Elektromagnetische Feldtheorie I (EFT I) / Electromagnetic Field Theory I (EFT I)

4th Lecture / 4. Vorlesung

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Exam EFT I / Prüfung EFT I

2511 Elektromagnetische Feldtheorie 23.09.2003, 14:00, R 1603

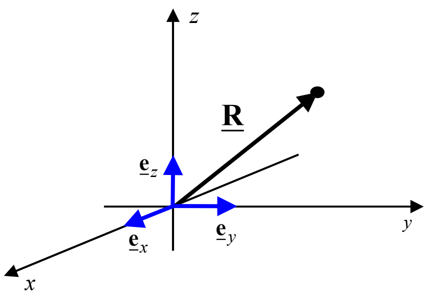
Metric Coefficients / Metrische Koeffizienten

Cartesian Coordinates / Kartesische Koordinaten

$$x, y, z = x_1, x_2, x_3$$

$$\underline{\mathbf{e}}_x, \underline{\mathbf{e}}_y, \underline{\mathbf{e}}_z = \underline{\mathbf{e}}_{x_1}, \underline{\mathbf{e}}_{x_2}, \underline{\mathbf{e}}_{x_3}$$

$$\underline{\mathbf{e}}_x, \perp \underline{\mathbf{e}}_y \perp \underline{\mathbf{e}}_z \;;\; \underline{\mathbf{e}}_{x_1} \perp \underline{\mathbf{e}}_{x_2} \perp \underline{\mathbf{e}}_{x_3}$$

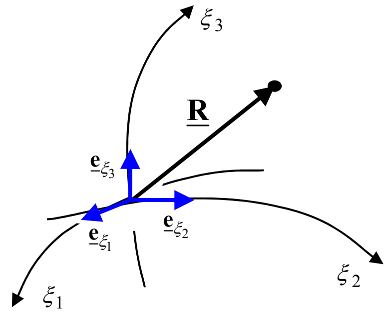


Orthogonal Curvilinear Coordinates / Orthogonale Krummlinige Koordinaten

$$\xi_1, \xi_2, \xi_3$$

$$\underline{\mathbf{e}}_{\xi_1}, \underline{\mathbf{e}}_{\xi_2}, \underline{\mathbf{e}}_{\xi_3}$$

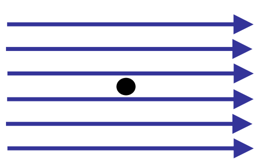
$$\underline{\mathbf{e}}_{\xi_1} \perp \underline{\mathbf{e}}_{\xi_2} \perp \underline{\mathbf{e}}_{\xi_3}$$



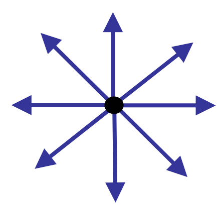
Cartesian Coordinates / Kartesische Koordinaten

$$x = x(\xi_1, \xi_2, \xi_3) \qquad \xi_1 = \xi_1(x, y, z)$$
$$y = y(\xi_1, \xi_2, \xi_3) \quad \longleftrightarrow \quad \xi_2 = \xi_2(x, y, z)$$
$$z = z(\xi_1, \xi_2, \xi_3) \qquad \xi_3 = \xi_3(x, y, z)$$

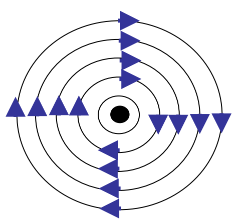
Div and Curl Examples / Div und Rot Beispiele



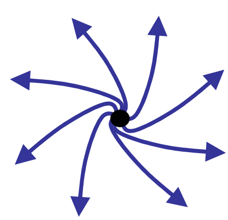
$$\text{div}\,\underline{\mathbf{A}} = \nabla \bullet \underline{\mathbf{A}} = 0$$
$$\text{curl / rot}\,\underline{\mathbf{A}} = \nabla \times \underline{\mathbf{A}} = \underline{\mathbf{0}}$$



$$\text{div}\,\underline{\mathbf{A}} = \nabla \bullet \underline{\mathbf{A}} \neq 0$$
$$\text{curl / rot}\,\underline{\mathbf{A}} = \nabla \times \underline{\mathbf{A}} = \underline{\mathbf{0}}$$



$$\text{div}\,\underline{\mathbf{A}} = \nabla \bullet \underline{\mathbf{A}} = 0$$
$$\text{curl / rot}\,\underline{\mathbf{A}} = \nabla \times \underline{\mathbf{A}} \neq \underline{\mathbf{0}}$$



$$\text{div}\,\underline{\mathbf{A}} = \nabla \bullet \underline{\mathbf{A}} \neq 0$$
$$\text{curl / rot}\,\underline{\mathbf{A}} = \nabla \times \underline{\mathbf{A}} \neq \underline{\mathbf{0}}$$

Grad, Div and Curl Examples / Grad, Div und Rot Beispiele

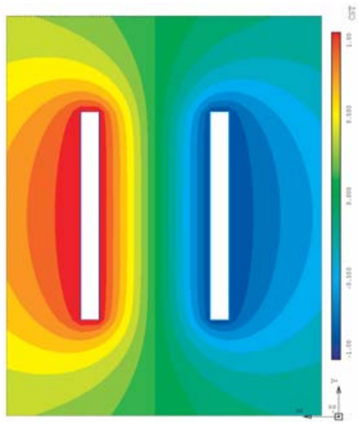
Scalar Field / Skalarfeld

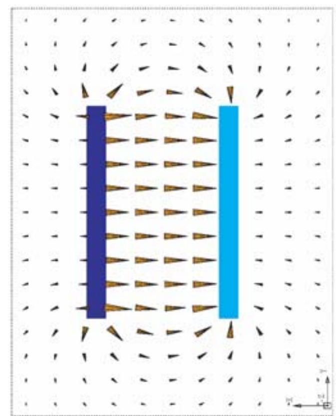
$$\Phi_{\mathrm{e}}(\underline{\mathbf{R}})$$

Vektor Field / Vektorfeld

$$\underline{\mathbf{E}}(\underline{\mathbf{R}}) = -\mathrm{grad}\ \Phi_{\mathrm{e}}(\underline{\mathbf{R}}) = -\nabla\Phi_{\mathrm{e}}(\underline{\mathbf{R}})$$

$$\mathrm{grad}\ \Phi_{\mathrm{e}}(\underline{\mathbf{R}}) = \nabla\,\Phi_{\mathrm{e}}(\underline{\mathbf{R}}) = \underline{\mathbf{e}}_x \frac{\partial}{\partial x}\Phi_{\mathrm{e}}(x,y,z) + \underline{\mathbf{e}}_y \frac{\partial}{\partial y}\Phi_{\mathrm{e}}(x,y,z) + \underline{\mathbf{e}}_z \frac{\partial}{\partial z}\Phi_{\mathrm{e}}(x,y,z)$$





Grad, Div and Curl Examples / Grad, Div und Rot Beispiele

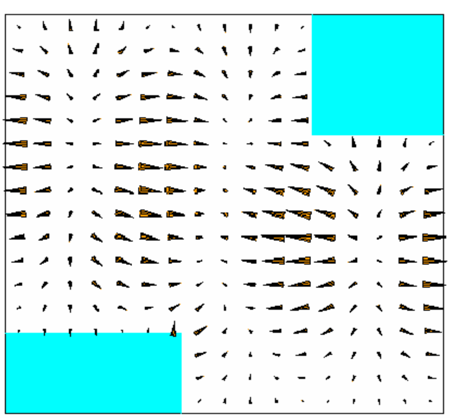
Vector Field / Vektorfeld

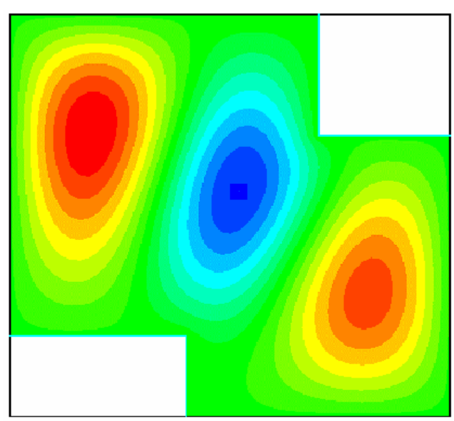
$$\underline{\mathbf{D}}(\underline{\mathbf{R}})$$

Scalar Field / Skalarfeld

$$\rho_{\mathrm{e}}(\underline{\mathbf{R}}) = \mathrm{div}\ \underline{\mathbf{D}}(\underline{\mathbf{R}}) = \nabla \bullet \underline{\mathbf{D}}(\underline{\mathbf{R}})$$

$$\mathrm{div}\ \underline{\mathbf{D}}(\underline{\mathbf{R}}) = \nabla\bullet\,\underline{\mathbf{D}}(\underline{\mathbf{R}}) = \frac{\partial}{\partial x}\mathrm{D}_x(x,y,z) + \frac{\partial}{\partial y}\mathrm{D}_y(x,y,z) + \frac{\partial}{\partial z}\mathrm{D}_z(x,y,z)$$





Grad, Div and Curl Examples / Grad, Div und Rot Beispiele

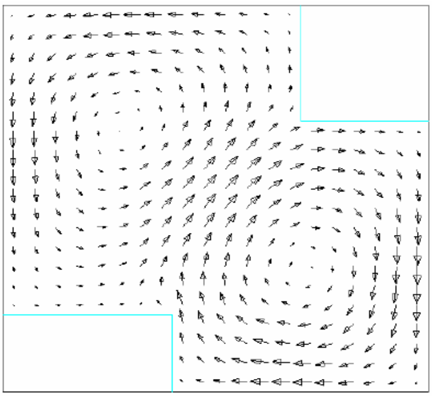
Vector Field / Vektorfeld

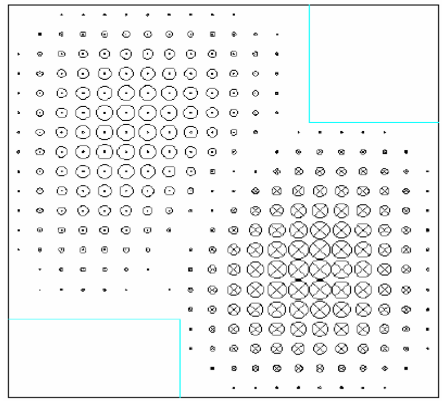
$$\underline{\mathbf{E}}(\underline{\mathbf{R}},t)$$

Vector Field / Vektorfeld

$$\text{curl/rot } \underline{\mathbf{E}}(\underline{\mathbf{R}},t) = \nabla \times \underline{\mathbf{E}}(\underline{\mathbf{R}},t) = -\frac{\partial}{\partial t}\underline{\mathbf{B}}(\underline{\mathbf{R}},t)$$

$$\text{curl / rot } \underline{\mathbf{E}}(\underline{\mathbf{R}},t) = \nabla \times \underline{\mathbf{E}}(\underline{\mathbf{R}},t) = \left(\underline{\mathbf{e}}_x \frac{\partial}{\partial x} + \underline{\mathbf{e}}_y \frac{\partial}{\partial y} + \underline{\mathbf{e}}_z \frac{\partial}{\partial z}\right) \times \underline{\mathbf{E}}(x,y,z,t)$$



