors / Orthonormale Einheitsvektoren $\underline{\mathbf{e}}_x, \underline{\mathbf{e}}_y, \underline{\mathbf{e}}_z$

$$\underline{\mathbf{e}}_x \perp \underline{\mathbf{e}}_y \perp \underline{\mathbf{e}}_z \quad |\underline{\mathbf{e}}_x| = |\underline{\mathbf{e}}_y| = |\underline{\mathbf{e}}_z| = 1$$

Scalar Vector Components / Skalare Vektorkomponenten

$$R_x(x,y,z) = x$$
$$R_y(x,y,z) = y$$
$$R_z(x,y,z) = z$$

Vectorial Vector Components / Vektorielle Vektorkomponenten

$$\underline{\mathbf{R}}_x(\underline{\mathbf{R}}) = \mathrm{R}_x(x,y,z)\underline{\mathbf{e}}_x = x\underline{\mathbf{e}}_x$$
$$\underline{\mathbf{R}}_y(\underline{\mathbf{R}}) = \mathrm{R}_y(x,y,z)\underline{\mathbf{e}}_y = y\underline{\mathbf{e}}_y$$
$$\underline{\mathbf{R}}_z(\underline{\mathbf{R}}) = \mathrm{R}_z(x,y,z)\underline{\mathbf{e}}_z = z\underline{\mathbf{e}}_z$$

Electric Field Strength / Elektrische Feldstärke

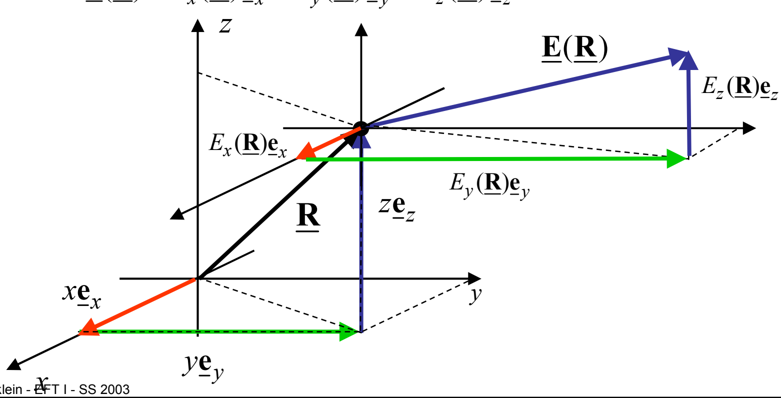
Cartesian Coordinate System / Kartesisches Koordinatensystem

Position Vector / Ortsvektor: $\underline{\mathbf{R}} = R_x\,\underline{\mathbf{e}}_x + R_y\,\underline{\mathbf{e}}_y + R_z\,\underline{\mathbf{e}}_z$

$$= x\,\underline{\mathbf{e}}_x + y\,\underline{\mathbf{e}}_y + z\,\underline{\mathbf{e}}_z$$
$$= x_1\,\underline{\mathbf{e}}_{x_1} + x_2\,\underline{\mathbf{e}}_{x_2} + x_3\,\underline{\mathbf{e}}_{x_3}$$

Electric Field Strength at the Position $\underline{\mathbf{R}}$ / Elektrische Feldstärke am Ort: $\underline{\mathbf{R}}$

$$\underline{\mathbf{E}}(\underline{\mathbf{R}}) = E_x(\underline{\mathbf{R}})\,\underline{\mathbf{e}}_x + E_y(\underline{\mathbf{R}})\,\underline{\mathbf{e}}_y + E_z(\underline{\mathbf{R}})\,\underline{\mathbf{e}}_z$$



Maxwell‘s Equations in Integral Form / Maxwellsche Gleichungen in Integralform

Faraday‘s Induction Law / Faradaysches Induktionsgesetz

$$\oint_{C(t)=\partial S(t)} \underline{\mathbf{E}}(\underline{\mathbf{R}},t)\bullet\underline{\mathbf{dR}} = -\frac{d}{dt}\iint_{S(t)} \underline{\mathbf{B}}(\underline{\mathbf{R}},t)\bullet\underline{\mathbf{dS}} - \iint_{S(t)} \underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},t)\bullet\underline{\mathbf{dS}}$$

Ampère-Maxwell‘s Circuital Law / Ampère-Maxwellsches Durchflutungsgesetz

$$\oint_{C(t)=\partial S(t)} \underline{\mathbf{H}}(\underline{\mathbf{R}},t)\bullet\underline{\mathbf{dR}} = \frac{d}{dt}\iint_{S(t)} \underline{\mathbf{D}}(\underline{\mathbf{R}},t)\bullet\underline{\mathbf{dS}} + \iint_{S(t)} \underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}},t)\bullet\underline{\mathbf{dS}}$$

Gauss‘ Magnetic Law / Gaußsches magnetisches Gesetz

$$\oiint_{S(t)=\partial V(t)} \underline{\mathbf{B}}(\underline{\mathbf{R}},t)\bullet\underline{\mathbf{dS}} = \iiint_{V(t)} \rho_{\mathrm{m}}(\underline{\mathbf{R}},t)\,\mathrm{d}V$$

Gauss‘ Electric Law / Gaußsches elektrisches Gesetz

$$\oiint_{S(t)=\partial V(t)} \underline{\mathbf{D}}(\underline{\mathbf{R}},t)\bullet\underline{\mathbf{dS}} = \iiint_{V(t)} \rho_{\mathrm{e}}(\underline{\mathbf{R}},t)\,\mathrm{d}V$$

Continuity Equations in Integral Form / Kontinuitätsgleichungen in Integralform

Continuity (Conservation) Equation for Electric Charges in Integral Form / Kontinuitätsgleichung (Erhaltungsgleichung) der elektrischen Ladungen in Integralform

$$\oiint_{S=\partial V} \underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}},t)\bullet\underline{\mathbf{dS}} = -\frac{d}{dt}\iiint_{V} \rho_{\mathrm{e}}(\underline{\mathbf{R}},t)\,\mathrm{d}V$$

Continuity (Conservation) Equation for Magnetic Charges in Integral Form / Kontinuitätsgleichung (Erhaltungsgleichung) für magnetische Ladungen in Integralform

$$\oiint_{S=\partial V} \underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},t)\bullet\underline{\mathbf{dS}} = -\frac{d}{dt}\iiint_{V} \rho_{\mathrm{m}}(\underline{\mathbf{R}},t)\,\mathrm{d}V$$

A negative time variation of charge, which means a decrease of charge in the volume V is equal to the total flux of current through the closed surface S.

Eine negative zeitliche Änderung der Ladung, d. h. eine Abnahme der Ladung im Volumen V, entspricht dem Gesamtfluss des Stromes durch die geschlossene Oberfläche S des Volumens.

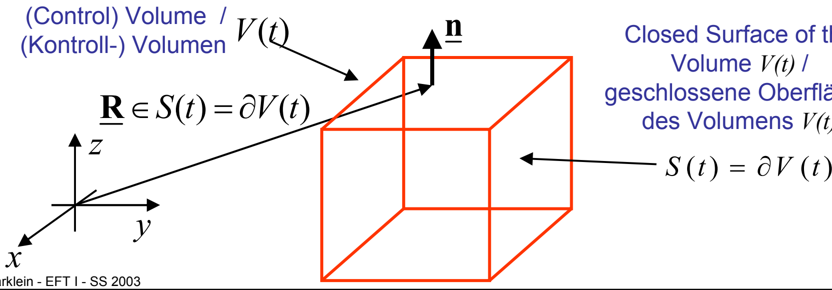
Maxwell‘s Equations in Integral Form / Maxwellsche Gleichungen in Integralform

Gauss‘ Electric Law / Gaußsches elektrisches Gesetz

$$\oiint_{S(t)=\partial V(t)} \underline{\mathbf{D}}(\underline{\mathbf{R}},t)\bullet \underline{\mathbf{dS}} = \iiint_{V(t)} \rho_{\mathrm{e}}(\underline{\mathbf{R}},t)\,\mathrm{d}V$$

Time Dependent Volume / Zeitabhängiges Volumen $V(t)$ $S(t) = \partial V(t)$ **Time Dependent Surface / Zeitabhängige Fläche**

$$\underline{\mathbf{dS}} = \underline{\mathbf{n}}\,\mathrm{d}S$$



Maxwell‘s Equations in Integral Form / Maxwellsche Gleichungen in Integralform

Faraday‘s Induction Law / Faradaysches Induktionsgesetz

$$\oint_{C(t)=\partial S(t)} \underline{\mathbf{E}}(\underline{\mathbf{R}},t)\bullet \underline{\mathbf{dR}} = -\frac{d}{dt}\iint_{S(t)} \underline{\mathbf{B}}(\underline{\mathbf{R}},t)\bullet \underline{\mathbf{dS}} - \iint_{S(t)} \underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},t)\bullet \underline{\mathbf{dS}}$$

