Elektromagnetische Feldtheorie I (EFT I) / Electromagnetic Field Theory I (EFT I)

6th Lecture / 6. Vorlesung

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Transition and Boundary Conditions / Übergangs- und Randbedingungen

Governing Equations in Integral Form / Grundgleichungen in Integralform

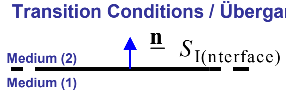
$$\oint_{C=\partial S} \underline{\mathbf{E}}(\underline{\mathbf{R}},t)\cdot \underline{\mathbf{dR}} = -\iint_S \frac{\partial}{\partial t}\underline{\mathbf{B}}(\underline{\mathbf{R}},t)\cdot\underline{\mathbf{dS}} - \iint_S \underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},t)\cdot\underline{\mathbf{dS}}$$

$$\oint_{C=\partial S} \underline{\mathbf{H}}(\underline{\mathbf{R}},t)\cdot \underline{\mathbf{dR}} = \iint_S \frac{\partial}{\partial t}\underline{\mathbf{D}}(\underline{\mathbf{R}},t)\cdot\underline{\mathbf{dS}} + \iint_S \underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}},t)\cdot\underline{\mathbf{dS}}$$

$$\oiint_{S=\partial V} \underline{\mathbf{D}}(\underline{\mathbf{R}},t)\cdot\underline{\mathbf{dS}} = \iiint_V \rho_{\mathrm{e}}(\underline{\mathbf{R}},t)\,\mathrm{d}V$$

$$\oiint_{S=\partial V} \underline{\mathbf{B}}(\underline{\mathbf{R}},t)\cdot\underline{\mathbf{dS}} = \iiint_V \rho_{\mathrm{m}}(\underline{\mathbf{R}},t)\,\mathrm{d}V$$

Transition Conditions / Übergangsbedingungen



For / Für $\underline{\mathbf{R}} \in S_{\mathrm{I}}$

$$\underline{\mathbf{n}}\times\left[\underline{\mathbf{E}}^{(2)}(\underline{\mathbf{R}},t)-\underline{\mathbf{E}}^{(1)}(\underline{\mathbf{R}},t)\right]=\begin{cases}-\underline{\mathbf{K}}_{\mathrm{m}}(\underline{\mathbf{R}},t) & \text{ws / mq}\\ \underline{\mathbf{0}} & \text{sf / qf}\end{cases}$$

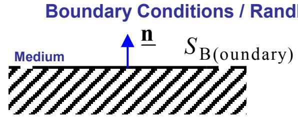
$$\underline{\mathbf{n}}\times\left[\underline{\mathbf{H}}^{(2)}(\underline{\mathbf{R}},t)-\underline{\mathbf{H}}^{(1)}(\underline{\mathbf{R}},t)\right]=\begin{cases}\underline{\mathbf{K}}_{\mathrm{e}}(\underline{\mathbf{R}},t) & \text{ws / mq}\\ \underline{\mathbf{0}} & \text{sf / qf}\end{cases}$$

$$\underline{\mathbf{n}}\cdot\left[\underline{\mathbf{D}}^{(2)}(\underline{\mathbf{R}},t)-\underline{\mathbf{D}}^{(1)}(\underline{\mathbf{R}},t)\right]=\begin{cases}\eta_{\mathrm{e}}(\underline{\mathbf{R}},t) & \text{ws / mq}\\ 0 & \text{sf / qf}\end{cases}$$

$$\underline{\mathbf{n}}\cdot\left[\underline{\mathbf{B}}^{(2)}(\underline{\mathbf{R}},t)-\underline{\mathbf{B}}^{(1)}(\underline{\mathbf{R}},t)\right]=\begin{cases}\eta_{\mathrm{m}}(\underline{\mathbf{R}},t) & \text{ws / mq}\\ 0 & \text{sf / qf}\end{cases}$$

ws: with sources; sf = source-free / mq = mit Quellen; qf = quellenfrei

Boundary Conditions / Randbedingungen



For / Für $\underline{\mathbf{R}} \in S_{\mathrm{B}}$

$$\underline{\mathbf{n}}\times\underline{\mathbf{E}}(\underline{\mathbf{R}},t)=\begin{cases}\underline{\mathbf{0}} & \text{pec / iel}\\ -\underline{\mathbf{K}}_{\mathrm{m}}(\underline{\mathbf{R}},t) & \text{pem / iml}\end{cases}$$

