Elektromagnetische Feldtheorie I (EFT I) / Electromagnetic Field Theory I (EFT I)

3rd Lecture / 3. Vorlesung

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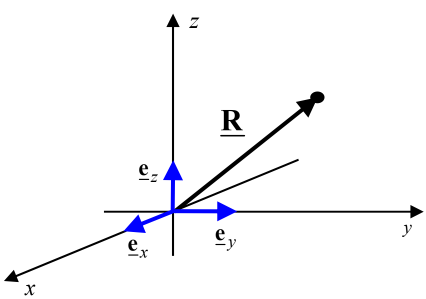
Metric Coefficients / Metrische Koeffizienten

Cartesian Coordinates / Kartesische Koordinaten

$$x, y, z = x_1, x_2, x_3$$

$$\underline{\mathbf{e}}_x, \underline{\mathbf{e}}_y, \underline{\mathbf{e}}_z = \underline{\mathbf{e}}_{x_1}, \underline{\mathbf{e}}_{x_2}, \underline{\mathbf{e}}_{x_3}$$

$$\underline{\mathbf{e}}_x, \perp \underline{\mathbf{e}}_y \perp \underline{\mathbf{e}}_z \; ; \; \underline{\mathbf{e}}_{x_1} \perp \underline{\mathbf{e}}_{x_2} \perp \underline{\mathbf{e}}_{x_3}$$

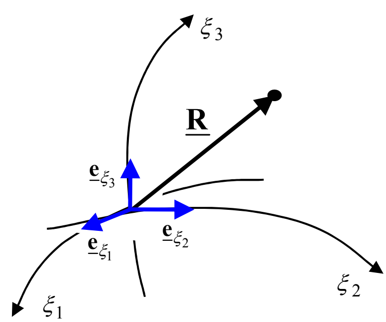


Orthogonal Curvilinear Coordinates / Orthogonale Krummlinige Koordinaten

$$\xi_1, \xi_2, \xi_3$$

$$\underline{\mathbf{e}}_{\xi_1}, \underline{\mathbf{e}}_{\xi_2}, \underline{\mathbf{e}}_{\xi_3}$$

$$\underline{\mathbf{e}}_{\xi_1} \perp \underline{\mathbf{e}}_{\xi_2} \perp \underline{\mathbf{e}}_{\xi_3}$$



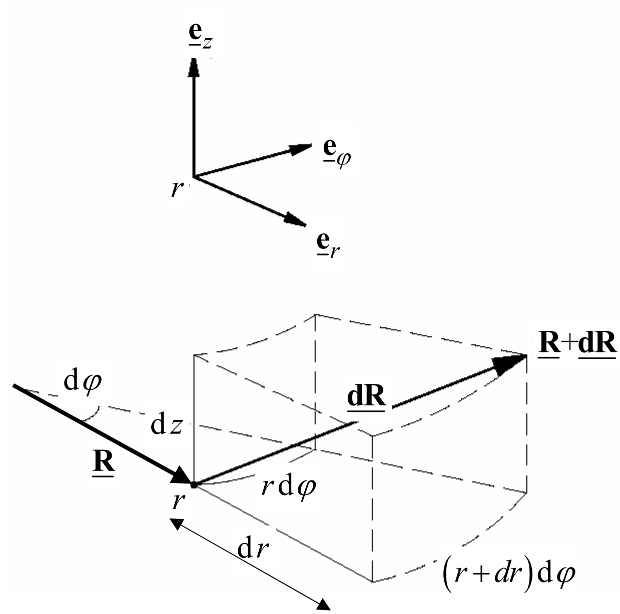
Cartesian Coordinates / Kartesische Koordinaten

$$x = x(\xi_1, \xi_2, \xi_3)$$
$$y = y(\xi_1, \xi_2, \xi_3)$$
$$z = z(\xi_1, \xi_2, \xi_3)$$

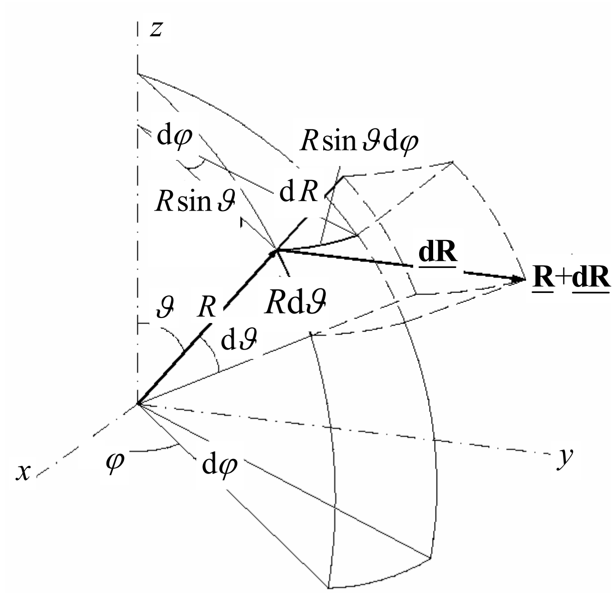
$\longleftrightarrow$

$$\xi_1 = \xi_1(x, y, z)$$
$$\xi_2 = \xi_2(x, y, z)$$
$$\xi_3 = \xi_3(x, y, z)$$

Metric Coefficients – Cylindrical Coordinate System / Metrische Koeffizienten – Zylinderkoordinatensystem



Metric Coefficients – Spherical Coordinate System / Metrische Koeffizienten – Kugelkoordinatensystem



Metric Coefficients / Metrische Koeffizienten

Cartesian Coordinates / Kartesische Koordinaten

$$x, y, z = x_1, x_2, x_3$$
$$x = x(\xi_1, \xi_2, \xi_3)$$
$$y = y(\xi_1, \xi_2, \xi_3)$$
$$z = z(\xi_1, \xi_2, \xi_3)$$

Orthogonal Curvilinear Coordinates / Orthogonale Krummlinige Koordinaten

$$\xi_1, \xi_2, \xi_3$$
$$\xi_1 = \xi_1(x, y, z)$$
$$\xi_2 = \xi_2(x, y, z)$$
$$\xi_3 = \xi_3(x, y, z)$$

$$\underline{\mathbf{R}} = x(\xi_1, \xi_2, \xi_3)\underline{\mathbf{e}}_x + y(\xi_1, \xi_2, \xi_3)\underline{\mathbf{e}}_y + z(\xi_1, \xi_2, \xi_3)\underline{\mathbf{e}}_z$$

$$\frac{\partial \underline{\mathbf{R}}}{\partial \xi_i} = \frac{\partial x(\xi_1, \xi_2, \xi_3)}{\partial \xi_i}\underline{\mathbf{e}}_x + \frac{\partial y(\xi_1, \xi_2, \xi_3)}{\partial \xi_i}\underline{\mathbf{e}}_y + \frac{\partial z(\xi_1, \xi_2, \xi_3)}{\partial \xi_i}\underline{\mathbf{e}}_z$$
$$i = 1, 2, 3$$

$$\frac{\partial \underline{\mathbf{R}}}{\partial \xi_i} = \underbrace{\underbrace{\left|\frac{\partial \underline{\mathbf{R}}(\xi_1, \xi_2, \xi_3)}{\partial \xi_i}\right|}_{\text{Magnitude / Betrag}}}_{= h_{\xi_i}} \underbrace{\underbrace{\frac{\dfrac{\partial \underline{\mathbf{R}}(\xi_1, \xi_2, \xi_3)}{\partial \xi_i}}{\left|\dfrac{\partial \underline{\mathbf{R}}(\xi_1, \xi_2, \xi_3)}{\partial \xi_i}\right|}}_{\text{Direction / Richtung}}}_{= \underline{\mathbf{e}}_{\xi_i}}, \quad i = 1, 2, 3$$