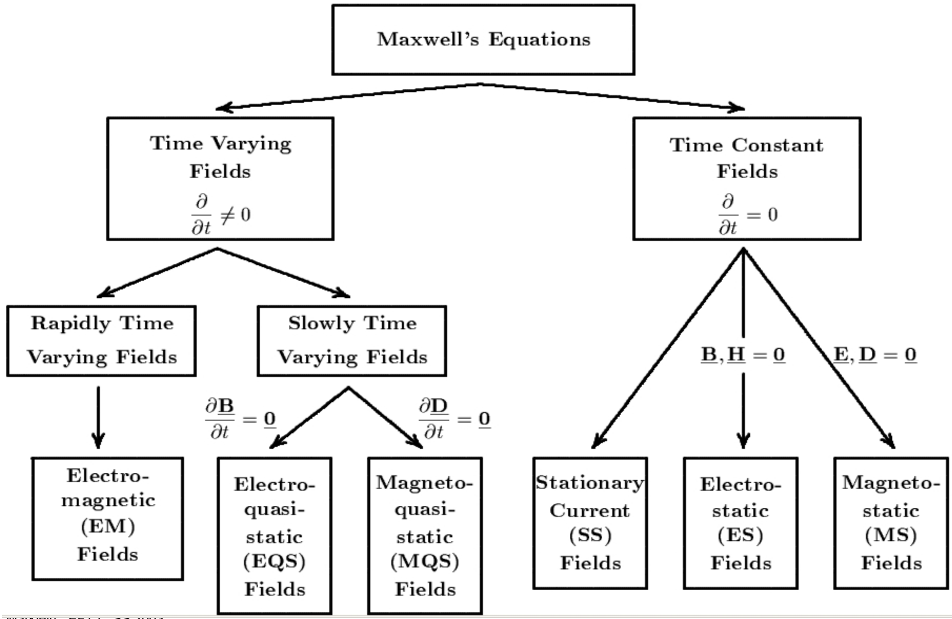


Classification of Maxwell‘s Equations / Klassifikation der Maxwellschen Gleichungen (1)

- **Time Varying Electromagnetic Fields / Zeitveränderliche elektromagnetische Felder**
  - **Rapidly Time Varying Fields / Zeitlich schnell veränderliche Felder**
    - **Electromagnetic (EM) Fields / Elektromagnetische (EM) Felder**
  - **Slowly Time Varying Fields / Zeitlich langsam veränderliche Felder**
    - **Electroquasistatic Fields (EQS) / Elektroquasistatische Felder (EQS)**
    - **Magnetoquasistatic Fields (MQS) / Magnetoquasistatische Felder (MQS)**

- **Time Constant Electromagnetic Fields / Zeitlich konstante elektromagnetische Felder**
  - **Stationary Current (SC) Fields / Stationäre Strömungsfelder (SS)**
  - **Electrostatic (ES) Fields / Elektrostatische (ES) Felder**
  - **Magnetostatic (MS) Fields / Magnetostatische (MS) Felder**

Classification of Maxwell's Equations / Klassifikation der Maxwellschen Gleichungen (2)



Classification … / Klassifikation ... (3)

Rapidly Time Varying Electromagnetic (EM) Fields / Zeitlich schnell veränderliche elektromagnetische (EM) Felder

Governing Equations in Integral Form / Grundgleichungen in Integralform

$$\oint_{C=\partial S} \underline{\mathbf{E}}(\underline{\mathbf{R}},t)\bullet\underline{\mathbf{dR}} = -\iint_S \frac{\partial}{\partial t}\underline{\mathbf{B}}(\underline{\mathbf{R}},t)\bullet\underline{\mathbf{dS}} - \iint_S \underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},t)\bullet\underline{\mathbf{dS}}$$

$$\oint_{C=\partial S} \underline{\mathbf{H}}(\underline{\mathbf{R}},t)\bullet\underline{\mathbf{dR}} = \iint_S \frac{\partial}{\partial t}\underline{\mathbf{D}}(\underline{\mathbf{R}},t)\bullet\underline{\mathbf{dS}} + \iint_S \underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}},t)\bullet\underline{\mathbf{dS}}$$

$$\oiint_{S=\partial V} \underline{\mathbf{D}}(\underline{\mathbf{R}},t)\bullet\underline{\mathbf{dS}} = \iiint_V \rho_{\mathrm{e}}(\underline{\mathbf{R}},t)\,\mathrm{d}V$$

$$\oiint_{S=\partial V} \underline{\mathbf{B}}(\underline{\mathbf{R}},t)\bullet\underline{\mathbf{dS}} = \iiint_V \rho_{\mathrm{m}}(\underline{\mathbf{R}},t)\,\mathrm{d}V$$

$$\oiint_{S=\partial V} \underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}},t)\bullet\underline{\mathbf{dS}} = -\iiint_V \frac{\partial}{\partial t}\rho_{\mathrm{e}}(\underline{\mathbf{R}},t)\,\mathrm{d}V$$

$$\oiint_{S=\partial V} \underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},t)\bullet\underline{\mathbf{dS}} = -\iiint_V \frac{\partial}{\partial t}\rho_{\mathrm{m}}(\underline{\mathbf{R}},t)\,\mathrm{d}V$$

Governing Equations in Differential Form / Grundgleichungen in Differentialform

$$\nabla\times\underline{\mathbf{E}}(\underline{\mathbf{R}},t) = -\frac{\partial}{\partial t}\underline{\mathbf{B}}(\underline{\mathbf{R}},t) - \underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},t)$$

$$\nabla\times\underline{\mathbf{H}}(\underline{\mathbf{R}},t) = \frac{\partial}{\partial t}\underline{\mathbf{D}}(\underline{\mathbf{R}},t) + \underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}},t)$$

$$\nabla\bullet\underline{\mathbf{D}}(\underline{\mathbf{R}},t) = \rho_{\mathrm{e}}(\underline{\mathbf{R}},t)$$

$$\nabla\bullet\underline{\mathbf{B}}(\underline{\mathbf{R}},t) = \rho_{\mathrm{m}}(\underline{\mathbf{R}},t)$$

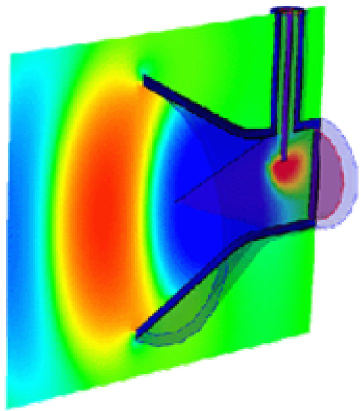
$$\nabla\bullet\underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}},t) = -\frac{\partial}{\partial t}\rho_{\mathrm{e}}(\underline{\mathbf{R}},t)$$

$$\nabla\bullet\underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},t) = -\frac{\partial}{\partial t}\rho_{\mathrm{m}}(\underline{\mathbf{R}},t)$$

Classification … / Klassifikation ... (4)

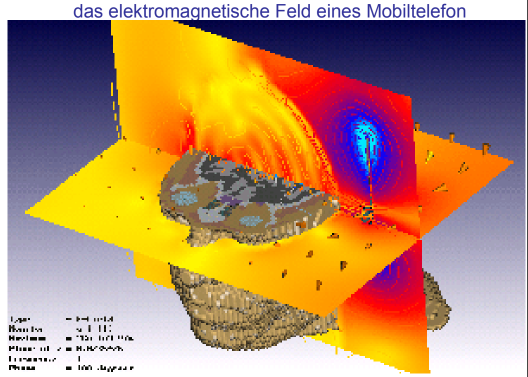
Rapidly Time Varying Electromagnetic (EM) Fields: Examples / Zeitlich schnell veränderliche elektromagnetische (EM) Felder: Beispiele

Horn Antenna: Contour Plot of Electric Field Strength Vector (E_y Component) / Hornantenne: Konturdarstellung des elektrischen Feldstärkevektors (E_y-Komponente)



(CST Microwave Studio, www.cst.de)

Biomedical Application: Human head model irradiated by the electromagnetic field of a mobile phone / Biomedizinische Anwendung: Menschliches Kopfmodell bei Bestrahlung durch das elektromagnetische Feld eines Mobiltelefon



(CST Microwave Studio, www.cst.de)

Classification … / Klassifikation ... (5)

Electroquasistatic (EQS) Fields / Elektroquasistatische (EQS) Felder

$$\frac{\partial}{\partial t}\underline{\mathbf{B}}(\underline{\mathbf{R}},t) = \underline{\mathbf{0}} \qquad \left(\underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},t) = \underline{\mathbf{0}}\right)$$

(Neglect Magnetic Induction / Vernachlässige magnetische Induktion)

With the typical length *L* / Mit der Typischen Länge *L*

$$L \ll \lambda_{\mathrm{em}} = cT$$

Magnetoquasistatic (MQS) Fields / Magnetoquasistatische (MQS) Felder

$$\frac{\partial}{\partial t}\underline{\mathbf{D}}(\underline{\mathbf{R}},t) = \underline{\mathbf{0}} \qquad \left(\underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},t) = \underline{\mathbf{0}}\right)$$

(Neglect Displacement Current / Vernachlässige Verschiebungsstrom)

λ_{em} : Electromagnetic Wavelength / Elektromagnetische Wellenlänge
c : Propagation Velocity / Ausbreitungsgeschwindigkeit
T : Length of Period / Periodenlänge

Governing Equations in Integral Form / Grundgleichungen in Integralform

EQS:

$$\oint_{C=\partial S}\underline{\mathbf{E}}(\underline{\mathbf{R}},t)\bullet\underline{\mathbf{dR}} = 0$$

$$\oiint_{S=\partial V}\underline{\mathbf{D}}(\underline{\mathbf{R}},t)\bullet\underline{\mathbf{dS}} = \iiint_V \rho_{\mathrm{e}}(\underline{\mathbf{R}},t)\,\mathrm{d}V$$

$$\oiint_{S=\partial V}\underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}},t)\bullet\underline{\mathbf{dS}} = -\iiint_V \frac{\partial}{\partial t}\rho_{\mathrm{e}}(\underline{\mathbf{R}},t)\,\mathrm{d}V$$

MQS:

$$\oint_{C=\partial S}\underline{\mathbf{E}}(\underline{\mathbf{R}},t)\bullet\underline{\mathbf{dR}} = -\iint_S \frac{\partial}{\partial t}\underline{\mathbf{B}}(\underline{\mathbf{R}},t)\bullet\underline{\mathbf{dS}}$$

$$\oint_{C=\partial S}\underline{\mathbf{H}}(\underline{\mathbf{R}},t)\bullet\underline{\mathbf{dR}} = \iint_S \underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}},t)\bullet\underline{\mathbf{dS}}$$

$$\oiint_{S=\partial V}\underline{\mathbf{B}}(\underline{\mathbf{R}},t)\bullet\underline{\mathbf{dS}} = 0$$

$$\oiint_{S=\partial V}\underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}},t)\bullet\underline{\mathbf{dS}} = 0$$

Governing Equations in Differential Form / Grundgleichungen in Differentialform

EQS:

$$\nabla\times\underline{\mathbf{E}}(\underline{\mathbf{R}},t) = \underline{\mathbf{0}}$$

$$\nabla\bullet\underline{\mathbf{D}}(\underline{\mathbf{R}},t) = \rho_{\mathrm{e}}(\underline{\mathbf{R}},t)$$

$$\nabla\bullet\underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}},t) = -\frac{\partial}{\partial t}\rho_{\mathrm{e}}(\underline{\mathbf{R}},t)$$