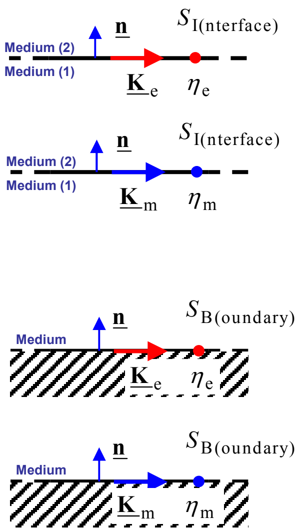
$$\underline{\mathbf{n}}\times\underline{\mathbf{H}}(\underline{\mathbf{R}},t)=\begin{cases}\underline{\mathbf{K}}_{\mathrm{e}}(\underline{\mathbf{R}},t) & \text{pec / iel}\\ \underline{\mathbf{0}} & \text{pem / iml}\end{cases}$$

$$\underline{\mathbf{n}}\cdot\underline{\mathbf{D}}(\underline{\mathbf{R}},t)=\begin{cases}\eta_{\mathrm{e}}(\underline{\mathbf{R}},t) & \text{pec / iel}\\ 0 & \text{pem / iml}\end{cases}$$

$$\underline{\mathbf{n}}\cdot\underline{\mathbf{B}}(\underline{\mathbf{R}},t)=\begin{cases}0 & \text{pec / iel}\\ \eta_{\mathrm{m}}(\underline{\mathbf{R}},t) & \text{pem / iml}\end{cases}$$

pec = perfectly electric conducting; pmc = perfectly magnetic conducting / iel = ideal elektrisch leitend; iml = ideal magnetisch leitend

Transition and Boundary Conditions / Übergangs- und Randbedingungen



For / Für $\underline{\mathbf{R}} \in S_{\mathrm{I}}$

$\underline{\mathbf{K}}_{\mathrm{e}}(\underline{\mathbf{R}},t)\,[\mathrm{A/m}]$: **Surface Density of Electric Current / Flächendichte des elektrischen Stromes**

$\underline{\mathbf{K}}_{\mathrm{m}}(\underline{\mathbf{R}},t)\,[\mathrm{V/m}]$: **Surface Density of Magnetic Current / Flächendichte des magnetischen Stromes**

$\eta_{\mathrm{e}}(\underline{\mathbf{R}},t)\,[\mathrm{As/m}]$: **Surface Density of Electric Charge / Flächendichte der elektrischen Ladung**

$\eta_{\mathrm{m}}(\underline{\mathbf{R}},t)\,[\mathrm{Vs/m}]$: **Surface Density of Magnetic Charge / Flächendichte der elektrischen Ladung**

Continuity Equations / Kontinuitätsgleichungen

$$\oiint_{S=\partial V} \underline{\mathbf{K}}_{\mathrm{e}}(\underline{\mathbf{R}},t)\cdot\underline{\mathbf{dS}} = -\iiint_V \frac{\partial}{\partial t}\eta_{\mathrm{e}}(\underline{\mathbf{R}},t)\,\mathrm{d}V$$