Time Dependent Surface / Zeitabhängige Fläche $S(t)$ $C(t) = \partial S(t)$ **Time Dependent Contour / Zeitabhängige Kontur**

Surface / Fläche S $C = \partial S$ **Contour / Kontur**

$$\oint_{C=\partial S} \underline{\mathbf{E}}(\underline{\mathbf{R}},t)\bullet \underline{\mathbf{dR}} = -\frac{d}{dt}\iint_{S} \underline{\mathbf{B}}(\underline{\mathbf{R}},t)\bullet \underline{\mathbf{dS}} - \iint_{S} \underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},t)\bullet \underline{\mathbf{dS}}$$

$$\oint_{C=\partial S} \underline{\mathbf{E}}(\underline{\mathbf{R}},t)\bullet \underline{\mathbf{dR}} = -\iint_{S} \frac{\partial}{\partial t}\underline{\mathbf{B}}(\underline{\mathbf{R}},t)\bullet \underline{\mathbf{dS}} - \iint_{S} \underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},t)\bullet \underline{\mathbf{dS}}$$

Maxwell‘s Equations in Integral Form / Maxwellsche Gleichungen in Integralform

Faraday‘s Induction Law / Faradaysches Induktionsgesetz

$$\oint_{C=\partial S} \underline{\mathbf{E}}(\underline{\mathbf{R}},t)\bullet\underline{\mathbf{dR}} = -\iint_S \frac{\partial}{\partial t}\underline{\mathbf{B}}(\underline{\mathbf{R}},t)\bullet\underline{\mathbf{dS}} - \iint_S \underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},t)\bullet\underline{\mathbf{dS}}$$

Ampère-Maxwell‘s Circuital Law / Ampère-Maxwellsches Durchflutungsgesetz

$$\oint_{C=\partial S} \underline{\mathbf{H}}(\underline{\mathbf{R}},t)\bullet\underline{\mathbf{dR}} = \iint_S \frac{\partial}{\partial t}\underline{\mathbf{D}}(\underline{\mathbf{R}},t)\bullet\underline{\mathbf{dS}} + \iint_S \underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}},t)\bullet\underline{\mathbf{dS}}$$

Gauss‘ Electric Law / Gaußsches elektrisches Gesetz

$$\oiint_{S=\partial V} \underline{\mathbf{D}}(\underline{\mathbf{R}},t)\bullet\underline{\mathbf{dS}} = \iiint_V \rho_{\mathrm{e}}(\underline{\mathbf{R}},t)\,\mathrm{d}V$$

Gauss‘ Magnetic Law / Gaußsches magnetisches Gesetz

$$\oiint_{S=\partial V} \underline{\mathbf{B}}(\underline{\mathbf{R}},t)\bullet\underline{\mathbf{dS}} = \iiint_V \rho_{\mathrm{m}}(\underline{\mathbf{R}},t)\,\mathrm{d}V$$

Continuity Equation in Integral Form / Kontinuitätsgleichungen in Integralform

Continuity (Conservation) Equation for Electric Charges in Integral Form / Kontinuitätsgleichung (Erhaltungsgleichung) der elektrischen Ladungen in Integralform

$$\oiint_{S=\partial V} \underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}},t)\bullet\underline{\mathbf{dS}} = -\iiint_V \frac{\partial}{\partial t}\rho_{\mathrm{e}}(\underline{\mathbf{R}},t)\,\mathrm{d}V$$

Continuity (Conservation) Equation for Magnetic Charges in Integral Form / Kontinuitätsgleichung (Erhaltungsgleichung) für magnetische Ladungen in Integralform

$$\oiint_{S=\partial V} \underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},t)\bullet\underline{\mathbf{dS}} = -\iiint_V \frac{\partial}{\partial t}\rho_{\mathrm{m}}(\underline{\mathbf{R}},t)\,\mathrm{d}V$$

A negative time variation of charge, which means a decrease of charge in the volume V is equal to the total flux through the closed surface S.

Eine negative zeitliche Änderung der Ladung, d. h. eine Abnahme der Ladung im Volumen V, entspricht dem Gesamtfluss durch die geschlossene Oberfläche S des Volumens.

Governing Equations in Integral Form / Grundgleichungen in Integralform

Faraday's Induction Law / Faradaysches Induktionsgesetz

$$\oint_{C=\partial S} \underline{\mathbf{E}}(\underline{\mathbf{R}},t)\bullet \underline{\mathbf{dR}} = -\iint_S \frac{\partial}{\partial t}\underline{\mathbf{B}}(\underline{\mathbf{R}},t)\bullet \underline{\mathbf{dS}} - \iint_S \underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},t)\bullet \underline{\mathbf{dS}}$$

Ampère-Maxwell's Circuital Law / Ampère-Maxwellsches Durchflutungsgesetz

$$\oint_{C=\partial S} \underline{\mathbf{H}}(\underline{\mathbf{R}},t)\bullet \underline{\mathbf{dR}} = \iint_S \frac{\partial}{\partial t}\underline{\mathbf{D}}(\underline{\mathbf{R}},t)\bullet \underline{\mathbf{dS}} + \iint_S \underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}},t)\bullet \underline{\mathbf{dS}}$$

Gauss' Electric Law / Gaußsches elektrisches Gesetz

$$\oiint_{S=\partial V} \underline{\mathbf{D}}(\underline{\mathbf{R}},t)\bullet \underline{\mathbf{dS}} = \iiint_V \rho_{\mathrm{e}}(\underline{\mathbf{R}},t)\,\mathrm{d}V$$

Gauss' Magnetic Law / Gaußsches magnetisches Gesetz

$$\oiint_{S=\partial V} \underline{\mathbf{B}}(\underline{\mathbf{R}},t)\bullet \underline{\mathbf{dS}} = \iiint_V \rho_{\mathrm{m}}(\underline{\mathbf{R}},t)\,\mathrm{d}V$$

Continuity Equation for Electric Charges / Kontinuitätsgleichung für die elektrischer Ladungen

$$\oiint_{S=\partial V} \underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}},t)\bullet \underline{\mathbf{dS}} = -\iiint_V \frac{\partial}{\partial t}\rho_{\mathrm{e}}(\underline{\mathbf{R}},t)\,\mathrm{d}V$$

Continuity Equation for Magnetic Charges / Kontinuitätsgleichung für die magnetischer Ladungen

$$\oiint_{S=\partial V} \underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},t)\bullet \underline{\mathbf{dS}} = -\iiint_V \frac{\partial}{\partial t}\rho_{\mathrm{m}}(\underline{\mathbf{R}},t)\,\mathrm{d}V$$

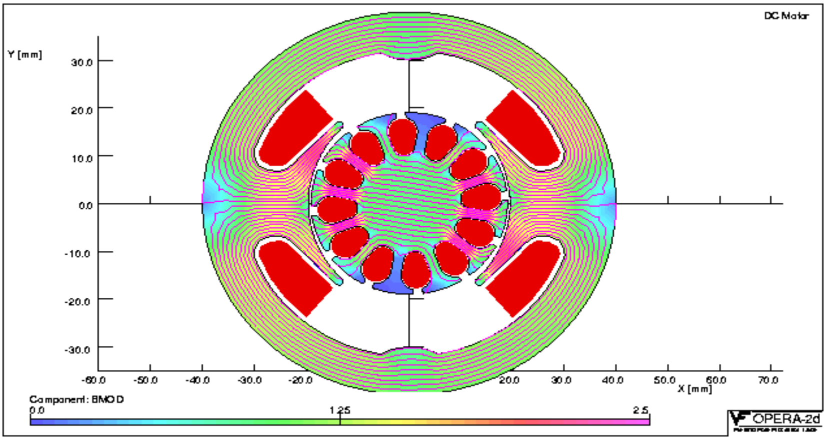
Different Coordinate Systems / Verschiedene Koordinatensysteme

- **Cartesian (Rectangular) Coordinate System / Kartesisches Koordinatensystem**
- **Cylindrical Coordinate System / Zylinderkoordinatensystem**
- **Spherical Coordinate System / Kugelkoordinatensystem**

What is the benefit of Different Coordinate Systems ? / Was ist der Nutzen von verschiedenen Koordinatensystemen ?