MQS:

$$\nabla\times\underline{\mathbf{E}}(\underline{\mathbf{R}},t) = -\frac{\partial}{\partial t}\underline{\mathbf{B}}(\underline{\mathbf{R}},t)$$

$$\nabla\times\underline{\mathbf{H}}(\underline{\mathbf{R}},t) = \underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}},t)$$

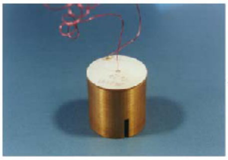
$$\nabla\bullet\underline{\mathbf{B}}(\underline{\mathbf{R}},t) = 0$$

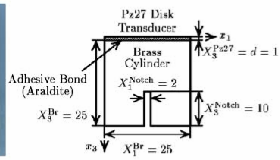
$$\nabla\bullet\underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}},t) = 0$$

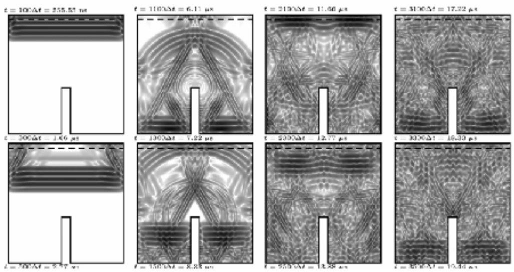
Classification … / Klassifikation ... (6)

Electroquasistatic (EQS) Fields: Example / Elektroquasistatische (EQS) Felder: Beispiel

Non-Destructive Testing: Piezoelectric Sensor /
Zerstörungsfreie Materialprüfung: Piezoelektrischer Sensor

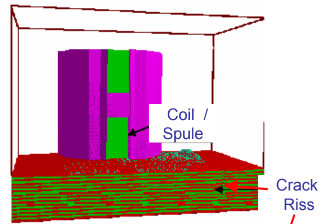




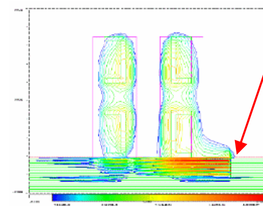


Magnetoquasistatic (MQS) Fields: Example / Magnetoquasistatische (MQS) Felder: Beispiel

Non-Destructive Testing: Eddy Current Sensor /
Zerstörungsfreie Materialprüfung: Wirbelstromsensor



Geometry / Geometrie



Electric Energy Density / Elektrische Energiedichte

Classification … / Klassifikation ... (7)

Stationary Electric Current (SEC) Fields / Stationäre elektrische Strömungsfelder (SEC) Felder

$$\frac{\partial}{\partial t}\{\underline{\mathbf{E}},\underline{\mathbf{B}},\underline{\mathbf{H}},\underline{\mathbf{D}},\underline{\mathbf{J}}_{\mathrm{e}},\underline{\mathbf{J}}_{\mathrm{m}},\rho_{\mathrm{e}},\rho_{\mathrm{m}}\}=\{\underline{\mathbf{0}},0\}$$

$$\{\underline{\mathbf{J}}_{\mathrm{m}},\rho_{\mathrm{m}}\}=\{\underline{\mathbf{0}},0\}$$

$$\nabla\times\underline{\mathbf{E}}(\underline{\mathbf{R}})=\underline{\mathbf{0}}$$

$$\underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}})=\underline{\mathbf{J}}_{\mathrm{ec}}(\underline{\mathbf{R}})=\sigma_{\mathrm{e}}(\underline{\mathbf{R}})\underline{\mathbf{E}}(\underline{\mathbf{R}})$$

$$\nabla\bullet\underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}},t)=0$$

Governing Equations in Integral Form / Grundgleichungen in Integralform

$$\oint_{C=\partial S}\underline{\mathbf{E}}(\underline{\mathbf{R}})\bullet\underline{\mathbf{dR}}=0$$

$$\oint_{C=\partial S}\underline{\mathbf{H}}(\underline{\mathbf{R}})\bullet\underline{\mathbf{dR}}=\oiint_{S=\partial V}\underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}})\bullet\underline{\mathbf{dS}}$$

$$\oiint_{S=\partial V}\underline{\mathbf{D}}(\underline{\mathbf{R}})\bullet\underline{\mathbf{dS}}=\iiint_V\rho_{\mathrm{e}}(\underline{\mathbf{R}})\,\mathrm{d}V$$

$$\oiint_{S=\partial V}\underline{\mathbf{B}}(\underline{\mathbf{R}})\bullet\underline{\mathbf{dS}}=0$$

Governing Equations in Differential Form / Grundgleichungen in Differentialform

$$\nabla\times\underline{\mathbf{E}}(\underline{\mathbf{R}})=\underline{\mathbf{0}}$$

$$\nabla\times\underline{\mathbf{H}}(\underline{\mathbf{R}})=\underline{\mathbf{J}}_{\mathrm{ec}}(\underline{\mathbf{R}})$$

$$\nabla\bullet\underline{\mathbf{D}}(\underline{\mathbf{R}})=\rho_{\mathrm{e}}(\underline{\mathbf{R}})$$

$$\nabla\bullet\underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}})=0$$

Classification … / Klassifikation ... (8)

Electrostatic (ES) Fields / Elektrostatische (ES) Felder

$$\frac{\partial}{\partial t}\left\{\underline{\mathbf{E}},\underline{\mathbf{D}},\rho_\mathrm{e}\right\}=\left\{\underline{\mathbf{0}},0\right\}$$

$$\left\{\underline{\mathbf{B}},\underline{\mathbf{H}},\underline{\mathbf{J}}_\mathrm{e},\underline{\mathbf{J}}_\mathrm{m},\rho_\mathrm{m}\right\}=\left\{\underline{\mathbf{0}},0\right\}$$

Magnetostatic (MS) Fields / Magnetostatische (MS) Felder

$$\frac{\partial}{\partial t}\left\{\underline{\mathbf{H}},\underline{\mathbf{B}},\underline{\mathbf{J}}_\mathrm{e}\right\}=\left\{\underline{\mathbf{0}}\right\}$$

$$\left\{\underline{\mathbf{E}},\underline{\mathbf{D}},\underline{\mathbf{J}}_\mathrm{m},\rho_\mathrm{e},\rho_\mathrm{m}\right\}=\left\{\underline{\mathbf{0}},0\right\}$$

with / mit

$$\underline{\mathbf{J}}_\mathrm{e}(\underline{\mathbf{R}})=\underline{\mathbf{J}}_\mathrm{ei}(\underline{\mathbf{R}})$$

Governing Equations in Integral Form / Grundgleichungen in Integralform

$$\oint_{C=\partial S}\underline{\mathbf{E}}(\underline{\mathbf{R}})\cdot\underline{\mathbf{dR}}=0$$

$$\oiint_{S=\partial V}\underline{\mathbf{D}}(\underline{\mathbf{R}})\cdot\underline{\mathbf{dS}}=\iiint_V\rho_\mathrm{e}(\underline{\mathbf{R}})\,\mathrm{d}V$$

$$\oint_{C=\partial S}\underline{\mathbf{H}}(\underline{\mathbf{R}})\cdot\underline{\mathbf{dR}}=\iint_S\underline{\mathbf{J}}_\mathrm{ei}(\underline{\mathbf{R}})\cdot\underline{\mathbf{dS}}$$

$$\oiint_{S=\partial V}\underline{\mathbf{B}}(\underline{\mathbf{R}})\cdot\underline{\mathbf{dS}}=0$$

Governing Equations in Differential Form / Grundgleichungen in Differentialform

$$\nabla\times\underline{\mathbf{E}}(\underline{\mathbf{R}})=\underline{\mathbf{0}}$$

$$\nabla\cdot\underline{\mathbf{D}}(\underline{\mathbf{R}})=\rho_\mathrm{e}(\underline{\mathbf{R}})$$

$$\nabla\times\underline{\mathbf{H}}(\underline{\mathbf{R}})=\underline{\mathbf{J}}_\mathrm{ei}(\underline{\mathbf{R}})$$

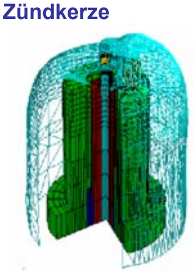
$$\nabla\cdot\underline{\mathbf{B}}(\underline{\mathbf{R}})=0$$

Classification … / Klassifikation ... (9)

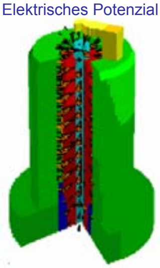
Electrostatic (ES) Fields: Example / Elektrostatische (ES) Felder: Beispiel

Spark Plug / Zündkerze

Elektric Field Strength / Elektrische Feldstärke



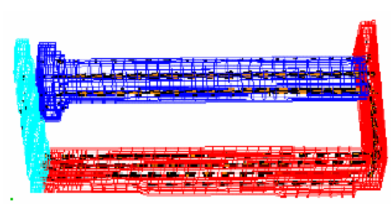
Elektric Potential / Elektrisches Potenzial



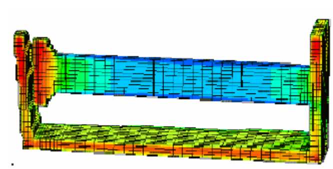
Elektric Field Strength / Elektrische Feldstärke

Magnetostatic (MS) Fields / Magnetostatische (MS) Felder

Relay / Relais



Magnetic Flux Density / Magnetische Flussdichte



Permeability / Permeabilität

Faraday‘s Induction Law / Faradaysches Induktionsgesetz

$$\nabla \times \underline{\mathbf{E}}(\underline{\mathbf{R}},t) = -\frac{\partial}{\partial t}\underline{\mathbf{B}}(\underline{\mathbf{R}},t) - \underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},t)$$

$$\underbrace{\oint_{C=\partial S} \underline{\mathbf{E}}(\underline{\mathbf{R}},t)\bullet \underline{\mathbf{dR}}}_{\text{Closed Contour Integral / Geschlossenes Kontur- oder Linienintegral}} = -\frac{d}{dt}\underbrace{\iint_S \underline{\mathbf{B}}(\underline{\mathbf{R}},t)\bullet \underline{\mathbf{dS}}}_{\text{(Open) Surface Integral / (Offenes) Flächenintegral}} - \underbrace{\iint_S \underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},t)\bullet \underline{\mathbf{dS}}}_{\text{(Open) Surface Integral / (Offenes Flächenintegral}}$$

$$\underbrace{\oint_{C=\partial S} \underbrace{\underline{\mathbf{E}}(\underline{\mathbf{R}},t)}_{[\mathrm{V/m}]}\bullet \underbrace{\underline{\mathbf{dR}}}_{[\mathrm{m}]}}_{[\mathrm{V}]} = \underbrace{-\underbrace{\frac{d}{dt}}_{[1/\mathrm{s}]} \iint_S \underbrace{\underline{\mathbf{B}}(\underline{\mathbf{R}},t)}_{[\mathrm{Vs/m^2}]}\bullet \underbrace{\underline{\mathbf{dS}}}_{[\mathrm{m}^2]}}_{[\mathrm{V}]} - \underbrace{\iint_S \underbrace{\underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},t)}_{[\mathrm{V/m^2}]}\bullet \underbrace{\underline{\mathbf{dS}}}_{[\mathrm{m}^2]}}_{[\mathrm{V}]}$$

$$\underbrace{\oint_{C=\partial S} \underline{\mathbf{E}}(\underline{\mathbf{R}},t)\bullet \underline{\mathbf{dR}}}_{\overset{\circ}{u}_{\mathrm{e}}(t)} = -\frac{d}{dt}\underbrace{\iint_S \underline{\mathbf{B}}(\underline{\mathbf{R}},t)\bullet \underline{\mathbf{dS}}}_{\psi_{\mathrm{m}}(t)} - \underbrace{\iint_S \underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},t)\bullet \underline{\mathbf{dS}}}_{i_{\mathrm{m}}(t)}$$