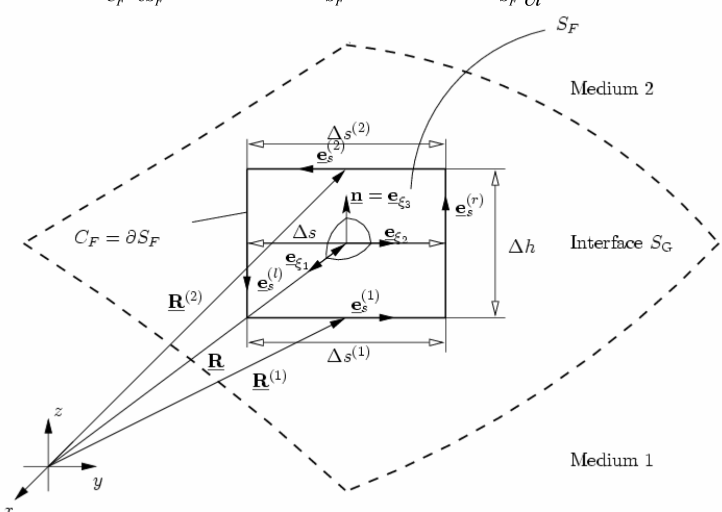
$$\oiint_{S=\partial V} \underline{\mathbf{K}}_{\mathrm{m}}(\underline{\mathbf{R}},t)\cdot\underline{\mathbf{dS}} = -\iiint_V \frac{\partial}{\partial t}\eta_{\mathrm{m}}(\underline{\mathbf{R}},t)\,\mathrm{d}V$$

$$\nabla\cdot\underline{\mathbf{K}}_{\mathrm{e}}(\underline{\mathbf{R}},t) = -\frac{\partial}{\partial t}\eta_{\mathrm{e}}(\underline{\mathbf{R}},t)$$

$$\nabla\cdot\underline{\mathbf{K}}_{\mathrm{m}}(\underline{\mathbf{R}},t) = -\frac{\partial}{\partial t}\eta_{\mathrm{m}}(\underline{\mathbf{R}},t)$$

Transition and Boundary Conditions / Übergangs- und Randbedingungen (...)

$$\oint_{C_F=\partial S_F} \underline{\mathbf{E}}(\underline{\mathbf{R}},t)\cdot\underline{\mathbf{dR}} = -\iint_{S_F} \underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},t)\cdot\underline{\mathbf{dS}} - \iint_{S_F} \frac{\partial}{\partial t}\underline{\mathbf{B}}(\underline{\mathbf{R}},t)\cdot\underline{\mathbf{dS}}$$



$$\underline{\mathbf{n}}\times\left[\underline{\mathbf{E}}^{(2)}(\underline{\mathbf{R}},t)-\underline{\mathbf{E}}^{(1)}(\underline{\mathbf{R}},t)\right] = \begin{cases} -\underline{\mathbf{K}}_{\mathrm{m}}(\underline{\mathbf{R}},t) & \mathrm{ws\,/\,mq} \\ \underline{\mathbf{0}} & \mathrm{sf\,/\,qf} \end{cases}$$

Transition and Boundary Conditions / Übergangs- und Randbedingungen (...)

For sufficiently small $\Delta s^{(1)}, \Delta s^{(1)}$ and S_F / Für genügend kleines $\Delta s^{(1)}, \Delta s^{(1)}$ und S_F

$$\oint_{C_F=\partial S_F} \underline{\mathbf{E}}(\underline{\mathbf{R}},t)\bullet \underline{\mathbf{dR}} \simeq \underline{\mathbf{E}}^{(r)}(\underline{\mathbf{R}}^{(r)},t)\bullet \underline{\mathbf{e}}_s^{(r)}\Delta h + \underline{\mathbf{E}}^{(2)}(\underline{\mathbf{R}}^{(2)},t)\bullet \underline{\mathbf{e}}_s^{(2)}\Delta s^{(2)}$$

$$+\underline{\mathbf{E}}^{(l)}(\underline{\mathbf{R}}^{(l)},t)\bullet \underline{\mathbf{e}}_s^{(l)}\Delta h + \underline{\mathbf{E}}^{(1)}(\underline{\mathbf{R}}^{(1)},t)\bullet \underline{\mathbf{e}}_s^{(1)}\Delta s^{(1)}$$

$$\iint_{S_F} \frac{\partial}{\partial t}\underline{\mathbf{B}}(\underline{\mathbf{R}},t)\bullet \underline{\mathbf{dS}} \simeq \frac{\partial}{\partial t}\underline{\mathbf{B}}^{(m)}(\underline{\mathbf{R}},t)\bullet \underline{\mathbf{n}}_{\Delta S_F}\Delta S_F$$

$$\iint_{S_F} \underline{\mathbf{J}}_\mathrm{m}(\underline{\mathbf{R}},t)\bullet \underline{\mathbf{dS}} \simeq \underline{\mathbf{J}}_\mathrm{m}^{(m)}(\underline{\mathbf{R}},t)\bullet \underline{\mathbf{n}}_{\Delta S_F}\Delta S_F$$