Example: Problem Matched Coordinate System / Beispiel: Problemangepasstes Koordinatensystem

Rotating DC-Motor / Rotierender Gleichspannungsmotor



http://helium.ee.tut.fi/Computational_Electromagnetics_files/rot_motor.html
http://helium.ee.tut.fi/p_research_ce.htm

Coordinate Systems / Koordinatensysteme

Cartesian Coordinate System / Kartesisches Koordinatensystem

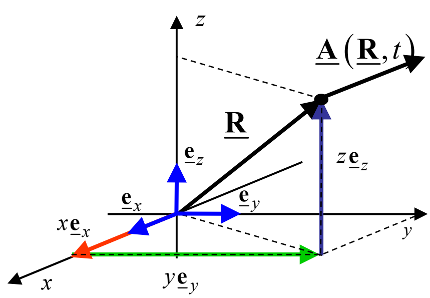
Coordinates / Koordinaten x, y, z

Limits / Grenzen

$-\infty < x < \infty$

$-\infty < y < \infty$

$-\infty < z < \infty$



Orthonormal Unit Vectors / Orthonormale Einheitsvektoren

$\underline{\mathbf{e}}_x, \underline{\mathbf{e}}_y, \underline{\mathbf{e}}_z$

$\underline{\mathbf{e}}_x \perp \underline{\mathbf{e}}_y \perp \underline{\mathbf{e}}_z$

$|\underline{\mathbf{e}}_x| = |\underline{\mathbf{e}}_y| = |\underline{\mathbf{e}}_z| = 1$

$\perp$: Perpendicular / Senkrecht

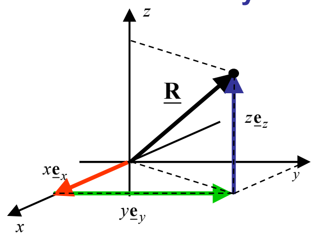
Arbitrary Vector Field / Beliebiges Vektorfeld

$$\underline{\mathbf{A}}(\underline{\mathbf{R}},t) = \underline{\mathbf{A}}_x(\underline{\mathbf{R}},t) + \underline{\mathbf{A}}_y(\underline{\mathbf{R}},t) + \underline{\mathbf{A}}_z(\underline{\mathbf{R}},t)$$
$$= \mathrm{A}_x(x,y,z,t)\underline{\mathbf{e}}_x + A_y(x,y,z,t)\underline{\mathbf{e}}_y + A_z(x,y,z,t)\underline{\mathbf{e}}_z$$

Position Vector / Ortsvektor (Positionsvektor)

Cartesian Coordinate System / Kartesisches Koordinatensystem

$$\underline{\mathbf{R}} = \underline{\mathbf{R}}_x(\underline{\mathbf{R}}) + \underline{\mathbf{R}}_y(\underline{\mathbf{R}}) + \underline{\mathbf{R}}_z(\underline{\mathbf{R}})$$
$$= R_x(\underline{\mathbf{R}})\underline{\mathbf{e}}_x + R_y(\underline{\mathbf{R}})\underline{\mathbf{e}}_y + R_z(\underline{\mathbf{R}})\underline{\mathbf{e}}_z$$
$$= x\underline{\mathbf{e}}_x + y\underline{\mathbf{e}}_y + z\underline{\mathbf{e}}_z$$



Coordinates / Koordinaten $x, y, z; \quad -\infty < x, y, z < \infty$

Orthonormal Unit Vectors / Orthonormale Einheitsvektoren $\underline{\mathbf{e}}_x, \underline{\mathbf{e}}_y, \underline{\mathbf{e}}_z$

$$\underline{\mathbf{e}}_x \perp \underline{\mathbf{e}}_y \perp \underline{\mathbf{e}}_z \quad |\underline{\mathbf{e}}_x| = |\underline{\mathbf{e}}_y| = |\underline{\mathbf{e}}_z| = 1$$

Scalar Vector Components / Skalare Vektorkomponenten

$$R_x(x,y,z) = x$$
$$R_y(x,y,z) = y$$
$$R_z(x,y,z) = z$$

Vectorial Vector Components / Vektorielle Vektorkomponenten

$$\underline{\mathbf{R}}_x(\underline{\mathbf{R}}) = R_x(x,y,z)\underline{\mathbf{e}}_x = x\underline{\mathbf{e}}_x$$
$$\underline{\mathbf{R}}_y(\underline{\mathbf{R}}) = R_y(x,y,z)\underline{\mathbf{e}}_y = y\underline{\mathbf{e}}_y$$
$$\underline{\mathbf{R}}_z(\underline{\mathbf{R}}) = R_z(x,y,z)\underline{\mathbf{e}}_z = z\underline{\mathbf{e}}_z$$

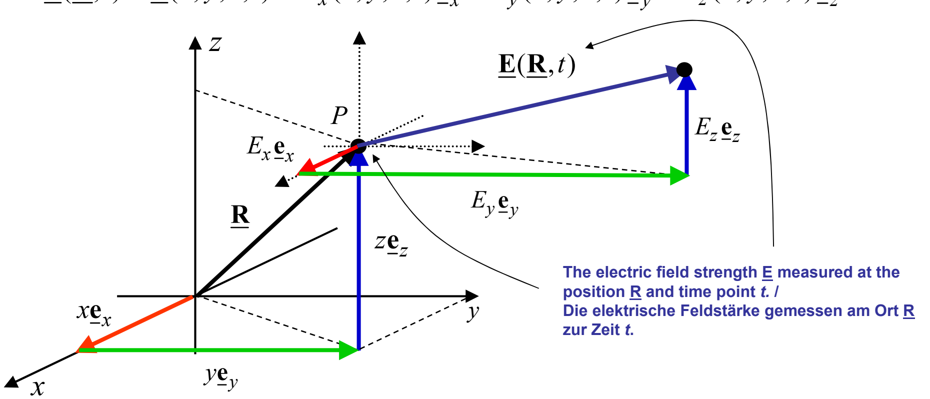
Position Vector and Electric Field Strength Vector / Ortsvektor und elektrischer Feldstärkevektor

Cartesian Coordinate System / Kartesisches Koordinatensystem

$$\underline{\mathbf{R}}(x,y,z) = R_x(x,y,z)\underline{\mathbf{e}}_x + R_y(x,y,z)\underline{\mathbf{e}}_y + R_z(x,y,z)\underline{\mathbf{e}}_z$$
$$= x\underline{\mathbf{e}}_x + y\underline{\mathbf{e}}_y + z\underline{\mathbf{e}}_z$$

Electric Field Strength at the Position: $\underline{\mathbf{R}}$ / Elektrische Feldstärke am Ort: $\underline{\mathbf{R}}$

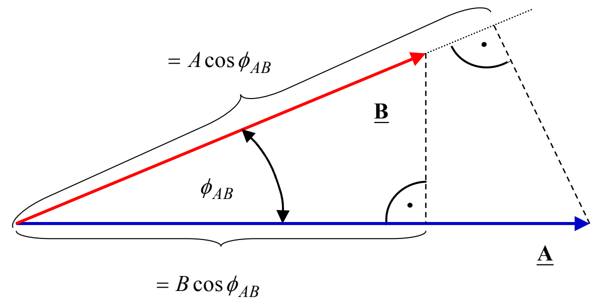
$$\underline{\mathbf{E}}(\underline{\mathbf{R}},t) = \underline{\mathbf{E}}(x,y,z,t) = E_x(x,y,z,t)\underline{\mathbf{e}}_x + E_y(x,y,z,t)\underline{\mathbf{e}}_y + E_z(x,y,z,t)\underline{\mathbf{e}}_z$$



Scalar Product (Dot or Inner Product) / Skalarprodukt (Punktprodukt oder inneres Produkt) (1)

$$\underline{\mathbf{A}} \cdot \underline{\mathbf{B}} = |\underline{\mathbf{A}}||\underline{\mathbf{B}}| \cos \underbrace{\angle(\underline{\mathbf{A}}, \underline{\mathbf{B}})}_{\phi_{AB}}$$

$$= AB \cos \phi_{AB}$$



Enclosed Angle / Eingeschlossener Winkel ϕ_{AB}

$$\underline{\mathbf{A}} \cdot \underline{\mathbf{B}} = \underline{\mathbf{B}} \cdot \underline{\mathbf{A}}$$

$$= BA \cos \phi_{BA}$$

$$= AB \cos \phi_{AB}$$

$$\cos(\phi_{AB}) = \cos(-\phi_{AB})$$

$$\cos \phi_{AB} = \frac{\underline{\mathbf{A}} \cdot \underline{\mathbf{B}}}{|\underline{\mathbf{A}}||\underline{\mathbf{B}}|}$$

$$\phi_{AB} = \arccos\left(\frac{\underline{\mathbf{A}} \cdot \underline{\mathbf{B}}}{|\underline{\mathbf{A}}||\underline{\mathbf{B}}|}\right)$$