$\overset{\circ}{u}_{\mathrm{e}}(t)$: Electric Contour Voltage / Elektrische Umlaufspannung; $\psi_{\mathrm{m}}(t)$: Magnetic Flux / Magnetischer Fluss; $i_{\mathrm{m}}(t)$: Magnetic Current / Magnetischer Strom

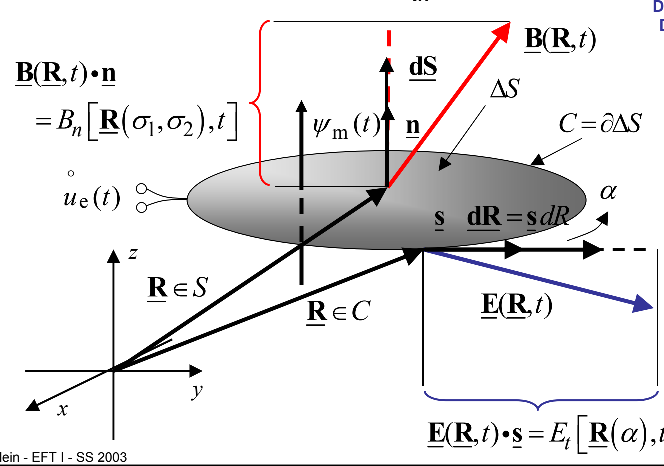
$$\overset{\circ}{u}_{\mathrm{e}}(t) = -\frac{d}{dt}\psi_{\mathrm{m}}(t) - i_{\mathrm{m}}(t)$$

Faraday‘s Induction Law / Faradaysches Induktionsgesetz

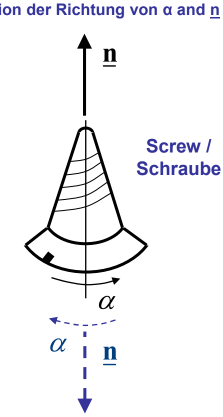
$$\underbrace{\oint_{C=\partial S} \underline{\mathbf{E}}(\underline{\mathbf{R}},t)\bullet \underline{\mathbf{dR}}}_{\text{Closed Contour Integral / Geschlossenes Kontur- oder Linienintegral}} = -\frac{d}{dt}\underbrace{\iint_S \underline{\mathbf{B}}(\underline{\mathbf{R}},t)\bullet \underline{\mathbf{dS}}}_{\text{(Open) Surface Integral / (Offenes) Flächenintegral}} - \underbrace{\iint_S \underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},t)\bullet \underline{\mathbf{dS}}}_{\text{(Open) Surface Integral / (Offenes) Flächenintegral}}$$

$$\overset{\circ}{u}_{\mathrm{e}}(t) = -\frac{d}{dt}\psi_{\mathrm{m}}(t) - i_{\mathrm{m}}(t)$$

$\underline{\mathbf{J}}_{\mathrm{m}}$ and $\underline{\mathbf{B}}$ are treated in a similar way / $\underline{\mathbf{J}}_{\mathrm{m}}$ und $\underline{\mathbf{B}}$ werden ähnlich behandelt



Definition of the Direction of α and $\underline{\mathbf{n}}$ / Definition der Richtung von α and $\underline{\mathbf{n}}$



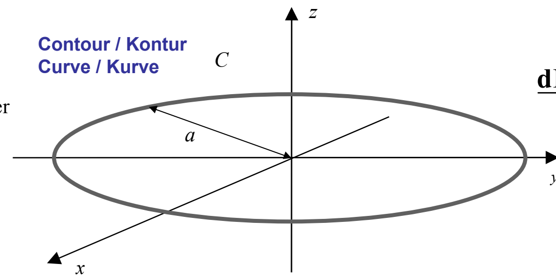
Example: Vector Differential Line and Surface Element / Beispiel: Vektorielles differentielles Linien- und Flächenelement (1)

Circle with Radius a in the xy Plane / Kreis mit dem Radius a in der xy-Ebene

$$\underbrace{\oint_{C=\partial S} \underline{\mathbf{E}}(\underline{\mathbf{R}},t)\bullet \underline{\mathbf{dR}}}$$

Closed Contour Integral / Geschlossenes Kontur- oder Linienintegral

Contour / Kontur
Curve / Kurve



$\underline{\mathbf{dR}} = \ ?$

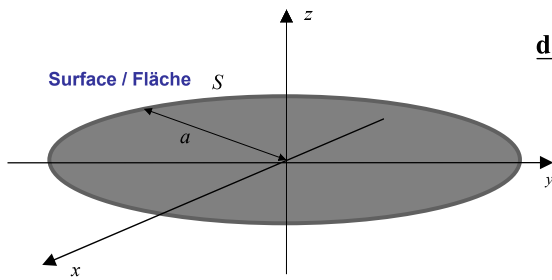
$$\underbrace{\iint_S \underline{\mathbf{B}}(\underline{\mathbf{R}},t)\bullet \underline{\mathbf{dS}}}$$

(Open) Surface Integral / (Offenes Flächenintegral

$$\underbrace{\iint_S \underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},t)\bullet \underline{\mathbf{dS}}}$$

(Open) Surface Integral / (Offenes Flächenintegral

Surface / Fläche

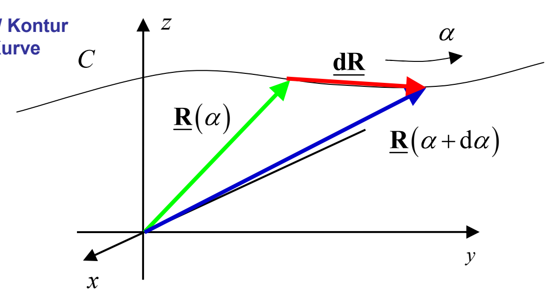


$\underline{\mathbf{dS}} = \ ?$

Vector Differential Line Element / Vektorielles differentielles Linienelement (1)

Definition: $$\underline{\mathbf{dR}} \overset{\text{def}}{=} \underline{\mathbf{R}}(\alpha + \mathrm{d}\alpha) - \underline{\mathbf{R}}(\alpha)$$

Contour / Kontur
Curve / Kurve



α Contour Parameter / Konturparameter

$\underline{\mathbf{R}}(\alpha)$ Position Vector / Ortsvektor

$\underline{\mathbf{R}}(\alpha+\mathrm{d}\alpha)$ Position Vector / Ortsvektor

$\underline{\mathbf{dR}}$ Difference Vector / Differenzvektor

$$\underline{\mathbf{dR}} = \underbrace{\underline{\mathbf{R}}(\alpha + \mathrm{d}\alpha)} - \underline{\mathbf{R}}(\alpha)$$

Taylor Expansion / Taylor-Entwicklung

$$= \underline{\mathbf{R}}(\alpha) + \frac{\mathrm{d}}{\mathrm{d}\alpha}\underline{\mathbf{R}}(\alpha)\mathrm{d}\alpha - \underline{\mathbf{R}}(\alpha)$$

$$\underline{\mathbf{dR}} = \underbrace{\frac{\mathrm{d}}{\mathrm{d}\alpha}\underline{\mathbf{R}}(\alpha)}_{\underline{\boldsymbol{\tau}}(\alpha)}\mathrm{d}\alpha$$

$$= \underline{\boldsymbol{\tau}}(\alpha)\mathrm{d}\alpha$$

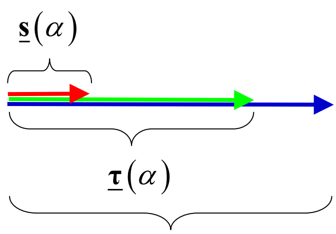
$$= \underline{\mathbf{s}}(\alpha)\mathrm{d}R$$

Vector Differential Line Element / Vektorielles differentielles Linienelement (2)

$$\underline{\mathbf{dR}} = \underbrace{\frac{\mathrm{d}}{\mathrm{d}\alpha}\underline{\mathbf{R}}(\alpha)}_{\underline{\boldsymbol{\tau}}(\alpha)}\mathrm{d}\alpha = \underline{\boldsymbol{\tau}}(\alpha)\,\mathrm{d}\alpha = \underbrace{\underline{\mathbf{s}}(\alpha)}_{|\underline{\mathbf{s}}(\alpha)|=1}\,\mathrm{d}R$$

$\underline{\boldsymbol{\tau}}(\alpha)$ **Tangential Vector / Tangentialvektor**

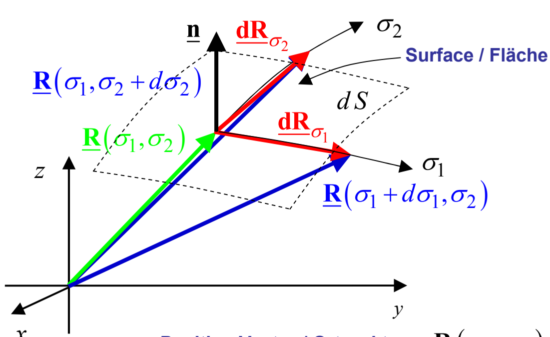
$\underline{\mathbf{s}}(\alpha)$ **Tangential Unit Vector / Tangentialeinheitsvektor**



$$\underline{\mathbf{dR}} = \underline{\mathbf{s}}(\alpha)\,\mathrm{d}R = \underline{\boldsymbol{\tau}}(\alpha)\,\mathrm{d}\alpha$$

Vector Differential Surface Element / Vektorielles differentielles Flächenelement (1)

Definition: $\underline{\mathbf{d\,S}} = \underline{\mathbf{n}}\,\mathrm{d}S$



σ_1, σ_2 **Surface Parameters / Flächenparameter**

$\underline{\mathbf{R}}(\sigma_1, \sigma_2)$ **Position Vector / Ortsvektor**

$\underline{\mathbf{R}}(\sigma_1 + \mathrm{d}\sigma_1, \sigma_2)$ **Position Vector / Ortsvektor**

$\underline{\mathbf{R}}(\sigma_1, \sigma_2 + \mathrm{d}\sigma_2)$ **Position Vector / Ortsvektor**

$\underline{\mathbf{dR}}_{\sigma_1}$, $\underline{\mathbf{dR}}_{\sigma_2}$ **Vector Differential Line Elements / Vektorielle differentielle Linienelemente**

Position Vector / Ortsvektor $\underline{\mathbf{R}}(\sigma_1, \sigma_2)$

Tangential Vectors / Tangentialvektoren

$$\underline{\boldsymbol{\sigma}}_1(\sigma_1, \sigma_2) = \frac{\partial}{\partial \sigma_1}\underline{\mathbf{R}}(\sigma_1, \sigma_2)$$

$$\underline{\boldsymbol{\sigma}}_2(\sigma_1, \sigma_2) = \frac{\partial}{\partial \sigma_2}\underline{\mathbf{R}}(\sigma_1, \sigma_2)$$