$\underline{\mathbf{n}}_{\Delta S_F} = \underline{\mathbf{e}}_{\xi_1}$ **Surface Normal of ΔS_F / Flächennormale von ΔS_F**

For the limit $\Delta h \to 0$ / Für den Grenzübergang $\Delta h \to 0$

$$\lim_{\Delta h\to 0}\left\{\underline{\mathbf{e}}_s^{(r)}, -\underline{\mathbf{e}}_s^{(l)}\right\} = \underline{\mathbf{e}}_{\xi_3} \qquad \lim_{\Delta h\to 0}\left\{\underline{\mathbf{R}}^{(r)}, \underline{\mathbf{R}}^{(2)}, \underline{\mathbf{R}}^{(l)}, \underline{\mathbf{R}}^{(1)}\right\} = \underline{\mathbf{R}}$$

$$\lim_{\Delta h\to 0}\left\{\underline{\mathbf{e}}_s^{(1)}, -\underline{\mathbf{e}}_s^{(1)}\right\} = \underline{\mathbf{e}}_{\xi_2} \qquad \lim_{\Delta h\to 0}\Delta S_F = \Delta s\Delta h$$

$$\lim_{\Delta h\to 0}\left\{\Delta s^{(1)}, \Delta s^{(2)}\right\} = \Delta s \qquad \lim_{\Delta h\to 0}\left\{\underline{\mathbf{E}}^{(r)}(\underline{\mathbf{R}},t), \underline{\mathbf{E}}^{(l)}(\underline{\mathbf{R}},t)\right\} = \underline{\mathbf{E}}^{(1,2)}(\underline{\mathbf{R}},t)$$

Transition and Boundary Conditions / Übergangs- und Randbedingungen (...)

$$\lim_{\Delta h\to 0}\oint_{C_F=\partial S_F} \underline{\mathbf{E}}(\underline{\mathbf{R}},t)\bullet \underline{\mathbf{dR}} \simeq \lim_{\Delta h\to 0}\Big[\underline{\mathbf{E}}^{(1,2)}(\underline{\mathbf{R}},t)\bullet \underline{\mathbf{e}}_{\xi_3}\Delta h - \underline{\mathbf{E}}^{(2)}(\underline{\mathbf{R}},t)\bullet \underline{\mathbf{e}}_{\xi_2}\Delta s$$

$$-\underline{\mathbf{E}}^{(1,2)}(\underline{\mathbf{R}},t)\bullet \underline{\mathbf{e}}_{\xi_3}\Delta h + \underline{\mathbf{E}}^{(1)}(\underline{\mathbf{R}},t)\bullet \underline{\mathbf{e}}_{\xi_2}\Delta s\Big]$$

$$\lim_{\Delta h\to 0}\iint_{S_F} \frac{\partial}{\partial t}\underline{\mathbf{B}}(\underline{\mathbf{R}},t)\bullet \underline{\mathbf{dS}} \simeq \lim_{\Delta h\to 0}\frac{\partial}{\partial t}\underline{\mathbf{B}}^{(m)}(\underline{\mathbf{R}},t)\bullet \underline{\mathbf{e}}_{\xi_1}\Delta s\Delta h$$

$$\simeq 0 \qquad \text{(because / weil } \left|\frac{\partial}{\partial t}\underline{\mathbf{B}}^{(m)}(\underline{\mathbf{R}},t)\right| \not\to \infty)$$

$$\lim_{\Delta h\to 0}\iint_{S_F} \underline{\mathbf{J}}_\mathrm{m}(\underline{\mathbf{R}},t)\bullet \underline{\mathbf{dS}} \simeq \lim_{\Delta h\to 0}\underline{\mathbf{J}}_\mathrm{m}^{(m)}(\underline{\mathbf{R}},t)\bullet \underline{\mathbf{e}}_{\xi_1}\Delta s\Delta h$$