Metric Coefficients / Metrische Koeffizienten

$$h_{\xi_i} = \left|\frac{\partial \underline{\mathbf{R}}(\xi_1, \xi_2, \xi_3)}{\partial \xi_i}\right|$$
$$= \sqrt{\frac{\partial \underline{\mathbf{R}}(\xi_1, \xi_2, \xi_3)}{\partial \xi_i} \bullet \frac{\partial \underline{\mathbf{R}}(\xi_1, \xi_2, \xi_3)}{\partial \xi_i}}$$
$$= \sqrt{\left(\frac{\partial x(\xi_1, \xi_2, \xi_3)}{\partial \xi_i}\right)^2 + \left(\frac{\partial y(\xi_1, \xi_2, \xi_3)}{\partial \xi_i}\right)^2 + \left(\frac{\partial z(\xi_1, \xi_2, \xi_3)}{\partial \xi_i}\right)^2}$$
$$i = 1, 2, 3$$

Metric Coefficients / Metrische Koeffizienten

$$\xi_1 = x$$
$$\xi_2 = y$$
$$\xi_3 = z$$

$$h_x = \left|\frac{\partial \underline{\mathbf{R}}(x, y, z)}{\partial x}\right|$$
$$= \sqrt{\frac{\partial \underline{\mathbf{R}}(x, y, z)}{\partial x} \bullet \frac{\partial \underline{\mathbf{R}}(x, y, z)}{\partial x}}$$
$$= \sqrt{\left(\frac{\partial x(x, y, z)}{\partial x}\right)^2 + \left(\frac{\partial y(x, y, z)}{\partial x}\right)^2 + \left(\frac{\partial z(x, y, z)}{\partial x}\right)^2}$$
$$= \sqrt{\underbrace{\left(\frac{\partial x}{\partial x}\right)^2}_{1} + \underbrace{\left(\frac{\partial y}{\partial x}\right)^2}_{=0} + \underbrace{\left(\frac{\partial z}{\partial x}\right)^2}_{=0}}$$
$$= 1$$
$$h_y = 1$$
$$h_z = 1$$

Metric Coefficients / Metrische Koeffizienten

Cartesian Coordinate System / Kartesisches Koordinatensystem

$$h_x = 1$$
$$h_y = 1$$
$$h_z = 1$$

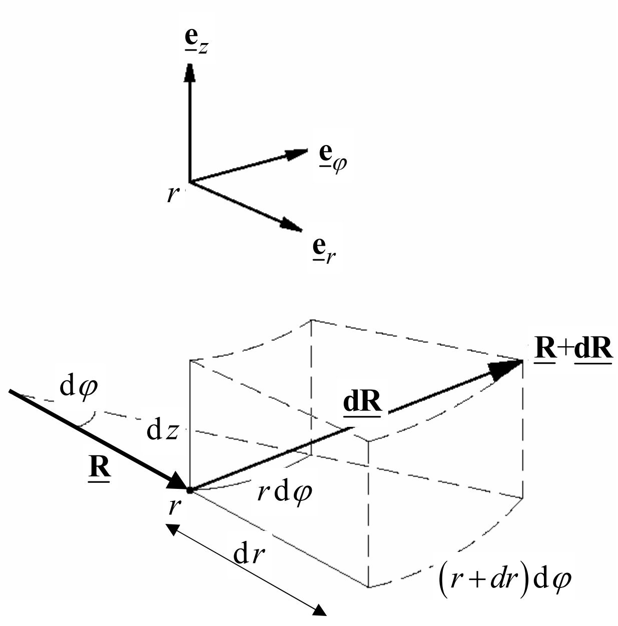
Cylindrical Coordinate System / Zylinderkoordinatensystem

$$h_r = 1$$
$$h_\varphi = r$$
$$h_z = 1$$

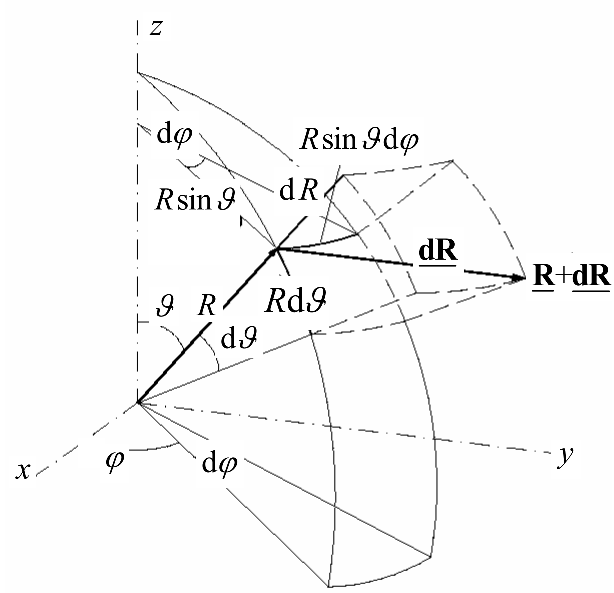
Spherical Coordinate System / Kugelkoordinatensystem

$$h_R = 1$$
$$h_\vartheta = R$$
$$h_\varphi = R \sin \vartheta$$

Metric Coefficients – Cylindrical Coordinate System / Metrische Koeffizienten – Zylinderkoordinatensystem



Metric Coefficients – Spherical Coordinate System / Metrische Koeffizienten – Kugelkoordinatensystem



Metric Coefficients and Vector Differential Line Elements / Metrische Koeffizienten und vektorielle differentielle Linienelemente

Cartesian Coordinate System / Kartesisches Koordinatensystem

$$h_x = 1, \quad h_y = 1, \quad h_z = 1$$

$$\underline{\mathbf{dR}}_x = \underline{\mathbf{s}}\,\mathrm{d}R = \underline{\mathbf{e}}_x\, h_x\, \mathrm{d}x = \underline{\mathbf{e}}_x\, \mathrm{d}x$$

$$\underline{\mathbf{dR}}_y = \underline{\mathbf{s}}\,\mathrm{d}R = \underline{\mathbf{e}}_y\, h_y\, \mathrm{d}y = \underline{\mathbf{e}}_y\, \mathrm{d}y$$

$$\underline{\mathbf{dR}}_z = \underline{\mathbf{n}}\,\mathrm{d}R = \underline{\mathbf{e}}_z\, h_z\, \mathrm{d}z = \underline{\mathbf{e}}_z\, \mathrm{d}z$$