Scalar Product (Dot or Inner Product) / Skalarprodukt (Punktprodukt oder inneres Produkt) (2)

$$\underline{\mathbf{A}} \cdot \underline{\mathbf{B}} = (A_x \underline{\mathbf{e}}_x + A_y \underline{\mathbf{e}}_y + A_z \underline{\mathbf{e}}_z) \cdot (B_x \underline{\mathbf{e}}_x + B_y \underline{\mathbf{e}}_y + B_z \underline{\mathbf{e}}_z)$$

$$= A_x B_x \underbrace{\underline{\mathbf{e}}_x \cdot \underline{\mathbf{e}}_x}_{=1} + A_x B_y \underbrace{\underline{\mathbf{e}}_x \cdot \underline{\mathbf{e}}_y}_{=0} + A_x B_z \underbrace{\underline{\mathbf{e}}_x \cdot \underline{\mathbf{e}}_z}_{=0}$$

$$+ A_y B_x \underbrace{\underline{\mathbf{e}}_y \cdot \underline{\mathbf{e}}_x}_{=0} + A_y B_y \underbrace{\underline{\mathbf{e}}_y \cdot \underline{\mathbf{e}}_y}_{=1} + A_y B_z \underbrace{\underline{\mathbf{e}}_y \cdot \underline{\mathbf{e}}_z}_{=0}$$

$$+ A_z B_x \underbrace{\underline{\mathbf{e}}_z \cdot \underline{\mathbf{e}}_x}_{=0} + A_z B_y \underbrace{\underline{\mathbf{e}}_z \cdot \underline{\mathbf{e}}_y}_{=0} + A_z B_z \underbrace{\underline{\mathbf{e}}_z \cdot \underline{\mathbf{e}}_z}_{=1}$$

$$= A_x B_x + A_y B_y + A_z B_z$$

Orthonormal Unit Vectors / Orthonormale Einheitsvektoren

$$\underline{\mathbf{e}}_x \perp \underline{\mathbf{e}}_y \perp \underline{\mathbf{e}}_z$$

$$\begin{array}{lll} \underline{\mathbf{e}}_x \cdot \underline{\mathbf{e}}_x = 1 & \underline{\mathbf{e}}_y \cdot \underline{\mathbf{e}}_x = 0 & \underline{\mathbf{e}}_z \cdot \underline{\mathbf{e}}_x = 0 \\ \underline{\mathbf{e}}_x \cdot \underline{\mathbf{e}}_y = 0 & \underline{\mathbf{e}}_y \cdot \underline{\mathbf{e}}_y = 1 & \underline{\mathbf{e}}_z \cdot \underline{\mathbf{e}}_y = 0 \\ \underline{\mathbf{e}}_x \cdot \underline{\mathbf{e}}_z = 0 & \underline{\mathbf{e}}_y \cdot \underline{\mathbf{e}}_z = 0 & \underline{\mathbf{e}}_z \cdot \underline{\mathbf{e}}_z = 1 \end{array}$$

$$\underline{\mathbf{A}} \cdot \underline{\mathbf{B}} = (A_x \underline{\mathbf{e}}_x + A_y \underline{\mathbf{e}}_y + A_z \underline{\mathbf{e}}_z) \cdot (B_x \underline{\mathbf{e}}_x + B_y \underline{\mathbf{e}}_y + B_z \underline{\mathbf{e}}_z)$$

$$= A_x B_x + A_y B_y + A_z B_z$$

$$= (A_{x_1} \underline{\mathbf{e}}_{x_1} + A_{x_2} \underline{\mathbf{e}}_{x_2} + A_{x_3} \underline{\mathbf{e}}_{x_3}) \cdot (B_{x_1} \underline{\mathbf{e}}_{x_1} + B_{x_2} \underline{\mathbf{e}}_{x_2} + B_{x_3} \underline{\mathbf{e}}_{x_3})$$

$$= A_{x_1} B_{x_1} + A_{x_2} B_{x_2} + A_{x_3} B_{x_3}$$

$$= \sum_{i=1}^{3} A_{x_i} B_{x_i}$$

Cartesian Coordinates / Kartesische Koordinaten

$$x = x_1$$
$$y = x_2$$
$$z = x_3$$

Scalar Product (Dot or Inner Product) / Skalarprodukt (Punktprodukt oder inneres Produkt) (3)

$$\underline{\mathbf{A}} \bullet \underline{\mathbf{B}} = (A_x \underline{\mathbf{e}}_x + A_y \underline{\mathbf{e}}_y + A_z \underline{\mathbf{e}}_z) \bullet (B_x \underline{\mathbf{e}}_x + B_y \underline{\mathbf{e}}_y + B_z \underline{\mathbf{e}}_z)$$
$$= \sum_{i=1}^{3} A_{x_i} \underline{\mathbf{e}}_{x_i} \bullet \sum_{j=1}^{3} B_{x_j} \underline{\mathbf{e}}_{x_j}$$
$$= \sum_{i=1}^{3} \sum_{j=1}^{3} A_{x_i} \underline{\mathbf{e}}_{x_i} \bullet B_{x_j} \underline{\mathbf{e}}_{x_j}$$
$$= \sum_{i=1}^{3} \sum_{j=1}^{3} A_{x_i} B_{x_j} \underbrace{\underline{\mathbf{e}}_{x_i} \bullet \underline{\mathbf{e}}_{x_j}}_{=\delta_{ij}}$$
$$= A_{x_i} B_{x_j} \underbrace{\underline{\mathbf{e}}_{x_i} \bullet \underline{\mathbf{e}}_{x_j}}_{=\delta_{ij}}$$
$$= A_{x_i} \underbrace{B_{x_j} \delta_{ij}}_{=B_{x_i}} \quad \left(\text{or/oder} \quad \underbrace{\underbrace{A_{x_i} \delta_{ij}}_{=A_{x_j}} B_{x_j}}_{=A_{x_j} B_{x_j}} \right)$$
$$= A_{x_i} B_{x_i}$$

Kronecker Delta / Kronecker-Delta

$$\delta_{ij} = \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases}$$

with Einstein's Summation Convention / mit Einsteinscher Summationskonvention

***Einstein's Summation Convention*: If a index appears two times at one side of an equation (and not at the other side), the index is automatically summed over 1 to 3. / *Einsteinsche Summenkonvention*: Wenn ein Index auf einer Seite einer Gleichung zweimal vorkommt (und auf der anderen nicht), wird darüber von 1 bis 3 summiert.**

Magnitude of a Vector / Betrag eines Vektors

$$|\underline{\mathbf{A}}| = \sqrt{\underline{\mathbf{A}} \bullet \underline{\mathbf{A}}}$$
$$= \sqrt{(\mathrm{A}_x \underline{\mathbf{e}}_x + \mathrm{A}_y \underline{\mathbf{e}}_y + \mathrm{A}_z \underline{\mathbf{e}}_z) \bullet (\mathrm{A}_x \underline{\mathbf{e}}_x + \mathrm{A}_y \underline{\mathbf{e}}_y + \mathrm{A}_z \underline{\mathbf{e}}_z)}$$
$$= \Bigg(\mathrm{A}_x \mathrm{A}_x \underbrace{\underline{\mathbf{e}}_x \bullet \underline{\mathbf{e}}_x}_{=1} + \mathrm{A}_x \mathrm{A}_y \underbrace{\underline{\mathbf{e}}_x \bullet \underline{\mathbf{e}}_y}_{=0} + \mathrm{A}_x \mathrm{A}_z \underbrace{\underline{\mathbf{e}}_x \bullet \underline{\mathbf{e}}_z}_{=0}$$
$$+ \mathrm{A}_y \mathrm{A}_x \underbrace{\underline{\mathbf{e}}_y \bullet \underline{\mathbf{e}}_x}_{=0} + \mathrm{A}_y \mathrm{A}_y \underbrace{\underline{\mathbf{e}}_y \bullet \underline{\mathbf{e}}_y}_{=1} + \mathrm{A}_y \mathrm{A}_z \underbrace{\underline{\mathbf{e}}_y \bullet \underline{\mathbf{e}}_z}_{=0}$$
$$+ \mathrm{A}_z \mathrm{A}_x \underbrace{\underline{\mathbf{e}}_z \bullet \underline{\mathbf{e}}_x}_{=0} + \mathrm{A}_z \mathrm{A}_y \underbrace{\underline{\mathbf{e}}_z \bullet \underline{\mathbf{e}}_y}_{=0} + \mathrm{A}_z \mathrm{A}_z \underbrace{\underline{\mathbf{e}}_z \bullet \underline{\mathbf{e}}_z}_{=1} \Bigg)^{\frac{1}{2}}$$
$$= \sqrt{\mathrm{A}_x \mathrm{A}_x + \mathrm{A}_y \mathrm{A}_y + \mathrm{A}_z \mathrm{A}_z}$$
$$= \sqrt{\mathrm{A}_x^2 + \mathrm{A}_y^2 + \mathrm{A}_z^2}$$
$$= \mathrm{A}$$

$$|\underline{\mathbf{A}}| = \sqrt{\underline{\mathbf{A}} \bullet \underline{\mathbf{A}}}$$
$$= \sqrt{\sum_{i=1}^{3} A_{x_i} \underline{\mathbf{e}}_{x_i} \bullet \sum_{j=1}^{3} B_{x_j} \underline{\mathbf{e}}_{x_j}}$$
$$= \sqrt{A_{x_i} \underline{\mathbf{e}}_{x_i} \bullet A_{x_j} \underline{\mathbf{e}}_{x_j}}$$
$$= \sqrt{A_{x_i} A_{x_j} \underbrace{\underline{\mathbf{e}}_{x_i} \bullet \underline{\mathbf{e}}_{x_j}}_{=\delta_{ij}}}$$
$$= \sqrt{A_{x_i}^2}$$