Vector Differential Surface Element / Vektorielles differentielles Flächenelement (2)

Vector Differential Line Elements / Vektorielles differentielles Linienelement

$$\underline{\mathbf{dR}}_{\sigma_1} = \underline{\boldsymbol{\sigma}}_1(\sigma_1,\sigma_2)\,\mathrm{d}\sigma_1$$

$$\underline{\mathbf{dR}}_{\sigma_2} = \underline{\boldsymbol{\sigma}}_2(\sigma_1,\sigma_2)\,\mathrm{d}\sigma_2$$

Scalar Differential Surface Elements / Skalares differentielles Flächenelement

$$\mathrm{d}S = \left|\underline{\mathbf{dR}}_{\sigma_1} \times \underline{\mathbf{dR}}_{\sigma_2}\right| = \left|\underline{\boldsymbol{\sigma}}_1(\sigma_1,\sigma_2)\times\underline{\boldsymbol{\sigma}}_2(\sigma_1,\sigma_2)\right|\mathrm{d}\sigma_1\mathrm{d}\sigma_2$$

Normal Unit-Vector / Normaleneinheitsvektor

$$\underline{\mathbf{n}} = \frac{\underline{\boldsymbol{\sigma}}_1(\sigma_1,\sigma_2)\times\underline{\boldsymbol{\sigma}}_2(\sigma_1,\sigma_2)}{\left|\underline{\boldsymbol{\sigma}}_1(\sigma_1,\sigma_2)\times\underline{\boldsymbol{\sigma}}_2(\sigma_1,\sigma_2)\right|}$$

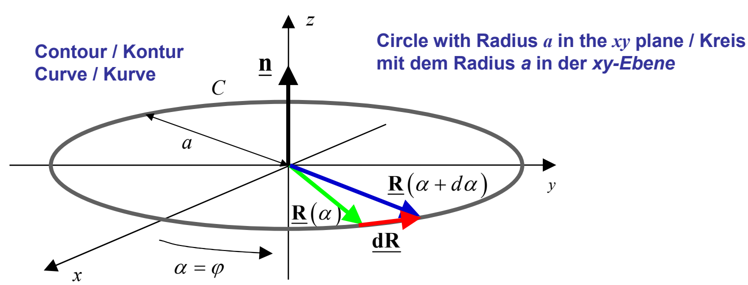
Vector Differential Surface Element / Vektorielles differentielles Flächenelement

$$\underline{\mathbf{dS}} = \underline{\mathbf{n}}\,\mathrm{d}S$$

$$= \frac{\underline{\boldsymbol{\sigma}}_1(\sigma_1,\sigma_2)\times\underline{\boldsymbol{\sigma}}_2(\sigma_1,\sigma_2)}{\left|\underline{\boldsymbol{\sigma}}_1(\sigma_1,\sigma_2)\times\underline{\boldsymbol{\sigma}}_2(\sigma_1,\sigma_2)\right|}\left|\underline{\boldsymbol{\sigma}}_1(\sigma_1,\sigma_2)\times\underline{\boldsymbol{\sigma}}_2(\sigma_1,\sigma_2)\right|\mathrm{d}\sigma_1\mathrm{d}\sigma_2$$

$$= \underline{\boldsymbol{\sigma}}_1(\sigma_1,\sigma_2)\times\underline{\boldsymbol{\sigma}}_2(\sigma_1,\sigma_2)\,\mathrm{d}\sigma_1\mathrm{d}\sigma_2$$

Example: Vector Differential Line Element / Beispiel: Vektorielles differentielles Linienelement (1)



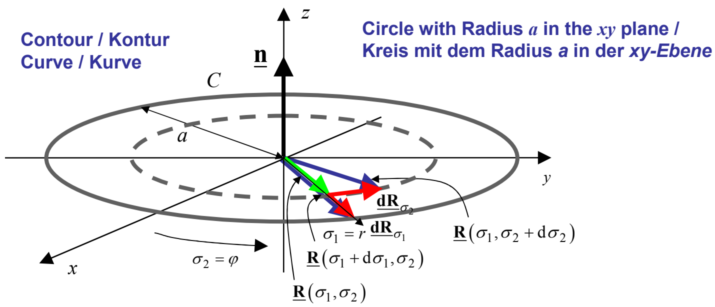
$$\underline{\mathbf{R}}(\alpha=\varphi) = a\cos\varphi\,\underline{\mathbf{e}}_x + a\sin\varphi\,\underline{\mathbf{e}}_y, \quad 0 \le \varphi < 2\pi$$

$$\frac{\mathrm{d}}{\mathrm{d}\varphi}\underline{\mathbf{R}}(\varphi) = \frac{\mathrm{d}}{\mathrm{d}\varphi}\left(a\cos\varphi\,\underline{\mathbf{e}}_x + a\sin\varphi\,\underline{\mathbf{e}}_y\right) = a\underbrace{\left(-\sin\varphi\,\underline{\mathbf{e}}_x + \cos\varphi\,\underline{\mathbf{e}}_y\right)}_{\underline{\mathbf{e}}_\varphi(\varphi)} = a\underline{\mathbf{e}}_\varphi(\varphi)$$

$$\underline{\mathbf{dR}} = \underbrace{\frac{\mathrm{d}}{\mathrm{d}\alpha}\underline{\mathbf{R}}(\alpha)}_{\underline{\boldsymbol{\tau}}(\alpha)}\mathrm{d}\alpha = \underline{\boldsymbol{\tau}}(\alpha)\,\mathrm{d}\alpha = \underline{\mathbf{s}}(\alpha)\,\mathrm{d}R$$

$$\underline{\mathbf{dR}} = \underbrace{\frac{\mathrm{d}}{\mathrm{d}\varphi}\underline{\mathbf{R}}(\varphi)}_{=\underline{\boldsymbol{\tau}}(\alpha)}\underbrace{\mathrm{d}\varphi}_{=\mathrm{d}\alpha} = \underbrace{a\underline{\mathbf{e}}_\varphi(\varphi)}_{=\underline{\boldsymbol{\tau}}(\alpha)}\underbrace{\mathrm{d}\varphi}_{=\mathrm{d}\alpha} = \underbrace{\underline{\mathbf{e}}_\varphi(\varphi)}_{=\underline{\mathbf{s}}(\alpha)}\underbrace{a\,\mathrm{d}\varphi}_{=\mathrm{d}R} = \underline{\mathbf{dR}}_\varphi$$

Example: Vector Differential Surface Element / Beispiel: Vektorielles differentielles Flächenelement (1)

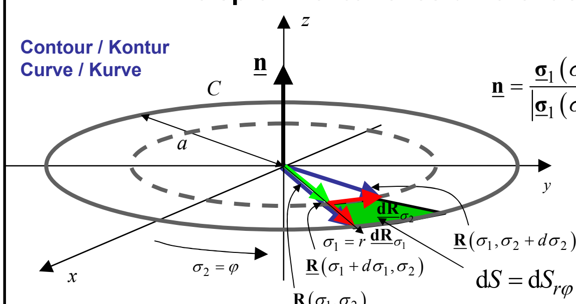


$$\underline{\mathbf{R}}(\sigma_1 = r, \sigma_2 = \varphi) = r\cos\varphi\,\underline{\mathbf{e}}_x + r\sin\varphi\,\underline{\mathbf{e}}_y, \qquad 0 \le \sigma_1 = r \le a,\quad 0 \le \sigma_2 = \varphi < 2\pi$$

$$\frac{\partial \underline{\mathbf{R}}(r,\varphi)}{\partial r} = \frac{\partial}{\partial r}\left(r\cos\varphi\,\underline{\mathbf{e}}_x + ra\sin\varphi\,\underline{\mathbf{e}}_y\right)$$
$$= \underbrace{\cos\varphi\,\underline{\mathbf{e}}_x + \sin\varphi\,\underline{\mathbf{e}}_y}_{\underline{\mathbf{e}}_r(r)}$$
$$= \underline{\mathbf{e}}_r(\varphi)$$

$$\frac{\partial \underline{\mathbf{R}}(r,\varphi)}{\partial \varphi} = \frac{\partial}{\partial \varphi}\left(r\cos\varphi\,\underline{\mathbf{e}}_x + r\sin\varphi\,\underline{\mathbf{e}}_y\right)$$
$$= r\underbrace{\left(-\sin\varphi\,\underline{\mathbf{e}}_x + \cos\varphi\,\underline{\mathbf{e}}_y\right)}_{\underline{\mathbf{e}}_\varphi(\varphi)}$$
$$= r\underline{\mathbf{e}}_\varphi(\varphi)$$

Example: Vector Differential Surface Element / Beispiel: Vektorielles differentielles Flächenelement (2)



$$\underline{\mathbf{n}} = \frac{\underline{\boldsymbol{\sigma}}_1(\sigma_1,\sigma_2)\times\underline{\boldsymbol{\sigma}}_2(\sigma_1,\sigma_2)}{\left|\underline{\boldsymbol{\sigma}}_1(\sigma_1,\sigma_2)\times\underline{\boldsymbol{\sigma}}_2(\sigma_1,\sigma_2)\right|} = \frac{\dfrac{\partial \underline{\mathbf{R}}(r,\varphi)}{\partial r}\times\dfrac{\partial \underline{\mathbf{R}}(r,\varphi)}{\partial \varphi}}{\left|\dfrac{\partial \underline{\mathbf{R}}(r,\varphi)}{\partial r}\times\dfrac{\partial \underline{\mathbf{R}}(r,\varphi)}{\partial \varphi}\right|}$$

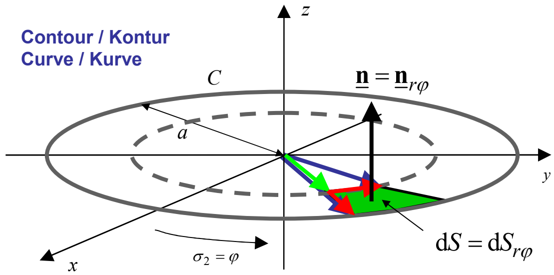
$$\frac{\partial \underline{\mathbf{R}}(r,\varphi)}{\partial r} = \underline{\mathbf{e}}_r(\varphi) \qquad \frac{\partial \underline{\mathbf{R}}(r,\varphi)}{\partial \varphi} = r\underline{\mathbf{e}}_\varphi(\varphi)$$

$$\underline{\boldsymbol{\sigma}}_1(\sigma_1,\sigma_2)\times\underline{\boldsymbol{\sigma}}_2(\sigma_1,\sigma_2)$$
$$= \frac{\partial \underline{\mathbf{R}}(r,\varphi)}{\partial r}\times\frac{\partial \underline{\mathbf{R}}(r,\varphi)}{\partial \varphi}$$
$$= \underline{\mathbf{e}}_r(\varphi)\times r\underline{\mathbf{e}}_\varphi(\varphi)$$
$$= r\underbrace{\left[\underline{\mathbf{e}}_r(\varphi)\times\underline{\mathbf{e}}_\varphi(\varphi)\right]}_{=\underline{\mathbf{e}}_z}$$
$$= r\underline{\mathbf{e}}_z$$

$$\left|\underline{\boldsymbol{\sigma}}_1(\sigma_1,\sigma_2)\times\underline{\boldsymbol{\sigma}}_2(\sigma_1,\sigma_2)\right|$$
$$= \left|\frac{\partial \underline{\mathbf{R}}(r,\varphi)}{\partial r}\times\frac{\partial \underline{\mathbf{R}}(r,\varphi)}{\partial \varphi}\right|$$
$$= \left|\underline{\mathbf{e}}_r(\varphi)\times r\underline{\mathbf{e}}_\varphi(\varphi)\right|$$
$$= r\underbrace{\left|\underbrace{\underline{\mathbf{e}}_r(\varphi)\times\underline{\mathbf{e}}_\varphi(\varphi)}_{\underline{\mathbf{e}}_z}\right|}_{=1}$$
$$= r$$

$$\Rightarrow \qquad \underline{\mathbf{n}} = \frac{r\underline{\mathbf{e}}_z}{r} = \underline{\mathbf{e}}_z$$