$$\neq 0 \qquad \text{(if / falls } \left|\underline{\mathbf{J}}_\mathrm{m}^{(m)}(\underline{\mathbf{R}},t)\right| \to \infty)$$

$$\oint_{C_F=\partial S_F} \underline{\mathbf{E}}(\underline{\mathbf{R}},t)\bullet \underline{\mathbf{dR}} = -\iint_{S_F} \underline{\mathbf{J}}_\mathrm{m}(\underline{\mathbf{R}},t)\bullet \underline{\mathbf{dS}} - \iint_{S_F}\frac{\partial}{\partial t}\underline{\mathbf{B}}(\underline{\mathbf{R}},t)\bullet \underline{\mathbf{dS}}$$

$$\Downarrow$$

$$\left[\underline{\mathbf{E}}^{(1)}(\underline{\mathbf{R}},t) - \underline{\mathbf{E}}^{(2)}(\underline{\mathbf{R}},t)\right]\bullet \underline{\mathbf{e}}_{\xi_2} = -\lim_{\Delta h\to 0}\left[\underline{\mathbf{J}}_\mathrm{m}(\underline{\mathbf{R}},t)\bullet \underline{\mathbf{e}}_{\xi_1}\Delta h\right]$$

Transition and Boundary Conditions / Übergangs- und Randbedingungen (...)

$$\left[\underline{\mathbf{E}}^{(1)}(\underline{\mathbf{R}},t)-\underline{\mathbf{E}}^{(2)}(\underline{\mathbf{R}},t)\right]\bullet\underline{\mathbf{e}}_{\xi_2} = -\lim_{\Delta h\to 0}\left[\underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},t)\bullet\underline{\mathbf{e}}_{\xi_1}\Delta h\right]$$

$$\underline{\mathbf{e}}_{\xi_2} = \underline{\mathbf{n}}\times\underline{\mathbf{e}}_{\xi_1}$$

$$\underline{\mathbf{A}}\bullet\left(\underline{\mathbf{B}}\times\underline{\mathbf{C}}\right) = \left(\underline{\mathbf{A}}\times\underline{\mathbf{B}}\right)\bullet\underline{\mathbf{C}}$$

$$\underline{\mathbf{A}} = \underline{\mathbf{E}}^{(i)}(\underline{\mathbf{R}},t);\underline{\mathbf{B}} = \underline{\mathbf{n}};\underline{\mathbf{C}} = \underline{\mathbf{e}}_{\xi_1}$$

$$\left\{\left[\underline{\mathbf{E}}^{(1)}(\underline{\mathbf{R}},t)-\underline{\mathbf{E}}^{(2)}(\underline{\mathbf{R}},t)\right]\times\underline{\mathbf{n}}\right\}\bullet\underline{\mathbf{e}}_{\xi_1} = -\lim_{\Delta h\to 0}\left[\underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},t)\Delta h\right]\bullet\underline{\mathbf{e}}_{\xi_1}$$

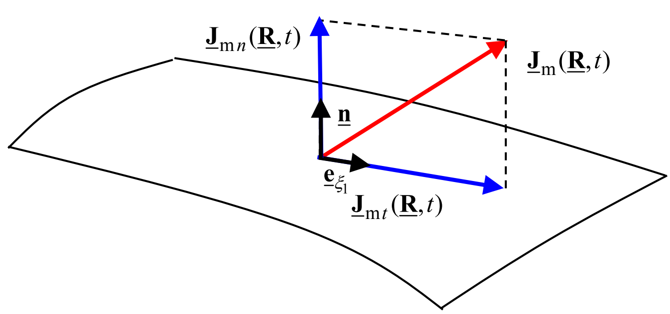
$$\left\{\underline{\mathbf{n}}\times\left[\underline{\mathbf{E}}^{(2)}(\underline{\mathbf{R}},t)-\underline{\mathbf{E}}^{(1)}(\underline{\mathbf{R}},t)\right]\right\}\bullet\underline{\mathbf{e}}_{\xi_1} = -\lim_{\Delta h\to 0}\left[\underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},t)\Delta h\right]\bullet\underline{\mathbf{e}}_{\xi_1}$$

Transition and Boundary Conditions / Übergangs- und Randbedingungen (...)