Example: Position Vector and Electric Field Strength Vector / Beispiel: Ortsvektor und elektrischer Feldstärkevektor

Cartesian Coordinate System / Kartesisches Koordinatensystem

Position Vector / Ortsvektor

$$\underline{\mathbf{R}}(x,y,z) = \mathrm{R}_x(x,y,z)\,\underline{\mathbf{e}}_x + \mathrm{R}_y(x,y,z)\,\underline{\mathbf{e}}_y + \mathrm{R}_z(x,y,z)\,\underline{\mathbf{e}}_z = x\,\underline{\mathbf{e}}_x + y\,\underline{\mathbf{e}}_y + z\,\underline{\mathbf{e}}_z$$

Electric Field Strength Vector / Elektrische Feldstärkevektor

$$\underline{\mathbf{E}}(\underline{\mathbf{R}},t) = \underline{\mathbf{E}}(x,y,z,t) = \mathrm{E}_x(x,y,z,t)\,\underline{\mathbf{e}}_x + \mathrm{E}_y(x,y,z,t)\,\underline{\mathbf{e}}_y + \mathrm{E}_z(x,y,z,t)\,\underline{\mathbf{e}}_z$$

Magnitude of the Position Vector (Distance) / Betrag des Ortsvektor (Abstand)

$$\left|\underline{\mathbf{R}}(x,y,z)\right| = \sqrt{\underline{\mathbf{R}}(x,y,z)\bullet\underline{\mathbf{R}}(x,y,z)} = \sqrt{\left(x\,\underline{\mathbf{e}}_x + y\,\underline{\mathbf{e}}_y + z\,\underline{\mathbf{e}}_z\right)\bullet\left(x\,\underline{\mathbf{e}}_x + y\,\underline{\mathbf{e}}_y + z\,\underline{\mathbf{e}}_z\right)} = \sqrt{x^2+y^2+z^2}$$

Magnitude of the Electric Field Strength Vector (Strength) / Betrag des elektrische Feldstärkevektors (Stärke)

$$\left|\underline{\mathbf{E}}(x,y,z)\right| = \sqrt{\underline{\mathbf{E}}(x,y,z)\bullet\underline{\mathbf{E}}(x,y,z)} = \sqrt{\left(\mathrm{E}_x\,\underline{\mathbf{e}}_x + \mathrm{E}_y\,\underline{\mathbf{e}}_y + \mathrm{E}_z\,\underline{\mathbf{e}}_z\right)\bullet\left(\mathrm{E}_x\,\underline{\mathbf{e}}_x + \mathrm{E}_y\,\underline{\mathbf{e}}_y + \mathrm{E}_z\,\underline{\mathbf{e}}_z\right)} = \sqrt{\mathrm{E}_x^2+\mathrm{E}_y^2+\mathrm{E}_z^2}$$

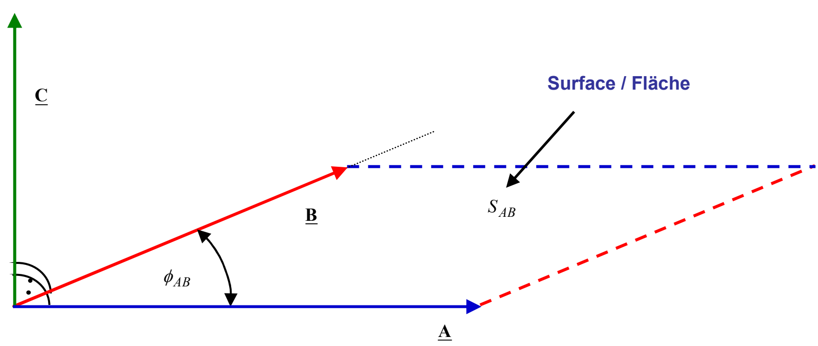
Position Unit Vector (Direction) / Ortseinheitsvektor (Richtung)

$$\hat{\underline{\mathbf{R}}}(x,y,z) = \frac{\underline{\mathbf{R}}(x,y,z)}{\left|\underline{\mathbf{R}}(x,y,z)\right|} = \frac{x\,\underline{\mathbf{e}}_x + y\,\underline{\mathbf{e}}_y + z\,\underline{\mathbf{e}}_z}{\sqrt{x^2+y^2+z^2}}$$

Electric Field Strength Unit Vector (Direction) / Elektrische Feldstärkeeinheitsvektor (Richtung)

$$\hat{\underline{\mathbf{E}}}(x,y,z) = \frac{\underline{\mathbf{E}}(x,y,z)}{\left|\underline{\mathbf{E}}(x,y,z)\right|} = \frac{\mathrm{E}_x\,\underline{\mathbf{e}}_x + \mathrm{E}_y\,\underline{\mathbf{e}}_y + \mathrm{E}_z\,\underline{\mathbf{e}}_z}{\sqrt{\mathrm{E}_x^2+\mathrm{E}_y^2+\mathrm{E}_z^2}}$$

Vector Product (Cross or Outer Product) / Vektorprodukt (Kreuzprodukt oder äußeres Produkt) (1)



$$\underline{\mathbf{C}} = \underline{\mathbf{A}} \times \underline{\mathbf{B}}$$

$$C = \left|\underline{\mathbf{A}}\right|\left|\underline{\mathbf{B}}\right| \sin \underbrace{\angle(\underline{\mathbf{A}},\underline{\mathbf{B}})}_{\phi_{AB}} = AB\sin\phi_{AB} = S_{AB}$$

$\underline{\mathbf{C}} \perp \underline{\mathbf{A}}$ and / und $\underline{\mathbf{C}} \perp \underline{\mathbf{B}}$

Vector Product (Cross or Outer Product) / Vektorprodukt (Kreuzprodukt oder äußeres Produkt) (2)

Orthonormal Unit Vectors / Orthonormale Einheitsvektoren

$$\underline{\mathbf{e}}_x \perp \underline{\mathbf{e}}_y \perp \underline{\mathbf{e}}_z$$

$$\underline{\mathbf{e}}_x \times \underline{\mathbf{e}}_x = \underline{\mathbf{0}}, \quad \underline{\mathbf{e}}_x \times \underline{\mathbf{e}}_y = \underline{\mathbf{e}}_z, \quad \underline{\mathbf{e}}_x \times \underline{\mathbf{e}}_z = -\underline{\mathbf{e}}_y$$

$$\underline{\mathbf{e}}_y \times \underline{\mathbf{e}}_x = -\underline{\mathbf{e}}_z, \quad \underline{\mathbf{e}}_y \times \underline{\mathbf{e}}_y = \underline{\mathbf{0}}, \quad \underline{\mathbf{e}}_y \times \underline{\mathbf{e}}_z = \underline{\mathbf{e}}_x$$

$$\underline{\mathbf{e}}_z \times \underline{\mathbf{e}}_x = \underline{\mathbf{e}}_y, \quad \underline{\mathbf{e}}_z \times \underline{\mathbf{e}}_y = -\underline{\mathbf{e}}_x, \quad \underline{\mathbf{e}}_z \times \underline{\mathbf{e}}_z = \underline{\mathbf{0}}$$