Cylindrical Coordinate System / Zylinderkoordinatensystem

$$h_r = 1, \quad h_\varphi = r, \quad h_z = 1$$

$$\underline{\mathbf{dR}}_r = \underline{\mathbf{s}}\,\mathrm{d}R = \underline{\mathbf{e}}_r\, h_r\, \mathrm{d}r = \underline{\mathbf{e}}_r\, \mathrm{d}r$$

$$\underline{\mathbf{dR}}_\varphi = \underline{\mathbf{s}}\,\mathrm{d}R = \underline{\mathbf{e}}_\varphi\, h_\varphi\, \mathrm{d}\varphi = \underline{\mathbf{e}}_\varphi\, r\, \mathrm{d}\varphi$$

$$\underline{\mathbf{dR}}_z = \underline{\mathbf{s}}\,\mathrm{d}R = \underline{\mathbf{e}}_z\, h_z\, \mathrm{d}z = \underline{\mathbf{e}}_z\, \mathrm{d}z$$

Spherical Coordinate System / Kugelkoordinatensystem

$$h_R = 1, \quad h_\vartheta = R, \quad h_\varphi = R \sin\vartheta$$

$$\underline{\mathbf{dR}}_R = \underline{\mathbf{s}}\,\mathrm{d}R = \underline{\mathbf{e}}_R\, h_R\, \mathrm{d}R = \underline{\mathbf{e}}_R\, \mathrm{d}R$$

$$\underline{\mathbf{dR}}_\vartheta = \underline{\mathbf{s}}\,\mathrm{d}R = \underline{\mathbf{e}}_\vartheta\, h_\vartheta\, \mathrm{d}\vartheta = \underline{\mathbf{e}}_\vartheta\, R\, \mathrm{d}\vartheta$$

$$\underline{\mathbf{dR}}_\varphi = \underline{\mathbf{s}}\,\mathrm{d}R = \underline{\mathbf{e}}_\varphi\, h_\varphi\, \mathrm{d}\varphi = \underline{\mathbf{e}}_\varphi\, R \sin\vartheta\, \mathrm{d}\varphi$$

Metric Coefficients and Differential Volume and Surface Elements / Metrische Koeffizienten und differentielle Volumen- und Flächenelemente

Cartesian Coordinate System / Kartesisches Koordinatensystem

$$h_x = 1, \quad h_y = 1, \quad h_z = 1$$

$$\mathrm{d}V = h_x\,\mathrm{d}x\,h_y\,\mathrm{d}y\,h_z\,\mathrm{d}z = h_x\,h_y\,h_z\,\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z = \mathrm{d}z\,\mathrm{d}x\,\mathrm{d}z$$

$$\underline{\mathbf{dS}}_{yz} = \underline{\mathbf{n}}\,\mathrm{d}S = (\underline{\mathbf{e}}_y \times \underline{\mathbf{e}}_z)\,h_y\,h_z\,\mathrm{d}y\,\mathrm{d}z = \underline{\mathbf{e}}_x\,\mathrm{d}y\,\mathrm{d}z$$

$$\underline{\mathbf{dS}}_{xz} = \underline{\mathbf{n}}\,\mathrm{d}S = (\underline{\mathbf{e}}_z \times \underline{\mathbf{e}}_x)\,h_x\,h_z\,\mathrm{d}x\,\mathrm{d}z = \underline{\mathbf{e}}_y\,\mathrm{d}x\,\mathrm{d}z$$

$$\underline{\mathbf{dS}}_{xy} = \underline{\mathbf{n}}\,\mathrm{d}S = (\underline{\mathbf{e}}_x \times \underline{\mathbf{e}}_y)\,h_x\,h_y\,\mathrm{d}x\,\mathrm{d}y = \underline{\mathbf{e}}_z\,\mathrm{d}x\,\mathrm{d}y$$

Cylindrical Coordinate System / Zylinderkoordinatensystem

$$h_r = 1, \quad h_\varphi = r, \quad h_z = 1$$

$$\mathrm{d}V = h_r\,\mathrm{d}r\,h_\varphi\,\mathrm{d}\varphi\,h_z\,\mathrm{d}z = h_r\,h_\varphi\,h_z\,\mathrm{d}r\,\mathrm{d}\varphi\,\mathrm{d}y = r\,\mathrm{d}r\,\mathrm{d}\varphi\,\mathrm{d}z$$

$$\underline{\mathbf{dS}}_{\varphi z} = \underline{\mathbf{n}}\,\mathrm{d}S = (\underline{\mathbf{e}}_\varphi \times \underline{\mathbf{e}}_z)\,h_\varphi\,h_z\,\mathrm{d}\varphi\,\mathrm{d}z = \underline{\mathbf{e}}_r\,r\,\mathrm{d}y\,\mathrm{d}z$$

$$\underline{\mathbf{dS}}_{rz} = \underline{\mathbf{n}}\,\mathrm{d}S = (\underline{\mathbf{e}}_z \times \underline{\mathbf{e}}_r)\,h_r\,h_z\,\mathrm{d}r\,\mathrm{d}z = \underline{\mathbf{e}}_\varphi\,\mathrm{d}r\,\mathrm{d}z$$

$$\underline{\mathbf{dS}}_{r\varphi} = \underline{\mathbf{n}}\,\mathrm{d}S = (\underline{\mathbf{e}}_r \times \underline{\mathbf{e}}_\varphi)\,h_r\,h_\varphi\,\mathrm{d}r\,\mathrm{d}\varphi = \underline{\mathbf{e}}_z\,r\,\mathrm{d}r\,\mathrm{d}\varphi$$

Spherical Coordinate System / Kugelkoordinatensystem

$$h_R = 1, \quad h_\vartheta = R, \quad h_\varphi = R\sin\vartheta$$

$$\mathrm{d}V = h_R\,\mathrm{d}R\,h_\vartheta\,\mathrm{d}\vartheta\,h_\varphi\,\mathrm{d}\varphi = h_R\,h_\vartheta\,h_\varphi\,\mathrm{d}R\,\mathrm{d}\vartheta\,\mathrm{d}\varphi = R^2\sin\vartheta\,\mathrm{d}R\,\mathrm{d}\vartheta\,\mathrm{d}\varphi$$

$$\underline{\mathbf{dS}}_{\vartheta\varphi} = \underline{\mathbf{n}}\,\mathrm{d}S = (\underline{\mathbf{e}}_\vartheta \times \underline{\mathbf{e}}_\varphi)\,h_\vartheta\,h_\varphi\,\mathrm{d}\vartheta\,\mathrm{d}\varphi = \underline{\mathbf{e}}_R\,R^2\sin\vartheta\,\mathrm{d}\vartheta\,\mathrm{d}\varphi$$

$$\underline{\mathbf{dS}}_{r\varphi} = \underline{\mathbf{n}}\,\mathrm{d}S = (\underline{\mathbf{e}}_\varphi \times \underline{\mathbf{e}}_R)\,h_R\,h_\varphi\,\mathrm{d}R\,\mathrm{d}\varphi = \underline{\mathbf{e}}_\vartheta\,R\sin\vartheta\,\mathrm{d}R\,\mathrm{d}\varphi$$

$$\underline{\mathbf{dS}}_{R\vartheta} = \underline{\mathbf{n}}\,\mathrm{d}S = (\underline{\mathbf{e}}_R \times \underline{\mathbf{e}}_\vartheta)\,h_R\,h_\vartheta\,\mathrm{d}R\,\mathrm{d}\vartheta = \underline{\mathbf{e}}_\varphi\,R\,\mathrm{d}R\,\mathrm{d}\vartheta$$