Example: Vector Differential Surface Element / Beispiel: Vektorielles differentielles Flächenelement (3)



$$\underline{\mathbf{dS}} = \underline{\mathbf{n}}\, dS$$

$$= \frac{\underline{\boldsymbol{\sigma}}_1(\sigma_1,\sigma_2)\times\underline{\boldsymbol{\sigma}}_2(\sigma_1,\sigma_2)}{\left|\underline{\boldsymbol{\sigma}}_1(\sigma_1,\sigma_2)\times\underline{\boldsymbol{\sigma}}_2(\sigma_1,\sigma_2)\right|}\left|\underline{\boldsymbol{\sigma}}_1(\sigma_1,\sigma_2)\times\underline{\boldsymbol{\sigma}}_2(\sigma_1,\sigma_2)\right|\mathrm{d}\sigma_1\,\mathrm{d}\sigma_2$$

$$= \underline{\boldsymbol{\sigma}}_1(\sigma_1,\sigma_2)\times\underline{\boldsymbol{\sigma}}_2(\sigma_1,\sigma_2)\,\mathrm{d}\sigma_1\,\mathrm{d}\sigma_2$$

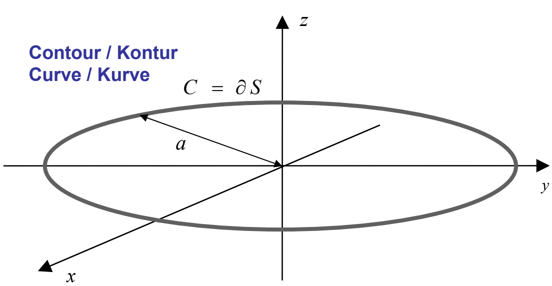
$$= \underbrace{\underline{\mathbf{e}}_z}_{=\underline{\mathbf{n}}_{r\varphi}}\ \underbrace{r\,\mathrm{d}r\,\mathrm{d}\varphi}_{=\mathrm{d}S_{r\varphi}}$$

$$= \underline{\mathbf{n}}_{r\varphi}\,\mathrm{d}S_{r\varphi}$$

$$= \underline{\mathbf{dS}}_{r\varphi}$$

Example: Vector Differential Line and Surface Element / Beispiel: Vektorielles differentielles Linien- und Flächenelement (2)

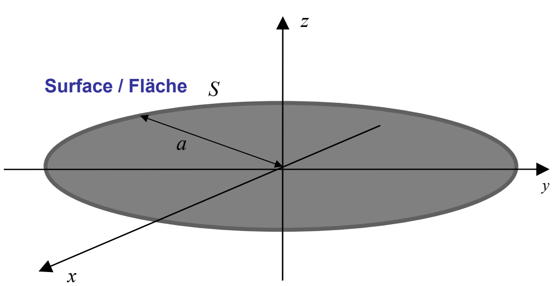
(Circle with Radius *a* in the *xy* Plane / Kreis mit dem Radius *a* in der *xy*-Ebene)



$$\underline{\mathbf{dR}} = \underline{\mathbf{dR}}_{\varphi} = a\,\underline{\mathbf{e}}_{\varphi}(\varphi)\,\mathrm{d}\varphi$$

$$\oint_{C=\partial S}\underline{\mathbf{E}}(\underline{\mathbf{R}},t)\bullet\underline{\mathbf{dR}}$$

$$= \int_{\varphi=0}^{2\pi}\underline{\mathbf{E}}\left[\underline{\mathbf{R}}\left(r=a,\varphi,z=0\right),t\right]\bullet a\,\underline{\mathbf{e}}_{\varphi}(\varphi)\,\mathrm{d}\varphi$$



$$\underline{\mathbf{dS}} = \underline{\mathbf{dS}}_{r\varphi} = \underline{\mathbf{e}}_z\, r\,\mathrm{d}r\,\mathrm{d}\varphi$$

$$\iint_S \underline{\mathbf{B}}(\underline{\mathbf{R}},t)\bullet\underline{\mathbf{dS}}$$

$$= \int_{\varphi=0}^{2\pi}\int_{r=0}^{a}\underline{\mathbf{B}}\left[\underline{\mathbf{R}}\left(r,\varphi,z=0\right),t\right]\bullet\underline{\mathbf{e}}_z\, r\,\mathrm{d}r\,\mathrm{d}\varphi$$

Example: Vector Differential Line and Surface Element / Beispiel: Vektorielles differentielles Linien- und Flächenelement (2)

(Circle with Radius *a* in the xy Plane / Kreis mit dem Radius *a* in der xy-Ebene)

$$a\int_{\varphi=0}^{2\pi}\underbrace{\underline{\mathbf{E}}\left[\underline{\mathbf{R}}\left(r=a,\varphi,z=0\right),t\right]\bullet\underline{\mathbf{e}}_{\varphi}(\varphi)}_{=E_{\varphi}\left[\underline{\mathbf{R}}(r=a,\varphi,z=0),t\right]}\mathrm{d}\varphi = a\int_{\varphi=0}^{2\pi}\underbrace{E_{\varphi}\left[\underline{\mathbf{R}}\left(r=a,\varphi,z=0\right),t\right]}_{\underbrace{E_t\left[\underline{\mathbf{R}}(r,\varphi,z=0),t\right]}_{\text{Tangential Component of }\underline{\mathbf{E}}\text{ / Tangentialkomponente von }\underline{\mathbf{E}}}}\mathrm{d}\varphi$$

$$\int_{\varphi=0}^{2\pi}\int_{r=0}^{a}\underbrace{\underline{\mathbf{B}}\left[\underline{\mathbf{R}}\left(r,\varphi,z=0\right),t\right]\bullet\underline{\mathbf{e}}_z}_{=B_z\left[\underline{\mathbf{R}}(r,\varphi,z=0),t\right]}\,r\,\mathrm{d}r\,\mathrm{d}\varphi = \int_{\varphi=0}^{2\pi}\int_{r=0}^{a}\underbrace{B_z\left[\underline{\mathbf{R}}\left(r,\varphi,z=0\right),t\right]}_{\underbrace{=B_n\left[\underline{\mathbf{R}}(r,\varphi,z=0),t\right]}_{\text{Normal Component of }\underline{\mathbf{B}}\text{ / Normalkomponente von }\underline{\mathbf{B}}}}r\,\mathrm{d}r\,\mathrm{d}\varphi$$

$$\int_{\varphi=0}^{2\pi}\underline{\mathbf{E}}\left[\underline{\mathbf{R}}\left(r=a,\varphi,z=0\right),t\right]\bullet a\underline{\mathbf{e}}_{\varphi}(\varphi)\mathrm{d}\varphi$$

$$=-\frac{\mathrm{d}}{\mathrm{d}t}\int_{\varphi=0}^{2\pi}\int_{r=0}^{a}\underline{\mathbf{B}}\left[\underline{\mathbf{R}}\left(r,\varphi,z=0\right),t\right]\bullet\underline{\mathbf{e}}_z\,r\,\mathrm{d}r\,\mathrm{d}\varphi$$

$$-\int_{\varphi=0}^{2\pi}\int_{r=0}^{a}\underline{\mathbf{J}}_{\mathrm{m}}\left[\underline{\mathbf{R}}\left(r,\varphi,z=0\right),t\right]\bullet\underline{\mathbf{e}}_z\,r\,\mathrm{d}r\,\mathrm{d}\varphi$$

Ampère-Maxwell's Circuital Law / Ampère-Maxwellsches Durchflutungsgesetz

Differential Form / Differentialform

$$\nabla\times\underline{\mathbf{H}}(\underline{\mathbf{R}},t)=\frac{\partial}{\partial t}\underline{\mathbf{D}}(\underline{\mathbf{R}},t)+\underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}},t)$$

Integral Form / Integralform

$$\underbrace{\oint_{C=\partial S}\underline{\mathbf{H}}(\underline{\mathbf{R}},t)\bullet\mathbf{d}\underline{\mathbf{R}}}_{\text{Closed Contour Integral / Geschlossenes Kontur- oder Linienintegral}}=\frac{\mathrm{d}}{\mathrm{d}t}\underbrace{\iint_S\underline{\mathbf{D}}(\underline{\mathbf{R}},t)\bullet\mathbf{d}\underline{\mathbf{S}}}_{\text{(Open) Surface Integral / (Offenes) Flächenintegral}}+\underbrace{\iint_S\underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}},t)\bullet\mathbf{d}\underline{\mathbf{S}}}_{\text{(Open) Surface Integral / (Offenes) Flächenintegral}}$$

$$\underbrace{\oint_{C=\partial S}\underbrace{\underline{\mathbf{H}}(\underline{\mathbf{R}},t)}_{[\mathrm{A/m}]}\bullet\underbrace{\mathbf{d}\underline{\mathbf{R}}}_{[\mathrm{m}]}}_{[\mathrm{A}]}=\underbrace{\underbrace{\frac{\mathrm{d}}{\mathrm{d}t}}_{[1/\mathrm{s}]}\iint_S\underbrace{\underline{\mathbf{D}}(\underline{\mathbf{R}},t)}_{[\mathrm{As/m^2}]}\bullet\underbrace{\mathbf{d}\underline{\mathbf{S}}}_{[\mathrm{m^2}]}}_{[\mathrm{A}]}-\underbrace{\iint_S\underbrace{\underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}},t)}_{[\mathrm{A/m^2}]}\bullet\underbrace{\mathbf{d}\underline{\mathbf{S}}}_{[\mathrm{m^2}]}}_{[\mathrm{A}]}$$

$$\underbrace{\oint_{C=\partial S}\underline{\mathbf{H}}(\underline{\mathbf{R}},t)\bullet\mathbf{d}\underline{\mathbf{R}}}_{\overset{\circ}{u}_{\mathrm{m}}(t)\,[\mathrm{A}]}=\frac{\mathrm{d}}{\mathrm{d}t}\underbrace{\iint_S\underline{\mathbf{D}}(\underline{\mathbf{R}},t)\bullet\mathbf{d}\underline{\mathbf{S}}}_{\psi_{\mathrm{e}}(t)\,[\mathrm{As}]}+\underbrace{\iint_S\underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}},t)\bullet\mathbf{d}\underline{\mathbf{S}}}_{i_{\mathrm{e}}(t)\,[\mathrm{A}]}$$

Magnetic Contour Voltage / Magnetische Umlaufspannung; Electric Flux / Elektrischer Fluss; Electric Current / Elektrischer Strom