$$\begin{aligned}
\underline{\mathbf{A}} \times \underline{\mathbf{B}} &= (A_x \underline{\mathbf{e}}_x + A_y \underline{\mathbf{e}}_y + A_z \underline{\mathbf{e}}_z) \times (B_x \underline{\mathbf{e}}_x + B_y \underline{\mathbf{e}}_y + B_z \underline{\mathbf{e}}_z) \\
&= A_x B_x \underbrace{\underline{\mathbf{e}}_x \times \underline{\mathbf{e}}_x}_{=0} + A_x B_y \underbrace{\underline{\mathbf{e}}_x \times \underline{\mathbf{e}}_y}_{=\underline{\mathbf{e}}_z} + A_x B_z \underbrace{\underline{\mathbf{e}}_x \times \underline{\mathbf{e}}_z}_{=-\underline{\mathbf{e}}_y} \\
&\quad + A_y B_x \underbrace{\underline{\mathbf{e}}_y \times \underline{\mathbf{e}}_x}_{=-\underline{\mathbf{e}}_z} + A_y B_y \underbrace{\underline{\mathbf{e}}_y \times \underline{\mathbf{e}}_y}_{=0} + A_y B_z \underbrace{\underline{\mathbf{e}}_y \times \underline{\mathbf{e}}_z}_{=\underline{\mathbf{e}}_x} \\
&\quad + A_z B_x \underbrace{\underline{\mathbf{e}}_z \times \underline{\mathbf{e}}_x}_{=\underline{\mathbf{e}}_y} + A_z B_y \underbrace{\underline{\mathbf{e}}_z \times \underline{\mathbf{e}}_y}_{=-\underline{\mathbf{e}}_x} + A_z B_z \underbrace{\underline{\mathbf{e}}_z \times \underline{\mathbf{e}}_z}_{=0} \\
&= (A_y B_z \underline{\mathbf{e}}_x - A_z B_y) \underline{\mathbf{e}}_x + (A_z B_x - A_x B_z) \underline{\mathbf{e}}_y + (A_x B_y - A_y B_x) \underline{\mathbf{e}}_z
\end{aligned}$$

$$\underline{\mathbf{A}} \times \underline{\mathbf{B}} = -\underline{\mathbf{B}} \times \underline{\mathbf{A}} \qquad \underline{\mathbf{A}} \times \underline{\mathbf{A}} = \underline{\mathbf{0}}$$

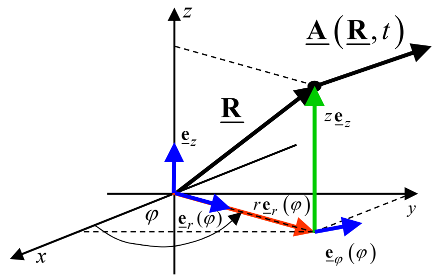
Vector Product (Cross or Outer Product) / Vektorprodukt (Kreuzprodukt oder äußeres Produkt) (2)

Add the first two Columns / Addiere die beiden ersten Spalten

$$\begin{aligned}
\underline{\mathbf{A}} \times \underline{\mathbf{B}} &= \begin{vmatrix} \underline{\mathbf{e}}_x & \underline{\mathbf{e}}_y & \underline{\mathbf{e}}_z \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} \\
&= \begin{vmatrix} \underline{\mathbf{e}}_x & \underline{\mathbf{e}}_y & \underline{\mathbf{e}}_z \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} \begin{vmatrix} \underline{\mathbf{e}}_x & \underline{\mathbf{e}}_y \\ A_x & A_y \\ B_x & B_y \end{vmatrix} \\
&= (A_y B_z - A_z B_y) \underline{\mathbf{e}}_x \\
&\quad + (A_z B_x - A_x B_z) \underline{\mathbf{e}}_y \\
&\quad + (A_x B_y - A_y B_x) \underline{\mathbf{e}}_z
\end{aligned}$$

Coordinate Systems / Koordinatensysteme (3)

Cylindrical Coordinate System / Zylinderkoordinatensystem



Coordinates / Koordinaten r, φ, z

Limits / Grenzen

$$0 \le r < \infty$$
$$0 \le \varphi < 2\pi$$
$$-\infty < z < \infty$$

Orthonormal Unit Vectors / Orthonormale Einheitsvektoren

$$\underline{\mathbf{e}}_r(\varphi), \underline{\mathbf{e}}_\varphi(\varphi), \underline{\mathbf{e}}_z$$
$$\underline{\mathbf{e}}_r(\varphi) \perp \underline{\mathbf{e}}_\varphi(\varphi) \perp \underline{\mathbf{e}}_z$$
$$\left|\underline{\mathbf{e}}_r(\varphi)\right| = \left|\underline{\mathbf{e}}_\varphi(\varphi)\right| = \left|\underline{\mathbf{e}}_z\right| = 1$$

$\perp$: Perpendicular / Senkrecht

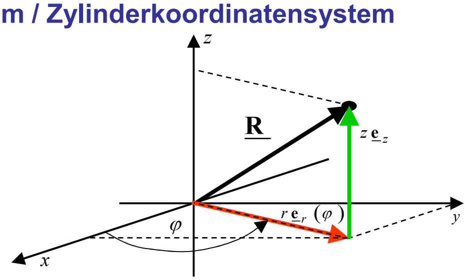
Arbitrary Vector Field / Beliebiges Vektorfeld

$$\underline{\mathbf{A}}(\underline{\mathbf{R}}, t) = \underline{\mathbf{A}}_r(\underline{\mathbf{R}}, t) + \underline{\mathbf{A}}_\varphi(\underline{\mathbf{R}}, t) + \underline{\mathbf{A}}_z(\underline{\mathbf{R}}, t)$$
$$= \mathrm{A}_r(r, \varphi, z, t)\,\underline{\mathbf{e}}_r(\varphi) + A_\varphi(r, \varphi, z, t)\,\underline{\mathbf{e}}_\varphi(\varphi) + A_z(r, \varphi, z, t)\,\underline{\mathbf{e}}_z$$

Position Vector / Ortsvektor (Positionsvektor)

Cylindrical Coordinate System / Zylinderkoordinatensystem

$$\underline{\mathbf{R}} = \underline{\mathbf{R}}_r(\underline{\mathbf{R}}) + \underline{\mathbf{R}}_\varphi(\underline{\mathbf{R}}) + \underline{\mathbf{R}}_z(\underline{\mathbf{R}})$$
$$= R_r(\underline{\mathbf{R}})\,\underline{\mathbf{e}}_r(\varphi) + R_\varphi(\underline{\mathbf{R}})\,\underline{\mathbf{e}}_\varphi(\varphi) + R_z(\underline{\mathbf{R}})\,\underline{\mathbf{e}}_z$$
$$= r\,\underline{\mathbf{e}}_r(\varphi) + z\,\underline{\mathbf{e}}_z$$



Coordinates / Koordinaten $r, \varphi, z; \quad 0 \le r < \infty, 0 \le \varphi < 2\pi, -\infty < z < \infty$

Orthonormal Unit Vectors / Orthonormale Einheitsvektoren

$$\underline{\mathbf{e}}_r(\varphi), \underline{\mathbf{e}}_\varphi(\varphi), \underline{\mathbf{e}}_z$$
$$\underline{\mathbf{e}}_r(\varphi) \perp \underline{\mathbf{e}}_\varphi(\varphi) \perp \underline{\mathbf{e}}_z \quad |\underline{\mathbf{e}}_r(\varphi)| = |\underline{\mathbf{e}}_\varphi(\varphi)| = |\underline{\mathbf{e}}_z| = 1$$

Scalar Vector Components / Skalare Vektorkomponenten

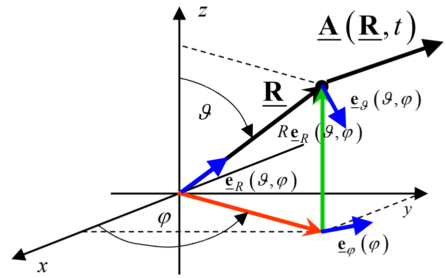
$$R_r(r, \varphi, z) = r\,\underline{\mathbf{e}}_r(\varphi)$$
$$R_\varphi(r, \varphi, z) = 0$$
$$R_z(r, \varphi, z) = z\,\underline{\mathbf{e}}_z$$

Vectorial Vector Components / Vektorielle Vektorkomponenten

$$\underline{\mathbf{R}}_r(\underline{\mathbf{R}}) = R_r(r)\,\underline{\mathbf{e}}_r(\varphi) = r\,\underline{\mathbf{e}}_r(\varphi)$$
$$\underline{\mathbf{R}}_\varphi(\underline{\mathbf{R}}) = \underline{\mathbf{0}}$$
$$\underline{\mathbf{R}}_z(\underline{\mathbf{R}}) = R_z(z)\,\underline{\mathbf{e}}_z = z\,\underline{\mathbf{e}}_z$$

Coordinate Systems / Koordinatensysteme (4)