Metric Coefficients and Differential Volume, Surface, and Line Elements / Metrische Koeffizienten und differentielle Volumen-, Flächen- und Linienelemente

Cartesian Coordinate System / Kartesisches Koordinatensystem

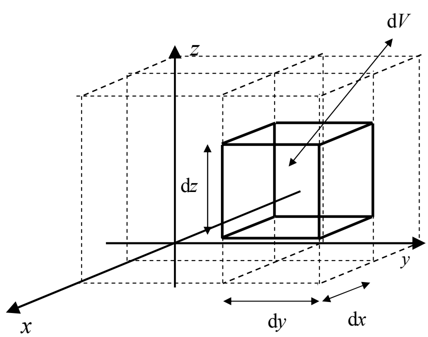
$$h_x = 1, \quad h_y = 1, \quad h_z = 1$$

$$\mathrm{d}V = h_x\,\mathrm{d}x\,h_y\,\mathrm{d}y\,h_z\,\mathrm{d}z = h_x\,h_y\,h_z\,\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z = \mathrm{d}z\,\mathrm{d}x\,\mathrm{d}z$$

$$\underline{\mathbf{dS}}_{yz} = \underline{\mathbf{n}}\,\mathrm{d}S = (\underline{\mathbf{e}}_y \times \underline{\mathbf{e}}_z)\,h_y\,h_z\,\mathrm{d}y\,\mathrm{d}z = \underline{\mathbf{e}}_x\,\mathrm{d}y\,\mathrm{d}z$$

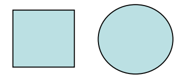
$$\underline{\mathbf{dS}}_{xz} = \underline{\mathbf{n}}\,\mathrm{d}S = (\underline{\mathbf{e}}_z \times \underline{\mathbf{e}}_x)\,h_x\,h_z\,\mathrm{d}x\,\mathrm{d}z = \underline{\mathbf{e}}_y\,\mathrm{d}x\,\mathrm{d}z$$

$$\underline{\mathbf{dS}}_{xy} = \underline{\mathbf{n}}\,\mathrm{d}S = (\underline{\mathbf{e}}_x \times \underline{\mathbf{e}}_y)\,h_x\,h_y\,\mathrm{d}x\,\mathrm{d}y = \underline{\mathbf{e}}_z\,\mathrm{d}x\,\mathrm{d}y$$

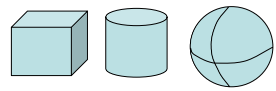


Governing Equations in Integral and Differential Form / Grundgleichungen in Integral- und Differentialform

Surface / Fläche



Volume / Volumen



Integral From / Integralform

(Global Formulation / Globale Formulierung)

Stokes' and Gauss' Integral Theorem / Stokesscher und Gaußscher Integralsatz

Differential Form / Differentialform

(Local Formulation / Lokale Formulierung)

Point / Punkt

•

Stokes' Integral Theorem / Stokesscher Integralsatz

Stokes' Integral Theorem / Stokesscher Integralsatz

$$\oint_{C=\partial S} \underline{\mathbf{A}}(\underline{\mathbf{R}},t)\bullet \underline{\mathbf{dR}} = \iint_S \text{curl/rot } \underline{\mathbf{A}}(\underline{\mathbf{R}},t)\bullet \underline{\mathbf{dS}} = \iint_S \underline{\mathbf{n}}\bullet \underbrace{\text{curl/rot } \underline{\mathbf{A}}(\underline{\mathbf{R}},t)}_{\nabla\times\underline{\mathbf{A}}(\underline{\mathbf{R}},t)} dS = \iint_S \underline{\mathbf{n}}\bullet \left[\nabla\times\underline{\mathbf{A}}(\underline{\mathbf{R}},t)\right] dS$$

with the normal component of circulation - curl - of the vector field $\underline{\mathbf{A}}(\underline{\mathbf{R}},t)$ at the position $\underline{\mathbf{R}}$ "in the middle" of ΔS /

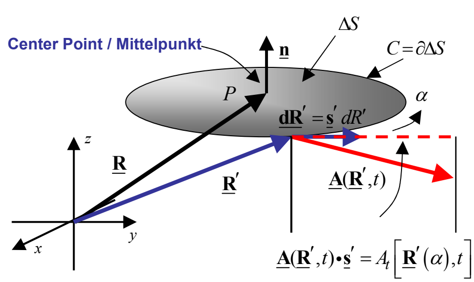
mit der Normalkomponente der Zirkulation/Rotation - rot - des Vektorfeldes $\underline{\mathbf{A}}(\underline{\mathbf{R}},t)$ am Ort $\underline{\mathbf{R}}$ "in der Mitte" von ΔS

$$\underline{\mathbf{n}}\bullet\left[\nabla\times\underline{\mathbf{A}}(\underline{\mathbf{R}},t)\right] = \lim_{\Delta S\to 0}\frac{1}{\Delta S}\oint_{C=\partial\Delta S}\underline{\mathbf{A}}(\underline{\mathbf{R}}',t)\bullet\underline{\mathbf{dR}}'$$

$$= \lim_{\Delta S\to 0}\frac{1}{\Delta S}\oint_{C=\partial\Delta S}\underbrace{\underline{\mathbf{A}}(\underline{\mathbf{R}}',t)\bullet\underline{\mathbf{s}}'}_{A_t(\underline{\mathbf{R}}',t)}\, dR'$$

= "Strength of circulation of $\underline{\mathbf{A}}(\underline{\mathbf{R}},t)$ around the normal unit-vector $\underline{\mathbf{n}}$ at the positon $\underline{\mathbf{R}}$"

= "Stärke der Zirkulation von $\underline{\mathbf{A}}(\underline{\mathbf{R}},t)$ um den Normaleneinheitsvektor $\underline{\mathbf{n}}$ am Ort $\underline{\mathbf{R}}$"



Gauss' Integral Theorem / Gaußscher Integralsatz

Gauss' Integral Theorem / Gaußscher Integralsatz

$$\oiint_{S=\partial V} \underline{\mathbf{A}}(\underline{\mathbf{R}},t)\bullet d\underline{\mathbf{S}} = \iiint_V \underbrace{\operatorname{div}\underline{\mathbf{A}}(\underline{\mathbf{R}},t)}_{\nabla\bullet\underline{\mathbf{A}}(\underline{\mathbf{R}},t)} dV = \iiint_V \nabla\bullet\underline{\mathbf{A}}(\underline{\mathbf{R}},t)dV$$

with the Divergence (Stength of Flux) - **div** - of the Vector Field $\underline{\mathbf{A}}(\underline{\mathbf{R}},\mathbf{t})$ at Position $\underline{\mathbf{R}}$ "in the Middle" of $\Delta\mathbf{S}$ /