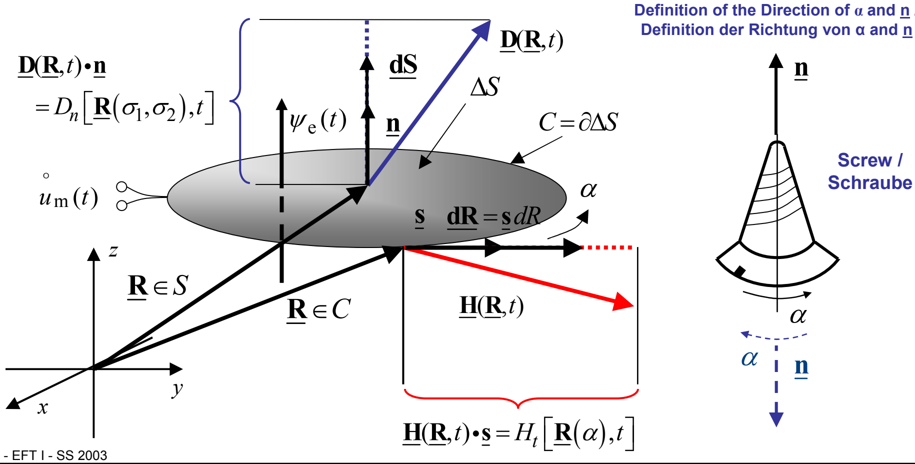
$$\overset{\circ}{u}_{\mathrm{m}}(t)=\frac{\mathrm{d}}{\mathrm{d}t}\psi_{\mathrm{e}}(t)+i_{\mathrm{e}}(t)$$

Ampère-Maxwell's Circuital Law / Ampère-Maxwellsches Durchflutungsgesetz

$$\underbrace{\oint_{C=\partial S} \underline{\mathbf{H}}(\underline{\mathbf{R}},t)\bullet \mathrm{d}\underline{\mathbf{R}}}_{\text{Closed Contour Integral / Geschlossenes Kontur- oder Linienintegral}} = \frac{\mathrm{d}}{\mathrm{d}t}\underbrace{\iint_S \underline{\mathbf{D}}(\underline{\mathbf{R}},t)\bullet \mathrm{d}\underline{\mathbf{S}}}_{\text{(Open) Surface Integral / (Offenes) Flächenintegral}} + \underbrace{\iint_S \underline{\mathbf{J}}_\mathrm{e}(\underline{\mathbf{R}},t)\bullet \mathrm{d}\underline{\mathbf{S}}}_{\text{(Open) Surface Integral / (Offenes) Flächenintegral}}$$

$$\overset{\circ}{u}_\mathrm{m}(t) = \frac{\mathrm{d}}{\mathrm{d}t}\psi_\mathrm{e}(t) + i_\mathrm{e}(t)$$

$\underline{J}_\mathrm{e}$ and $\underline{D}$ are treated in a similar way / $\underline{J}_\mathrm{e}$ und $\underline{D}$ werden ähnlich behandelt



Gauss' Electric Law / Gaußsches elektrisches Gesetz

Differential Form / Differentialform

$$\nabla \bullet \underline{\mathbf{D}}(\underline{\mathbf{R}},t) = \rho_\mathrm{e}(\underline{\mathbf{R}},t)$$

Integral Form / Integralform

$$\underbrace{\oiint_{S=\partial V} \underline{\mathbf{D}}(\underline{\mathbf{R}},t)\bullet \mathrm{d}\underline{\mathbf{S}}}_{\text{Closed Surface Integral / Geschlossenes Flächenintegral}} = \underbrace{\iiint_V \rho_\mathrm{e}(\underline{\mathbf{R}},t)\mathrm{d}V}_{\text{Volume Integral / Volumenintegral}}$$

$$\underbrace{\oiint_{S=\partial V} \underbrace{\underline{\mathbf{D}}(\underline{\mathbf{R}},t)}_{[\mathrm{As/m^2}]}\bullet \underbrace{\mathrm{d}\underline{\mathbf{S}}}_{[\mathrm{m^2}]}}_{[\mathrm{As}]} = \underbrace{\iiint_V \underbrace{\rho_\mathrm{e}(\underline{\mathbf{R}},t)}_{[\mathrm{As/m^3}]}\,\underbrace{\mathrm{d}V}_{[\mathrm{m^3}]}}_{[\mathrm{As}]}$$

$$\underbrace{\oiint_{S=\partial V} \underline{\mathbf{D}}(\underline{\mathbf{R}},t)\bullet \mathrm{d}\underline{\mathbf{S}}}_{=\psi_\mathrm{e}(t)\ [\mathrm{As}]\ \text{Electric Flux / Elektrischer Fluss}} = \underbrace{\iiint_V \rho_\mathrm{e}(\underline{\mathbf{R}},t)\mathrm{d}V}_{=q_\mathrm{e}(t)\ [\mathrm{As}]\ \text{Electric Charge / Elektrische Ladung}}$$

$$\psi_\mathrm{e}(t) = q_\mathrm{e}(t)$$

Gauss' Magnetic Law / Gaußes magnetisches Gesetz

Differential Form / Differentialform

$$\nabla \bullet \underline{\mathbf{B}}(\underline{\mathbf{R}},t) = \rho_{\mathrm{m}}(\underline{\mathbf{R}},t)$$

Integral Form / Integralform

$$\underbrace{\oint\!\!\!\oint_{S=\partial V} \underline{\mathbf{B}}(\underline{\mathbf{R}},t)\bullet \underline{\mathbf{dS}}}_{\text{Closed Surface Integral / Geschlossenes Flächenintegral}} = \underbrace{\iiint_V \rho_{\mathrm{m}}(\underline{\mathbf{R}},t)\,\mathrm{d}V}_{\text{Volume Integral / Volumenintegral}}$$

$$\oint\!\!\!\oint_{S=\partial V} \underbrace{\underbrace{\underline{\mathbf{B}}(\underline{\mathbf{R}},t)}_{[\mathrm{Vs/m^2}]}\bullet \underbrace{\underline{\mathbf{dS}}}_{[\mathrm{m^2}]}}_{[\mathrm{Vs}]} = \iiint_V \underbrace{\underbrace{\rho_{\mathrm{m}}(\underline{\mathbf{R}},t)}_{[\mathrm{Vs/m^3}]}\,\underbrace{\mathrm{d}V}_{[\mathrm{m^3}]}}_{[\mathrm{Vs}]}$$

$$\underbrace{\oint\!\!\!\oint_{S=\partial V} \underline{\mathbf{B}}(\underline{\mathbf{R}},t)\bullet \underline{\mathbf{dS}}}_{=\psi_{\mathrm{m}}(t)\ [\mathrm{Vs}]} = \underbrace{\iiint_V \rho_{\mathrm{m}}(\underline{\mathbf{R}},t)\,\mathrm{d}V}_{=q_{\mathrm{m}}(t)\ [\mathrm{Vs}]}$$

Magnetic Flux / Magnetischer Fluss — Magnetic Charge / Magnetische Ladung

$$\psi_{\mathrm{m}}(t) = q_{\mathrm{m}}(t)$$

Gauss' Electric Law / Gaußsches elektrisches Gesetz

$$\underbrace{\oint\!\!\!\oint_{S=\partial V} \underline{\mathbf{D}}(\underline{\mathbf{R}},t)\bullet \underline{\mathbf{dS}}}_{\text{Closed Surface Integral / Geschlossenes Flächenintegral}} = \underbrace{\oint\!\!\!\oint_{S=\partial V} \underbrace{\underline{\mathbf{D}}(\underline{\mathbf{R}},t)\bullet \underline{\mathbf{n}}}_{=D_n(\underline{\mathbf{R}},t)}\,\mathrm{d}S}_{=\psi_{\mathrm{e}}(t)} = \underbrace{\iiint_V \rho_{\mathrm{e}}(\underline{\mathbf{R}},t)\,\mathrm{d}V}_{=q_{\mathrm{e}}(t)}$$

Summation of all Normal Componentes of $\underline{\mathbf{D}}$ at the Closed Surface $S=\partial V$ of the Volume V / Summation aller Normalkomponenten von $\underline{\mathbf{D}}$ auf der geschlossenen Oberfläche $S=\partial V$ des Volumens V

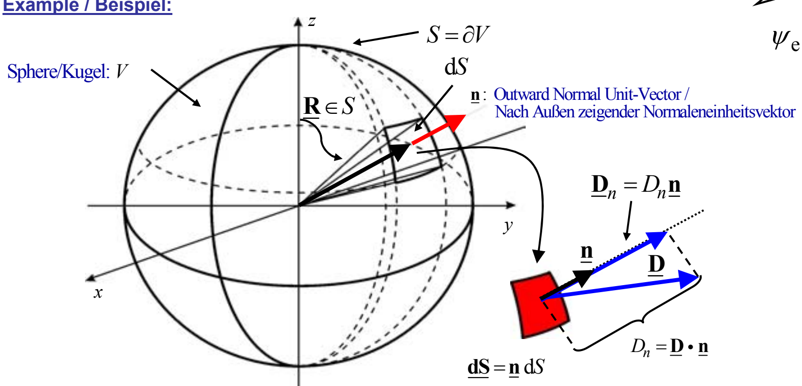
Flux Through the Colsed Surface / Fluss durch die geschlossene Oberfläche

Volume Integral / Volumenintegral

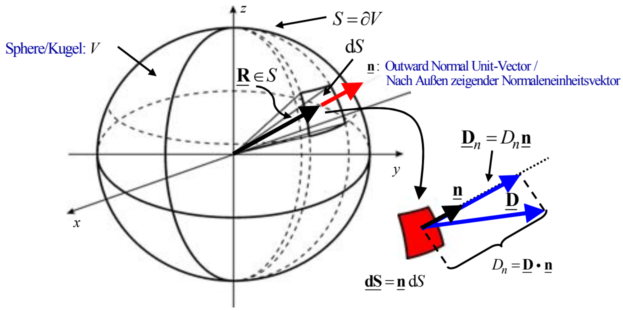
Summation of all charges inside the Volume V / Summation aller Ladungen in dem Volumen V

$$\psi_{\mathrm{e}}(t) = q_{\mathrm{e}}(t)$$

Example / Beispiel:



Example: Sphere with Radius *a* / Beispiel: Kugel mit Radius *a* (1)



$$\oiint_{S=\partial V} \underbrace{\underline{\mathbf{D}}(\underline{\mathbf{R}},t)\bullet\underline{\mathbf{n}}}_{=D_n(\underline{\mathbf{R}},t)}\,\mathrm{d}S$$

$$= \iiint_V \rho_\mathrm{e}(\underline{\mathbf{R}},t)\,\mathrm{d}V$$

$$\underline{\mathbf{dS}} = \underline{\mathbf{n}}dS \quad (= \underline{\mathbf{n}}_{\vartheta\varphi}\; h_\vartheta\; h_\varphi\; d\vartheta\; d\varphi)$$

$$0 \le \vartheta \le \pi$$

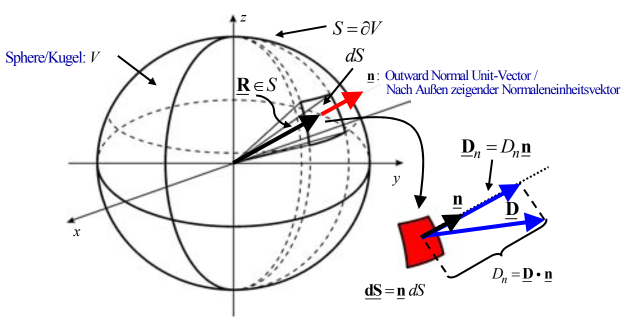
$$0 \le \varphi < 2\pi$$

$$= \underbrace{\underline{\mathbf{e}}_R\left(\vartheta,\varphi\right)}_{\underline{\mathbf{n}}}\underbrace{R^2 \sin\vartheta\; d\vartheta\; d\varphi}_{dS}\Bigg|_{R=a} \quad = \underbrace{\underline{\mathbf{e}}_R\left(\vartheta,\varphi\right)}_{\underline{\mathbf{n}}}\underbrace{a^2 \sin\vartheta\; d\vartheta\; d\varphi}_{dS}$$

$$\oiint_{S=\partial V} \underline{\mathbf{D}}(\underline{\mathbf{R}},t)\bullet\underline{\mathbf{dS}} = \oiint_{S=\partial V} \underbrace{\underline{\mathbf{D}}(\underline{\mathbf{R}},t)\bullet\underline{\mathbf{n}}}_{=D_n(\underline{\mathbf{R}},t)}\,\mathrm{d}S = \int_{\varphi=0}^{2\pi}\int_{\vartheta=0}^{\pi} \underbrace{\underline{\mathbf{D}}[\underline{\mathbf{R}}(R=a,\vartheta,\varphi),t]\bullet\underline{\mathbf{e}}_R\left(\vartheta,\varphi\right)}_{\substack{=D_R[\underline{\mathbf{R}}(R=a,\vartheta,\varphi),t] \\ =D_n[\underline{\mathbf{R}}(R=a,\vartheta,\varphi),t]}} a^2 \sin\vartheta\;\;\mathrm{d}\vartheta\;\;\mathrm{d}\varphi$$

$$=\psi_\mathrm{e}(t)$$

Example: Sphere with Radius *a* / Beispiel: Kugel mit Radius *a* (2)



$$\oiint_{S=\partial V} \underbrace{\underline{\mathbf{D}}(\underline{\mathbf{R}},t)\bullet\underline{\mathbf{n}}}_{D_n(\underline{\mathbf{R}},t)}\,\mathrm{d}S$$

$$= \iiint_V \rho_\mathrm{e}(\underline{\mathbf{R}},t)\,\mathrm{d}V$$

$$0 \le R \le a$$

$$0 \le \vartheta \le \pi$$

$$0 \le \varphi < 2\pi$$

$$\mathrm{d}V = R^2 \sin\vartheta\;\mathrm{d}R\;\mathrm{d}\vartheta\;\mathrm{d}\varphi \;\left(= h_R h_\vartheta h_\varphi\;\mathrm{d}R\;\mathrm{d}\vartheta\;\mathrm{d}\varphi\right)$$

$$\iiint_V \rho_\mathrm{e}(\underline{\mathbf{R}},t)\,\mathrm{d}V = \int_{\varphi=0}^{2\pi}\int_{\vartheta=0}^{\pi}\int_{R=0}^{a} \rho_\mathrm{e}[\underline{\mathbf{R}}(R,\vartheta,\varphi),t]\; R^2 \sin\vartheta\;\mathrm{d}R\;\mathrm{d}\vartheta\;\mathrm{d}\varphi$$

$$= q_\mathrm{e}(t)$$

Example: Electric Field Due to Spherically Symmetric Charge Distribution / Beispiel: Elektrisches Feld einer kugelsymmetrischen Raumladungsdichte

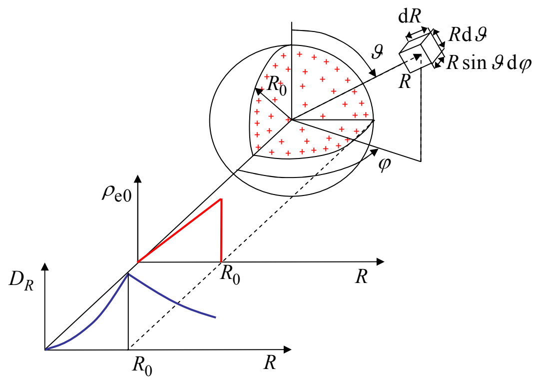
Consider the Electrostatic (ES) Case / Betrachte den elektrostatischen Fall

$$\oint\!\!\!\oint_{S=\partial V} \underbrace{\underline{\mathbf{D}}(\underline{\mathbf{R}})\bullet\underline{\mathbf{n}}}_{=D_n(\underline{\mathbf{R}})}\,\mathrm{d}S = \iiint_V \rho_\mathrm{e}(\underline{\mathbf{R}})\,\mathrm{d}V$$

Electric Charge Density / Elektrische Raumladungsdichte

$$\rho_\mathrm{e}(\underline{\mathbf{R}}) = \begin{cases} \rho_{\mathrm{e}0}\dfrac{R}{R_0} & R < R_0 \\ 0 & R > R_0 \end{cases}$$

Radial Symmetry / Radialsymmetrisch **!**



Continuity Equations for Electric and Magnetic Charges / Kontinuitätsgleichung für elektrische und magnetische Ladungen

(Conservation of Electric Charge / Erhaltung der elektrischen Ladung)

$$\underbrace{\oint\!\!\!\oint_{S=\partial V} \underline{\mathbf{J}}_\mathrm{e}(\underline{\mathbf{R}},t)\bullet\mathrm{d}\underline{\mathbf{S}}}_{\text{Closed Surface Integral / Geschlossenes Flächenintegral}} = -\frac{\mathrm{d}}{\mathrm{d}t}\underbrace{\iiint_V \rho_\mathrm{e}(\underline{\mathbf{R}},t)\,\mathrm{d}V}_{\text{Volume Integral / Volumenintegral}}$$

$$\underbrace{\oint\!\!\!\oint_{S=\partial V} \underbrace{\underline{\mathbf{J}}_\mathrm{e}(\underline{\mathbf{R}},t)}_{[\mathrm{A/m^2}]}\bullet\underbrace{\mathrm{d}\underline{\mathbf{S}}}_{[\mathrm{m^2}]}}_{[\mathrm{A}]} = \underbrace{-\underbrace{\frac{\mathrm{d}}{\mathrm{d}t}}_{[1/\mathrm{s}]}\underbrace{\iiint_V \underbrace{\rho_\mathrm{e}(\underline{\mathbf{R}},t)}_{[\mathrm{As/m^3}]}\underbrace{\mathrm{d}V}_{[\mathrm{m^3}]}}_{[\mathrm{As}]}}_{[\mathrm{A}]}$$

$$\underbrace{\oint\!\!\!\oint_{S=\partial V} \underline{\mathbf{J}}_\mathrm{e}(\underline{\mathbf{R}},t)\bullet\mathrm{d}\underline{\mathbf{S}}}_{=i_\mathrm{e}(t)\,[\mathrm{A}]} = -\frac{\mathrm{d}}{\mathrm{d}t}\underbrace{\iiint_V \rho_\mathrm{e}(\underline{\mathbf{R}},t)\,\mathrm{d}V}_{=q_\mathrm{e}(t)\,[\mathrm{As}]}$$

Electric Current / Elektrischer Strom — Electric Charge / Elektrische Ladung

$$i_\mathrm{e}(t) = -\frac{\mathrm{d}}{\mathrm{d}t}q_\mathrm{e}(t)$$

(Conservation of Magnetic Charge / Erhaltung der magnetischen Ladung)

$$\underbrace{\oint\!\!\!\oint_{S=\partial V} \underline{\mathbf{J}}_\mathrm{m}(\underline{\mathbf{R}},t)\bullet\mathrm{d}\underline{\mathbf{S}}}_{\text{Closed Surface Integral / Geschlossenes Flächenintegral}} = -\frac{\mathrm{d}}{\mathrm{d}t}\underbrace{\iiint_V \rho_\mathrm{m}(\underline{\mathbf{R}},t)\,\mathrm{d}V}_{\text{Volume Integral / Volumenintegral}}$$

$$\underbrace{\oint\!\!\!\oint_{S=\partial V} \underbrace{\underline{\mathbf{J}}_\mathrm{m}(\underline{\mathbf{R}},t)}_{[\mathrm{V/m^2}]}\bullet\underbrace{\mathrm{d}\underline{\mathbf{S}}}_{[\mathrm{m^2}]}}_{[\mathrm{V}]} = \underbrace{-\underbrace{\frac{\mathrm{d}}{\mathrm{d}t}}_{[1/\mathrm{s}]}\underbrace{\iiint_V \underbrace{\rho_\mathrm{m}(\underline{\mathbf{R}},t)}_{[\mathrm{Vs/m^3}]}\underbrace{\mathrm{d}V}_{[\mathrm{m^3}]}}_{[\mathrm{Vs}]}}_{[\mathrm{V}]}$$

$$\underbrace{\oint\!\!\!\oint_{S=\partial V} \underline{\mathbf{J}}_\mathrm{m}(\underline{\mathbf{R}},t)\bullet\mathrm{d}\underline{\mathbf{S}}}_{=i_\mathrm{m}(t)\,[\mathrm{V}]} = -\frac{\mathrm{d}}{\mathrm{d}t}\underbrace{\iiint_V \rho_\mathrm{m}(\underline{\mathbf{R}},t)\,\mathrm{d}V}_{=q_\mathrm{m}(t)\,[\mathrm{Vs}]}$$

Magnetic Current / Magnetischer Strom — Magnetic Charge / Magnetische Ladung

$$i_\mathrm{m}(t) = -\frac{\mathrm{d}}{\mathrm{d}t}q_\mathrm{m}(t)$$

Field and Circuit Relations / Feld- und Schaltungsrelationen

Field Relation / Feldrelation

$$\oiint_{S=\partial V} \underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}},t)\bullet\underline{\mathbf{dS}} = -\frac{\mathrm{d}}{\mathrm{d}t}\iiint_V \rho_{\mathrm{e}}(\underline{\mathbf{R}},t)\,\mathrm{d}V$$

(Continuity Equation for Electric Charges / Kontinuitätsgleichung für elektrische Ladungen)

$$\oint_{C=\partial S} \underline{\mathbf{E}}(\underline{\mathbf{R}},t)\bullet\underline{\mathbf{dR}} = -\frac{\mathrm{d}}{\mathrm{d}t}\iint_S \underline{\mathbf{B}}(\underline{\mathbf{R}},t)\bullet\underline{\mathbf{dS}}$$

(Faraday's Induction Law / Faradaysches Induktionsgesetz)

Circuit Relation / Schaltungsrelation

$$\sum_n i_{\mathrm{e}}^{(n)}(t) = -\frac{\mathrm{d}}{\mathrm{d}t} q_{\mathrm{e}}(t) = -C_s \frac{\mathrm{d}}{\mathrm{d}t} u_{\mathrm{e}}(t)$$

(Kirchhoff's Current Law / 1. Kirchhoffsches Gesetz, Knotenpunktregel)

$$\sum_n u_{\mathrm{e}}^{(n)}(t) = -\frac{\mathrm{d}}{\mathrm{d}t} \psi_{\mathrm{m}}(t) = -L_s \frac{\mathrm{d}}{\mathrm{d}t} i_{\mathrm{e}}(t)$$

(Kirchhoff's Loop Voltage Law / 2. Kirchhoffsches Gesetz, Maschenregel)

End of 4th Lecture / Ende der 4. Vorlesung