Spherical Coordinate System / Kugelkoordinatensystem



Coordinates / Koordinaten R,ϑ,φ

Limits / Grenzen

$$0 \le R < \infty$$
$$0 \le \vartheta \le \pi$$
$$0 \le \varphi < 2\pi$$

Orthonormal Unit Vectors / Orthonormale Einheitsvektoren

$$\underline{\mathbf{e}}_R(\vartheta,\varphi), \underline{\mathbf{e}}_\vartheta(\vartheta,\varphi), \underline{\mathbf{e}}_\varphi(\varphi)$$

$$\underline{\mathbf{e}}_R(\vartheta,\varphi) \perp \underline{\mathbf{e}}_\vartheta(\vartheta,\varphi) \perp \underline{\mathbf{e}}_\varphi(\varphi)$$

$\perp$: Perpendicular / Senkrecht

$$|\underline{\mathbf{e}}_R(\vartheta,\varphi)| = |\underline{\mathbf{e}}_\vartheta(\vartheta,\varphi)| = |\underline{\mathbf{e}}_\varphi(\varphi)| = 1$$

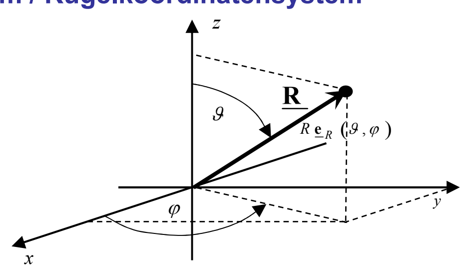
Arbitrary Vector Field / Beliebiges Vektorfeld

$$\underline{\mathbf{A}}(\underline{\mathbf{R}},t) = \underline{\mathbf{A}}_R(\underline{\mathbf{R}},t) + \underline{\mathbf{A}}_\vartheta(\underline{\mathbf{R}},t) + \underline{\mathbf{A}}_\varphi(\underline{\mathbf{R}},t)$$
$$= A_R(R,\vartheta,\varphi,t)\underline{\mathbf{e}}_R(\vartheta,\varphi) + A_\vartheta(R,\vartheta,\varphi,t)\underline{\mathbf{e}}_\vartheta(\vartheta,\varphi) + A_\varphi(R,\vartheta,\varphi,t)\underline{\mathbf{e}}_\varphi(\varphi)$$

Position Vector / Ortsvektor (Positionsvektor)

Spherical Coordinate System / Kugelkoordinatensystem

$$\underline{\mathbf{R}} = \underline{\mathbf{R}}_R(\underline{\mathbf{R}}) + \underline{\mathbf{R}}_\vartheta(\underline{\mathbf{R}}) + \underline{\mathbf{R}}_\varphi(\underline{\mathbf{R}})$$
$$= R_R(\underline{\mathbf{R}})\underline{\mathbf{e}}_R(\vartheta,\varphi) + R_\vartheta(\underline{\mathbf{R}})\underline{\mathbf{e}}_\vartheta(\vartheta,\varphi)$$
$$+ R_\varphi(\underline{\mathbf{R}})\underline{\mathbf{e}}_\varphi(\varphi)$$
$$= R\underline{\mathbf{e}}_R(\vartheta,\varphi)$$



Coordinates / Koordinaten $R,\vartheta,\varphi; \quad 0 \le R < \infty, 0 \le \vartheta \le \pi; 0 \le \varphi < 2\pi$

Orthonormal Unit Vectors / Orthonormale Einheitsvektoren $\underline{\mathbf{e}}_R, \underline{\mathbf{e}}_\vartheta, \underline{\mathbf{e}}_\varphi$

$$\underline{\mathbf{e}}_R \perp \underline{\mathbf{e}}_\vartheta \perp \underline{\mathbf{e}}_\varphi \qquad |\underline{\mathbf{e}}_R| = |\underline{\mathbf{e}}_\vartheta| = |\underline{\mathbf{e}}_\varphi| = 1$$

Scalar Vector Components / Skalare Vektorkomponenten $R_R(R,\vartheta,\varphi), R_\vartheta(R,\vartheta,\varphi), R_\varphi(R,\vartheta,\varphi)$

Vectorial Vector Components / Vektorielle Vektorkomponenten

$$\underline{\mathbf{R}}_R(\underline{\mathbf{R}}) = R_R(R,\vartheta,\varphi)\underline{\mathbf{e}}_R(\vartheta,\varphi) = R\underline{\mathbf{e}}_R(\vartheta,\varphi)$$
$$\underline{\mathbf{R}}_\vartheta(\underline{\mathbf{R}}) = R_\vartheta(R,\vartheta,\varphi)\underline{\mathbf{e}}_\vartheta(\vartheta,\varphi) = \underline{\mathbf{0}}$$
$$\underline{\mathbf{R}}_\varphi(\underline{\mathbf{R}}) = R_\varphi(R,\vartheta,\varphi)\underline{\mathbf{e}}_\varphi(\varphi) = \underline{\mathbf{0}}$$

Notation and Field Quantities / Notation und Feldgrößen

Rank / Rang	Tensor of... / Tensor des...	Name / Name	Example / Beispiel	Symbol / Symbol	Notation / Schreibweise
$n = 0$	Zeroth Rank / nullten Ranges	Scalar / Skalar	Electric Potential / Elektrisches Potential	Φ_e	Roman with no Underline / Roman mit keinem Unterstrich
$n = 1$	First Rank / ersten Ranges	Vector / Vektor	Electric Field Strength / Elektrische Feldstärke	$\underline{\mathbf{E}} = E_i \underline{\mathbf{e}}_i$	Bold Face with one Underline / Fett mit einem Unterstrich
$n = 2$	Second Rank / zweiten Ranges	Dyad / Dyade	Electric Permittivity Tensor / Elektrischer Permittivitätstensor	$\underline{\underline{\boldsymbol{\varepsilon}}} = \varepsilon_{ij} \underline{\mathbf{e}}_i \underline{\mathbf{e}}_j$	Bold Face with two Underlines / Fett mit zwei Unterstrichen
$n = 3$	Third Rank / dritten Ranges	Triad / Triade	Piezoelectric Coupling Tensor / Piezoelektrischer Koppeltensor	$\underline{\underline{\underline{\mathbf{d}}}} = d_{ijk} \underline{\mathbf{e}}_i \underline{\mathbf{e}}_j \underline{\mathbf{e}}_k$	Bold Face with three Underlines / Fett mit drei Unterstrichen
$n = 4$	Fourth Rank / vierten Ranges	Tetrad / Tetrade	Elastic Stiffness Tensor / Elastischer Steifigkeitstensor	$\underline{\underline{\underline{\underline{\mathbf{c}}}}} = c_{ijkl} \underline{\mathbf{e}}_i \underline{\mathbf{e}}_j \underline{\mathbf{e}}_k \underline{\mathbf{e}}_l$	Bold Face with four Underlines / Fett mit vier Unterstrichen

Coordinate Systems / Koordinatensysteme
Notation and Field Quantities / Notation und Feldgrößen

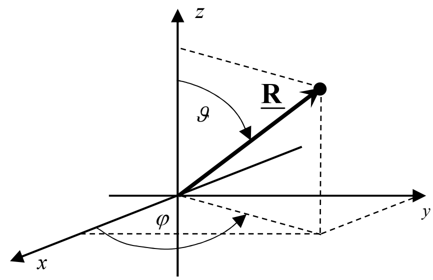
Vector / Vektor:
Electric Field Strength / Elektrische Feldstärke

$$\begin{aligned}\underline{\mathbf{E}}(\underline{\mathbf{R}},t) &= \underline{\mathbf{E}}_x(\underline{\mathbf{R}},t) + \underline{\mathbf{E}}_y(\underline{\mathbf{R}},t) + \underline{\mathbf{E}}_z(\underline{\mathbf{R}},t) \\ &= \mathrm{E}_x(x,y,z,t)\underline{\mathbf{e}}_x + \mathrm{E}_y(x,y,z,t)\underline{\mathbf{e}}_y + \mathrm{E}_z(x,y,z,t)\underline{\mathbf{e}}_z \\ &= \sum_{i=1}^{3} \mathrm{E}_{x_i}(x_1,x_2,x_3,t)\underline{\mathbf{e}}_{x_i} \\ &= \mathrm{E}_{x_i}(x_1,x_2,x_3,t)\underline{\mathbf{e}}_{x_i}\end{aligned}$$