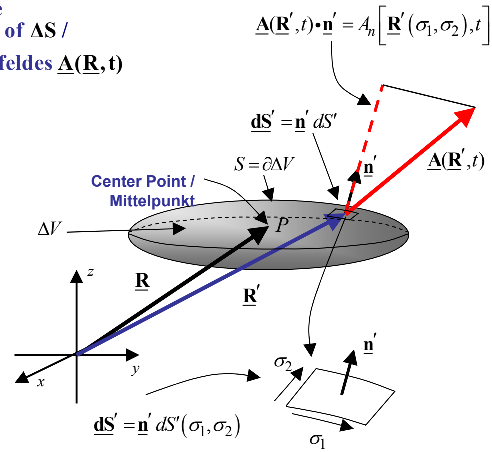
mit der Divergenz (Quellstärke) - **div** - des Vektorfeldes $\underline{\mathbf{A}}(\underline{\mathbf{R}},\mathbf{t})$ am Ort $\underline{\mathbf{R}}$ "in der Mitte" von $\Delta\mathbf{S}$

$$\nabla\bullet\underline{\mathbf{A}}(\underline{\mathbf{R}},t) = \lim_{\Delta V\to 0}\frac{1}{\Delta V}\oiint_{S=\partial\Delta V}\underline{\mathbf{A}}(\underline{\mathbf{R}}',t)\bullet d\underline{\mathbf{S}}'$$

$$= \lim_{\Delta V\to 0}\frac{1}{\Delta V}\oiint_{S=\partial\Delta V}\underbrace{\underline{\mathbf{A}}(\underline{\mathbf{R}}',t)\bullet\underline{\mathbf{n}}'}_{A_n(\underline{\mathbf{R}}',t)}\,dS'$$

= Strength of Flux (Divergence) of $\underline{\mathbf{A}}(\underline{\mathbf{R}},t)$ at the Position $\underline{\mathbf{R}}$

= Stärke des Flusses (Divergenz) von $\underline{\mathbf{A}}(\underline{\mathbf{R}},t)$ am Ort $\underline{\mathbf{R}}$



Del (Nabla) Operator in Orthogonal Curvilinear Coordinate System / Nabla-Operator im orthogonal krummlinigen Koordinatensystem

Del (Nabla) Operator / Nabla-Operator

$$\nabla = \underline{\mathbf{e}}_{\xi_1}\frac{1}{h_{\xi_1}}\frac{\partial}{\partial\xi_1} + \underline{\mathbf{e}}_{\xi_2}\frac{1}{h_{\xi_2}}\frac{\partial}{\partial\xi_2} + \underline{\mathbf{e}}_{\xi_3}\frac{1}{h_{\xi_3}}\frac{\partial}{\partial\xi_3}$$

$$= \sum_{i=1}^{3}\underline{\mathbf{e}}_{\xi_i}\frac{1}{h_{\xi_i}}\frac{\partial}{\partial\xi_i}$$

$$= \underline{\mathbf{e}}_{\xi_i}\frac{1}{h_{\xi_i}}\frac{\partial}{\partial\xi_i}$$

Generalized Curvilinear Coordinates /
Verallgemeinerte krummlinige Koordinaten

$$\xi_1,\xi_2,\xi_3 \quad \text{or} \quad \xi_i, i = 1,2,3$$

The del Operator / Der Nabla-Operator ∇ is a Vector / ist ein Vektor

Del (Nabla), Grad, Div, and Curl Operator in Cartesian Coordinate System / Nabla-, Grad-, Div- und Rot-Operator im Kartesischen Koordinatensystem

Del (Nabla) Operator / Nabla-Operator

$$\nabla = \underline{\mathbf{e}}_x \frac{\partial}{\partial x} + \underline{\mathbf{e}}_y \frac{\partial}{\partial y} + \underline{\mathbf{e}}_z \frac{\partial}{\partial z}$$

Gradient / Gradient

$$\text{grad} = \nabla = \underline{\mathbf{e}}_x \frac{\partial}{\partial x} + \underline{\mathbf{e}}_y \frac{\partial}{\partial y} + \underline{\mathbf{e}}_z \frac{\partial}{\partial z}$$

Divergence / Divergenz

$$\text{div} = \nabla\bullet = \left(\underline{\mathbf{e}}_x \frac{\partial}{\partial x} + \underline{\mathbf{e}}_y \frac{\partial}{\partial y} + \underline{\mathbf{e}}_z \frac{\partial}{\partial z} \right)\bullet$$

Curl / Rotation

$$\text{curl}\,/\,\text{rot} = \nabla\times = \left(\underline{\mathbf{e}}_x \frac{\partial}{\partial x} + \underline{\mathbf{e}}_y \frac{\partial}{\partial y} + \underline{\mathbf{e}}_z \frac{\partial}{\partial z} \right)\times$$

Vector-analytical Expressions in the Different Coordinate Systems / Vektoranalytische Ausdrücke in den verschiedenen Koordinatensystemen

	Cartesian Coordinates / Kartesische Koordinaten	Cylindrical Coordinates / Zylinderkoordinaten	Spherical Coordinates / Kugelkoordinaten
$\underline{\mathbf{dR}}$	$dx\,\underline{\mathbf{e}}_x + dy\,\underline{\mathbf{e}}_y + dz\,\underline{\mathbf{e}}_z$	$dr\,\underline{\mathbf{e}}_r + r d\varphi\,\underline{\mathbf{e}}_\varphi + dz\,\underline{\mathbf{e}}_z$	$dR\,\underline{\mathbf{e}}_R + R d\vartheta\,\underline{\mathbf{e}}_\vartheta + R\sin\vartheta\,d\varphi\,\underline{\mathbf{e}}_\varphi$
$\text{grad}\,\Phi$ $=\nabla\Phi$	$\frac{\partial\Phi}{\partial x}\underline{\mathbf{e}}_x + \frac{\partial\Phi}{\partial y}\underline{\mathbf{e}}_y + \frac{\partial\Phi}{\partial z}\underline{\mathbf{e}}_z$	$\frac{\partial\Phi}{\partial r}\underline{\mathbf{e}}_r + \frac{1}{r}\frac{\partial\Phi}{\partial\varphi}\underline{\mathbf{e}}_\varphi + \frac{\partial\Phi}{\partial z}\underline{\mathbf{e}}_z$	$\frac{\partial\Phi}{\partial R}\underline{\mathbf{e}}_R + \frac{1}{R}\frac{\partial\Phi}{\partial\vartheta}\underline{\mathbf{e}}_\vartheta + \frac{1}{R\sin\vartheta}\frac{\partial\Phi}{\partial\varphi}\underline{\mathbf{e}}_\varphi$
$\text{div}\,\underline{\mathbf{A}}$ $=\nabla\bullet\underline{\mathbf{A}}$	$\frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$	$\frac{1}{r}\frac{\partial(r A_r)}{\partial r} + \frac{1}{r}\frac{\partial A_\varphi}{\partial\varphi} + \frac{\partial A_z}{\partial z}$	$\frac{1}{R^2}\frac{\partial(R^2 A_R)}{\partial R} + \frac{1}{R\sin\vartheta}\frac{\partial(\sin\vartheta\,A_\vartheta)}{\partial\vartheta} + \frac{1}{R\sin\vartheta}\frac{\partial A_\varphi}{\partial\varphi}$
$\text{curl}\,\underline{\mathbf{A}}$ $=\text{rot}\,\underline{\mathbf{A}}$ $=\nabla\times\underline{\mathbf{A}}$	$\left[\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right]\underline{\mathbf{e}}_x + \left[\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right]\underline{\mathbf{e}}_y + \left[\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right]\underline{\mathbf{e}}_z$	$\left[\frac{1}{r}\frac{\partial A_z}{\partial\varphi} - \frac{\partial A_\varphi}{\partial z}\right]\underline{\mathbf{e}}_r + \left[\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r}\right]\underline{\mathbf{e}}_\varphi + \frac{1}{r}\left[\frac{\partial(r A_\varphi)}{\partial r} - \frac{\partial A_r}{\partial\varphi}\right]\underline{\mathbf{e}}_z$	$\frac{1}{R\sin\vartheta}\left[\frac{\partial(\sin\vartheta\,A_\varphi)}{\partial\vartheta} - \frac{\partial A_\vartheta}{\partial\varphi}\right]\underline{\mathbf{e}}_R + \frac{1}{R}\left[\frac{1}{\sin\vartheta}\frac{\partial A_R}{\partial\varphi} - \frac{\partial(R A_\varphi)}{\partial R}\right]\underline{\mathbf{e}}_\vartheta + \frac{1}{R}\left[\frac{\partial(R A_\vartheta)}{\partial R} - \frac{\partial A_R}{\partial\vartheta}\right]\underline{\mathbf{e}}_\varphi$