Dyad / Dyade:
Permittivity Dyad / Permittivitätsdyade

$$\begin{aligned}\underline{\underline{\varepsilon}}(\underline{\mathbf{R}},t) &= \underline{\underline{\varepsilon}}_{xx}(\underline{\mathbf{R}},t) + \underline{\underline{\varepsilon}}_{xy}(\underline{\mathbf{R}},t) + \underline{\underline{\varepsilon}}_{zz}(\underline{\mathbf{R}},t) \\ &= \varepsilon_{xx}(x,y,z,t)\underline{\mathbf{e}}_x\underline{\mathbf{e}}_x + \varepsilon_{xy}(x,y,z,t)\underline{\mathbf{e}}_x\underline{\mathbf{e}}_y + \varepsilon_{xz}(x,y,z,t)\underline{\mathbf{e}}_x\underline{\mathbf{e}}_z \\ &+ \varepsilon_{yx}(x,y,z,t)\underline{\mathbf{e}}_y\underline{\mathbf{e}}_x + \varepsilon_{yy}(x,y,z,t)\underline{\mathbf{e}}_y\underline{\mathbf{e}}_y + \varepsilon_{yz}(x,y,z,t)\underline{\mathbf{e}}_y\underline{\mathbf{e}}_z \\ &+ \varepsilon_{zx}(x,y,z,t)\underline{\mathbf{e}}_z\underline{\mathbf{e}}_x + \varepsilon_{zy}(x,y,z,t)\underline{\mathbf{e}}_z\underline{\mathbf{e}}_y + \varepsilon_{zz}(x,y,z,t)\underline{\mathbf{e}}_z\underline{\mathbf{e}}_z \\ &= \sum_{i=1}^{3}\sum_{j=1}^{3} \varepsilon_{x_i x_j}(x_1,x_2,x_3,t)\underline{\mathbf{e}}_{x_i}\underline{\mathbf{e}}_{x_j} \\ &= \varepsilon_{x_i x_j}(x_1,x_2,x_3,t)\underline{\mathbf{e}}_{x_i}\underline{\mathbf{e}}_{x_j}\end{aligned}$$

Coordinates of Different Coordinate Systems / Koordinaten verschiedenen Koordinatensystemen



Transformation Table / Umrechnungstabelle

Cartesian Coordinates / Kartesische Koordinaten	Cylindrical Coordinates / Zylinderkoordinaten	Spherical Coordinates / Kugelkoordinaten
x y z	$r\cos\varphi$ $r\sin\varphi$ z	$R\sin\vartheta\cos\varphi$ $R\sin\vartheta\sin\varphi$ $R\cos\vartheta$
$\sqrt{x^2+y^2}$ $\arctan\frac{y}{x}$ z	r φ z	$R\sin\vartheta$ φ $R\cos\vartheta$
$\sqrt{x^2+y^2+z^2}$ $\arctan\frac{\sqrt{x^2+y^2}}{z}$ $\arctan\frac{y}{x}$	$\sqrt{r^2+z^2}$ $\arctan\frac{r}{z}$ φ	R ϑ φ

Examples / Beispiele

1. Formulate x as a function of the cylinder and spherical coordinates. / Formuliere x als Funktion der Zylinder- und Kugelkoordinaten.

$$x = r\cos\varphi = R\sin\vartheta\cos\varphi$$

2. Formulate r as a function of the Cartesian and spherical coordinates. / Formuliere r als Funktion der Kartesischen und Kugelkoordinaten.

$$r = \sqrt{x^2+y^2} = R\sin\vartheta$$

3. Formulate $\sqrt{x^2+y^2}$ as a function of the cylinder coordinates. / Formuliere $\sqrt{x^2+y^2}$ als Funktion der Zylinderkoordinaten.

$$\sqrt{x^2+y^2} = \sqrt{(r\cos\varphi)^2+(r\sin\varphi)^2} = r\sqrt{\underbrace{\cos^2\varphi+\sin^2\varphi}_{=1}} = r$$

Scalar Vector Components in Different Coordinate Systems / Skalare Vektorkomponenten in verschiedenen Koordinatensystemen

Transformation Table / Umrechnungstabelle

Cartesian Coordinates / Kartesische Koordinaten	Cylindrical Coordinates / Zylinderkoordinaten	Spherical Coordinates / Kugelkoordinaten
$\underline{\mathbf{A}} = A_x \underline{\mathbf{e}}_x + A_y \underline{\mathbf{e}}_y + A_z \underline{\mathbf{e}}_z$	$\underline{\mathbf{A}} = A_r \underline{\mathbf{e}}_r + A_\varphi \underline{\mathbf{e}}_\varphi + A_z \underline{\mathbf{e}}_z$	$\underline{\mathbf{A}} = A_R \underline{\mathbf{e}}_R + A_\vartheta \underline{\mathbf{e}}_\vartheta + A_\varphi \underline{\mathbf{e}}_\varphi$
A_x A_y A_z	$A_r \cos\varphi - A_\varphi \sin\varphi$ $A_r \sin\varphi + A_\varphi \cos\varphi$ A_z	$A_R \sin\vartheta\cos\varphi + A_\vartheta \cos\vartheta\cos\varphi - A_\varphi \sin\varphi$ $A_R \sin\vartheta\sin\varphi + A_\vartheta \cos\vartheta\sin\varphi + A_\varphi \cos\varphi$ $A_R \cos\vartheta - A_\vartheta \sin\vartheta$
$A_x \cos\varphi + A_y \sin\varphi$ $-A_x \sin\varphi + A_y \cos\varphi$ A_z	A_r A_φ A_z	$A_R \sin\vartheta + A_\vartheta \cos\vartheta$ A_φ $A_R \cos\vartheta - A_\vartheta \sin\vartheta$
$A_x \sin\vartheta\cos\varphi + A_y \sin\vartheta\sin\varphi + A_z \cos\vartheta$ $A_x \cos\vartheta\cos\varphi + A_y \cos\vartheta\sin\varphi - A_z \sin\vartheta$ $-A_x \sin\varphi + A_y \cos\varphi$	$A_r \sin\vartheta + A_z \cos\vartheta$ $A_r \cos\vartheta - A_z \sin\vartheta$ A_φ	A_R A_ϑ A_φ

Example: Coordinate Transformation of the Position Vector / Beispiel: Koordinatentransformation des Ortsvektor

Position Vector in the Cartesian Coordinate System / Ortsvektor im Kartesischen Koordinatensystem

$$\underline{\mathbf{R}} = \underbrace{x}_{R_x(x,y,z)} \underline{\mathbf{e}}_x + \underbrace{y}_{R_y(x,y,z)} \underline{\mathbf{e}}_y + \underbrace{z}_{R_z(x,y,z)} \underline{\mathbf{e}}_z$$

Transformation of the Coordinates / Transformation der Koordinaten

$$R_x(r,\varphi,z) = x(r,\varphi,z) = r\cos\varphi$$
$$R_y(r,\varphi,z) = y(r,\varphi,z) = r\sin\varphi$$
$$R_z(r,\varphi,z) = z(r,\varphi,z) = z$$

Position Vector in the Cylinder Coordinate System / Ortsvektor im Zylinderkoordinatensystem ?

$$\underline{\mathbf{R}}(r,\varphi,y,R_r,R_\varphi,R_z)$$
$$= R_r(r,\varphi,y)\underline{\mathbf{e}}_r(\varphi) + R_\varphi(r,\varphi,y)\underline{\mathbf{e}}_\varphi(\varphi) + R_z(r,\varphi,y)\underline{\mathbf{e}}_z$$

Position Vector in the Cartesian Coordinate System as a Function of Cylinder Coordinates / Ortsvektor im Kartesischen Koordinatensystem als Funktion der Zylinderkoordinaten

$$\underline{\mathbf{R}} = \underbrace{r\cos\varphi}_{R_x(r,\varphi,z)} \underline{\mathbf{e}}_x + \underbrace{r\sin\varphi}_{R_y(r,\varphi,z)} \underline{\mathbf{e}}_y + \underbrace{z}_{R_z(r,\varphi,z)} \underline{\mathbf{e}}_z$$

Transformation of the Scalar Vector Components / Transformation der skalaren Vektorkomponenten

$$R_r(r,\varphi,z,R_x,R_y,R_z) = R_x\cos\varphi + R_y\sin\varphi$$
$$R_\varphi(r,\varphi,z,R_x,R_y,R_z) = -R_x\sin\varphi + R_y\cos\varphi$$
$$R_z(r,\varphi,z,R_x,R_y,R_z) = R_z$$

$$\downarrow$$

$$R_r = r\cos\varphi\cos\varphi + r\sin\varphi\sin\varphi$$
$$= r(\underbrace{\cos^2\varphi + \sin^2\varphi}_{=1}) = r$$
$$R_\varphi = -r\cos\varphi\sin\varphi + r\sin\varphi\cos\varphi$$
$$= 0$$
$$R_z = R_z$$

Position Vector in the Cylinder Coordinate System / Ortsvektor im Zylinderkoordinatensystem

$$\underline{\mathbf{R}} = \underbrace{r}_{R_r} \underline{\mathbf{e}}_r(\varphi) + \underbrace{z}_{R_z} \underline{\mathbf{e}}_z$$

End of 2nd Lecture /
Ende der 2. Vorlesung