Vector-Analytical Expressions in the Different Coordinate Systems / Vektoranalytische Ausdrücke in den verschiedenen Koordinatensystemen

	Cartesian Coordinates / Kartesische Koordinaten	Cylindrical Coordinates / Zylinderkoordinaten	Spherical Coordinates / Kugelkoordinaten
$\operatorname{div}\operatorname{grad}\Phi = \nabla\cdot\nabla\Phi = \nabla^2\Phi = \Delta\Phi$	$\frac{\partial^2\Phi}{\partial x^2}+\frac{\partial^2\Phi}{\partial y^2}+\frac{\partial^2\Phi}{\partial z^2}$	$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\Phi}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2\Phi}{\partial\varphi^2}+\frac{\partial^2\Phi}{\partial z^2}$	$\frac{1}{R^2}\frac{\partial}{\partial R}\left(R^2\frac{\partial\Phi}{\partial R}\right)+\frac{1}{R^2\sin\vartheta}\frac{\partial}{\partial\vartheta}\left(\sin\vartheta\frac{\partial\Phi}{\partial\vartheta}\right)+\frac{1}{R^2\sin^2\vartheta}\left(\frac{\partial^2\Phi}{\partial\varphi^2}\right)$
$\operatorname{div}\operatorname{grad}\underline{\mathbf{A}} = \nabla\cdot\nabla\underline{\mathbf{A}} = \nabla^2\underline{\mathbf{A}} = \Delta\underline{\mathbf{A}}$	$\Delta A_x\underline{\mathbf{e}}_x + \Delta A_y\underline{\mathbf{e}}_y + \Delta A_z\underline{\mathbf{e}}_z$	$\left[\Delta A_r-\frac{1}{r^2}A_r-\frac{2}{r^2}\frac{\partial A_\varphi}{\partial\varphi}\right]\underline{\mathbf{e}}_r + \left[\Delta A_\varphi-\frac{1}{r^2}A_\varphi+\frac{2}{r^2}\frac{\partial A_r}{\partial\varphi}\right]\underline{\mathbf{e}}_\varphi + \Delta A_z\underline{\mathbf{e}}_z$	$\left[\Delta A_R-\frac{2}{R^2}A_R-\frac{2\cot\vartheta}{R^2}A_\vartheta-\frac{2}{R^2}\frac{\partial A_\vartheta}{\partial\vartheta}-\frac{2}{R^2\sin\vartheta}\frac{\partial A_\varphi}{\partial\varphi}\right]\underline{\mathbf{e}}_R + \left[\Delta A_\vartheta+\frac{2}{R^2}\frac{\partial A_R}{\partial\vartheta}-\frac{1}{R^2\sin^2\vartheta}A_\vartheta-\frac{2\cos\vartheta}{R^2\sin^2\vartheta}\frac{\partial A_\varphi}{\partial\varphi}\right]\underline{\mathbf{e}}_\vartheta + \left[\Delta A_\varphi+\frac{2}{R^2\sin\vartheta}\frac{\partial A_R}{\partial\varphi}-\frac{1}{R^2\sin^2\vartheta}A_\varphi+\frac{2\cos\vartheta}{R^2\sin^2\vartheta}\frac{\partial A_\vartheta}{\partial\varphi}\right]\underline{\mathbf{e}}_\varphi$

Governing Equations in Integral Form / Grundgleichungen in Integralform

Faraday's Induction Law / Faradaysches Induktionsgesetz

$$\oint_{C=\partial S}\underline{\mathbf{E}}(\underline{\mathbf{R}},t)\cdot\underline{\mathbf{dR}} = -\iint_S\frac{\partial}{\partial t}\underline{\mathbf{B}}(\underline{\mathbf{R}},t)\cdot\underline{\mathbf{dS}} - \iint_S\underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},t)\cdot\underline{\mathbf{dS}}$$

Integral Form / Integralform

$$\oint_{C=\partial S}\underline{\mathbf{E}}(\underline{\mathbf{R}},t)\cdot\underline{\mathbf{dR}} = \iint_S[\nabla\times\underline{\mathbf{E}}(\underline{\mathbf{R}},t)]\cdot\underline{\mathbf{dS}}$$

Stokes' Integral Theorem / Stokesscher Integralsatz

$$\iint_S[\nabla\times\underline{\mathbf{E}}(\underline{\mathbf{R}},t)]\cdot\underline{\mathbf{dS}} = -\iint_S\frac{\partial}{\partial t}\underline{\mathbf{B}}(\underline{\mathbf{R}},t)\cdot\underline{\mathbf{dS}} - \iint_S\underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},t)\cdot\underline{\mathbf{dS}}$$

$$\nabla\times\underline{\mathbf{E}}(\underline{\mathbf{R}},t) = -\frac{\partial}{\partial t}\underline{\mathbf{B}}(\underline{\mathbf{R}},t) - \underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},t)$$

Differential Form / Differentialform

Gauss' Electric Law / Gaußsches elektrisches Gesetz

$$\oiint_{S=\partial V}\underline{\mathbf{D}}(\underline{\mathbf{R}},t)\cdot\underline{\mathbf{dS}} = \iiint_V\rho_{\mathrm{e}}(\underline{\mathbf{R}},t)dV$$

Integral Form / Integralform

$$\oiint_{S=\partial V}\underline{\mathbf{D}}(\underline{\mathbf{R}},t)\cdot\underline{\mathbf{dS}} = \iiint_V\nabla\cdot\underline{\mathbf{D}}(\underline{\mathbf{R}},t)dV$$

Gauss' Integral Theorem / Gaußscher Integralsatz

$$\iiint_V\nabla\cdot\underline{\mathbf{D}}(\underline{\mathbf{R}},t)dV = \iiint_V\rho_{\mathrm{e}}(\underline{\mathbf{R}},t)dV$$

$$\nabla\cdot\underline{\mathbf{D}}(\underline{\mathbf{R}},t) = \rho_{\mathrm{e}}(\underline{\mathbf{R}},t)$$

Differential Form / Differentialform

Governing Equations in Differential Form / Grundgleichungen in Differentialform

Faraday's Induction Law / Faradaysches Induktionsgesetz

$$\text{curl/rot}\ \underline{\mathbf{E}}(\underline{\mathbf{R}},t) = -\frac{\partial}{\partial t}\underline{\mathbf{B}}(\underline{\mathbf{R}},t) - \underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},t)$$

Ampère-Maxwell's Circuital Law / Ampère-Maxwellsches Durchflutungsgesetz

$$\text{curl/rot}\ \underline{\mathbf{H}}(\underline{\mathbf{R}},t) = \frac{\partial}{\partial t}\underline{\mathbf{D}}(\underline{\mathbf{R}},t) + \underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}},t)$$

Gauss' Electric Law / Gaußsches elektrisches Gesetz

$$\text{div}\ \underline{\mathbf{D}}(\underline{\mathbf{R}},t) = \rho_{\mathrm{e}}(\underline{\mathbf{R}},t)$$

Gauss' Magnetic Law / Gaußsches magnetisches Gesetz

$$\text{div}\ \underline{\mathbf{B}}(\underline{\mathbf{R}},t) = \rho_{\mathrm{m}}(\underline{\mathbf{R}},t)$$

Continuity Equation for Electric Charges / Kontinuitätsgleichung für die elektrischer Ladungen

$$\text{div}\,\underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}},t) = -\frac{\partial}{\partial t}\rho_{\mathrm{e}}(\underline{\mathbf{R}},t)$$

Continuity Equation for Magnetic Charges / Kontinuitätsgleichung für die magnetischer Ladungen

$$\text{div}\,\underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},t) = -\frac{\partial}{\partial t}\rho_{\mathrm{m}}(\underline{\mathbf{R}},t)$$

Del (Nabla), Grad, Div, and Curl Operator in Cartesian Coordinate System / Nabla-, Grad-, Div- und Rot-Operator im Kartesischen Koordinatensystem

Del (Nabla) Operator / Nabla-Operator

$$\nabla = \underline{\mathbf{e}}_x\frac{\partial}{\partial x} + \underline{\mathbf{e}}_y\frac{\partial}{\partial y} + \underline{\mathbf{e}}_z\frac{\partial}{\partial z}$$

Gradient / Gradient

$$\text{grad} = \nabla = \underline{\mathbf{e}}_x\frac{\partial}{\partial x} + \underline{\mathbf{e}}_y\frac{\partial}{\partial y} + \underline{\mathbf{e}}_z\frac{\partial}{\partial z}$$

Divergence / Divergenz

$$\text{div} = \nabla\bullet = \left(\underline{\mathbf{e}}_x\frac{\partial}{\partial x} + \underline{\mathbf{e}}_y\frac{\partial}{\partial y} + \underline{\mathbf{e}}_z\frac{\partial}{\partial z}\right)\bullet$$

Curl / Rotation

$$\text{curl/rot} = \nabla\times = \left(\underline{\mathbf{e}}_x\frac{\partial}{\partial x} + \underline{\mathbf{e}}_y\frac{\partial}{\partial y} + \underline{\mathbf{e}}_z\frac{\partial}{\partial z}\right)\times$$

Governing Equations in Differential Form / Grundgleichungen in Differentialform

Faraday's Induction Law / Faradaysches Induktionsgesetz

$$\nabla \times \underline{\mathbf{E}}(\underline{\mathbf{R}},t) = -\frac{\partial}{\partial t}\underline{\mathbf{B}}(\underline{\mathbf{R}},t) - \underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},t)$$

Ampère-Maxwell's Circuital Law / Ampère-Maxwellsches Durchflutungsgesetz

$$\nabla \times \underline{\mathbf{H}}(\underline{\mathbf{R}},t) = \frac{\partial}{\partial t}\underline{\mathbf{D}}(\underline{\mathbf{R}},t) + \underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}},t)$$

Gauss' Electric Law / Gaußsches elektrisches Gesetz

$$\nabla \bullet \underline{\mathbf{D}}(\underline{\mathbf{R}},t) = \rho_{\mathrm{e}}(\underline{\mathbf{R}},t)$$

Gauss' Magnetic Law / Gaußsches magnetisches Gesetz

$$\nabla \bullet \underline{\mathbf{B}}(\underline{\mathbf{R}},t) = \rho_{\mathrm{m}}(\underline{\mathbf{R}},t)$$

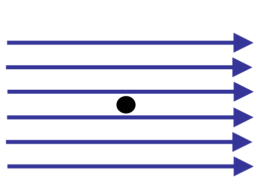
Continuity Equation for Electric Charges / Kontinuitätsgleichung für die elektrischer Ladungen

$$\nabla \bullet \underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}},t) = -\frac{\partial}{\partial t}\rho_{\mathrm{e}}(\underline{\mathbf{R}},t)$$

Continuity Equation for Magnetic Charges / Kontinuitätsgleichung für die magnetischer Ladungen

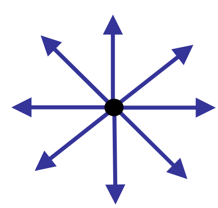
$$\nabla \bullet \underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},t) = -\frac{\partial}{\partial t}\rho_{\mathrm{m}}(\underline{\mathbf{R}},t)$$

Div and Curl Examples / Div und Rot Beispiele



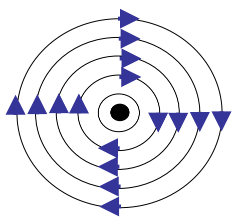
$$\mathrm{div}\,\underline{\mathbf{A}} = \nabla \bullet \underline{\mathbf{A}} = 0$$

$$\mathrm{curl}/\mathrm{rot}\,\underline{\mathbf{A}} = \nabla \times \underline{\mathbf{A}} = \underline{\mathbf{0}}$$



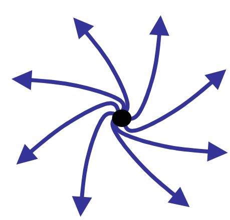
$$\mathrm{div}\,\underline{\mathbf{A}} = \nabla \bullet \underline{\mathbf{A}} \neq 0$$

$$\mathrm{curl}/\mathrm{rot}\,\underline{\mathbf{A}} = \nabla \times \underline{\mathbf{A}} = \underline{\mathbf{0}}$$



$$\mathrm{div}\,\underline{\mathbf{A}} = \nabla \bullet \underline{\mathbf{A}} = 0$$

$$\mathrm{curl}/\mathrm{rot}\,\underline{\mathbf{A}} = \nabla \times \underline{\mathbf{A}} \neq \underline{\mathbf{0}}$$



$$\mathrm{div}\,\underline{\mathbf{A}} = \nabla \bullet \underline{\mathbf{A}} \neq 0$$

$$\mathrm{curl}/\mathrm{rot}\,\underline{\mathbf{A}} = \nabla \times \underline{\mathbf{A}} \neq \underline{\mathbf{0}}$$

Grad, Div and Curl Examples / Grad, Div und Rot Beispiele

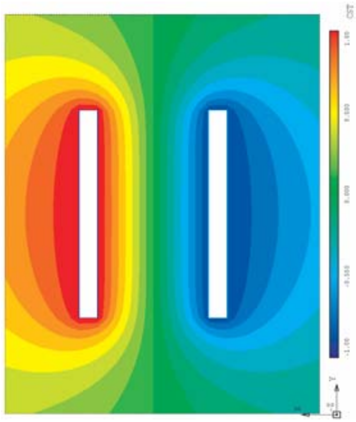
Scalar Field / Skalarfeld

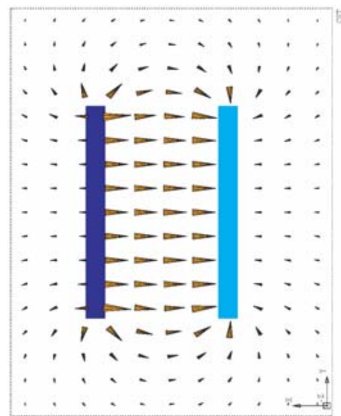
$$\Phi_{\mathrm{e}}(\underline{\mathbf{R}})$$

Vektor Field / Vektorfeld

$$\underline{\mathbf{E}}(\underline{\mathbf{R}}) = -\mathrm{grad}\;\Phi_{\mathrm{e}}(\underline{\mathbf{R}}) = -\nabla\Phi_{\mathrm{e}}(\underline{\mathbf{R}})$$

$$\mathrm{grad}\;\Phi_{\mathrm{e}}(\underline{\mathbf{R}}) = \nabla\,\Phi_{\mathrm{e}}(\underline{\mathbf{R}}) = \underline{\mathbf{e}}_x\,\frac{\partial}{\partial x}\Phi_{\mathrm{e}}(x,y,z) + \underline{\mathbf{e}}_y\,\frac{\partial}{\partial y}\Phi_{\mathrm{e}}(x,y,z) + \underline{\mathbf{e}}_z\,\frac{\partial}{\partial z}\Phi_{\mathrm{e}}(x,y,z)$$





Grad, Div and Curl Examples / Grad, Div und Rot Beispiele

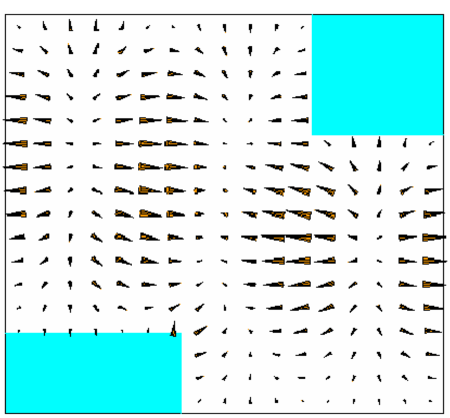
Vector Field / Vektorfeld

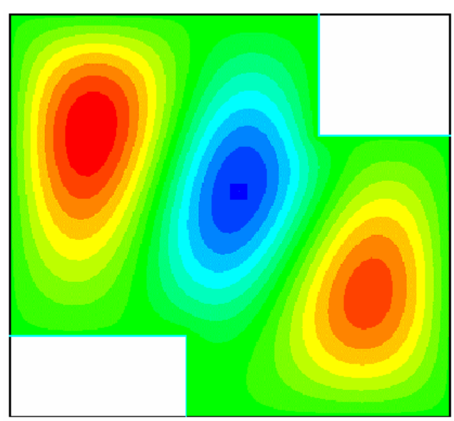
$$\underline{\mathbf{D}}(\underline{\mathbf{R}})$$

Scalar Field / Skalarfeld

$$\rho_{\mathrm{e}}(\underline{\mathbf{R}}) = \mathrm{div}\;\underline{\mathbf{D}}(\underline{\mathbf{R}}) = \nabla \bullet \underline{\mathbf{D}}(\underline{\mathbf{R}})$$

$$\mathrm{div}\;\underline{\mathbf{D}}(\underline{\mathbf{R}}) = \nabla\bullet\,\underline{\mathbf{D}}(\underline{\mathbf{R}}) = \frac{\partial}{\partial x}\mathrm{D}_x(x,y,z) + \frac{\partial}{\partial y}\mathrm{D}_y(x,y,z) + \frac{\partial}{\partial z}\mathrm{D}_z(x,y,z)$$





Grad, Div and Curl Examples / Grad, Div und Rot Beispiele

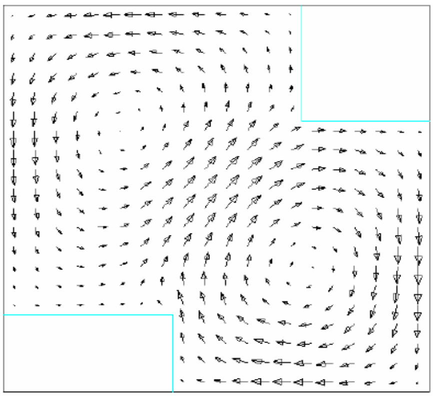
Vector Field / Vektorfeld

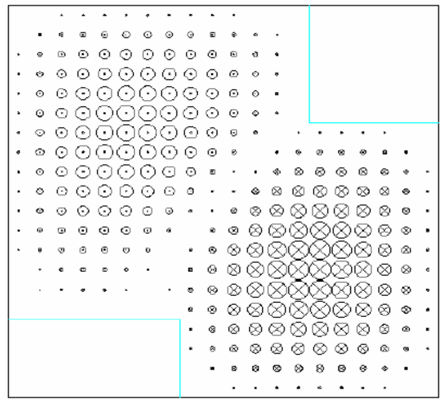
$\underline{\mathbf{E}}(\underline{\mathbf{R}},t)$

Vector Field / Vektorfeld

$$\text{curl/rot } \underline{\mathbf{E}}(\underline{\mathbf{R}},t) = \nabla \times \underline{\mathbf{E}}(\underline{\mathbf{R}},t) = -\frac{\partial}{\partial t}\underline{\mathbf{B}}(\underline{\mathbf{R}},t)$$

$$\text{curl / rot } \underline{\mathbf{E}}(\underline{\mathbf{R}},t) = \nabla \times \underline{\mathbf{E}}(\underline{\mathbf{R}},t) = \left(\underline{\mathbf{e}}_x \frac{\partial}{\partial x} + \underline{\mathbf{e}}_y \frac{\partial}{\partial y} + \underline{\mathbf{e}}_z \frac{\partial}{\partial z}\right) \times \underline{\mathbf{E}}(x,y,z,t)$$





End of 3rd Lecture / Ende der 3. Vorlesung