With the Decomposition of $\underline{\mathbf{J}}$ in a Normal and Tangential Component with Regard to the Interface / Mit der Zerlegung von $\underline{\mathbf{J}}$ in eine Normal - und Tangentialkomponente bezüglich der Trennfläche

$$\underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},t) = \underbrace{\underbrace{\underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},t)\bullet\underline{\mathbf{n}}}_{=J_{\mathrm{m}n}(\underline{\mathbf{R}},t)}\,\underline{\mathbf{n}}}_{=\underline{\mathbf{J}}_{\mathrm{m}n}(\underline{\mathbf{R}},t)} + \underbrace{\underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},t)\bullet\left(\underline{\underline{\mathbf{I}}}-\underline{\mathbf{n}\mathbf{n}}\right)}_{=\underline{\mathbf{J}}_{\mathrm{m}t}(\underline{\mathbf{R}},t)}$$

$$= \underbrace{\underline{\mathbf{J}}_{\mathrm{m}n}(\underline{\mathbf{R}},t)}_{\text{Normal Component / Normalkomponente}} + \underbrace{\underline{\mathbf{J}}_{\mathrm{m}t}(\underline{\mathbf{R}},t)}_{\text{Tangential Component / Tangentialkomponente}}$$



$$\underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},t)\bullet\underline{\mathbf{e}}_{\xi_1} = \underline{\mathbf{J}}_{\mathrm{m}t}(\underline{\mathbf{R}},t)\bullet\underline{\mathbf{e}}_{\xi_1}$$

Transition and Boundary Conditions / Übergangs- und Randbedingungen (...)

$$\left\{\underline{\mathbf{n}}\times\left[\underline{\mathbf{E}}^{(2)}(\underline{\mathbf{R}},t)-\underline{\mathbf{E}}^{(1)}(\underline{\mathbf{R}},t)\right]\right\}\bullet\underline{\mathbf{e}}_{\xi_1}=-\lim_{\Delta h\to 0}\left[\underline{\mathbf{J}}_{\mathrm{m}t}(\underline{\mathbf{R}},t)\Delta h\right]\bullet\underline{\mathbf{e}}_{\xi_1}$$

$$\underline{\mathbf{n}}\times\left[\underline{\mathbf{E}}^{(2)}(\underline{\mathbf{R}},t)-\underline{\mathbf{E}}^{(1)}(\underline{\mathbf{R}},t)\right]=-\lim_{\Delta h\to 0}\left[\underline{\mathbf{J}}_{\mathrm{m}t}(\underline{\mathbf{R}},t)\Delta h\right]$$

$$\lim_{\Delta h\to 0}\left[\underline{\mathbf{J}}_{\mathrm{m}t}(\underline{\mathbf{R}},t)\Delta h\right]\begin{cases}=0 & \left|\underline{\mathbf{J}}_{\mathrm{m}t}(\underline{\mathbf{R}},t)\right|=\text{finite / endlich}\\ \neq 0 & \left|\underline{\mathbf{J}}_{\mathrm{m}t}(\underline{\mathbf{R}},t)\right|=\text{infinite / unendlich}\end{cases}$$

$$\lim_{\substack{\Delta h\to 0\\ \left|\underline{\mathbf{J}}_{\mathrm{m}t}(\underline{\mathbf{R}},t)\right|\to\infty}}\left[\underline{\mathbf{J}}_{\mathrm{m}t}(\underline{\mathbf{R}},t)\Delta h\right]=\underline{\mathbf{K}}_{\mathrm{m}}(\underline{\mathbf{R}},t)$$

$\underline{\mathbf{J}}_{\mathrm{m}t}(\underline{\mathbf{R}},t)$ $[\mathrm{V/m^2}]$: **Tangential Component of the Volume Density of Magnetic Current / Tangentialkomponente der Volumendichte des magnetischen Stromes**

$\underline{\mathbf{K}}_{\mathrm{m}}(\underline{\mathbf{R}},t)$ $[\mathrm{V/m}]$: **Surface Density of Magnetic Current / Flächendichte des magnetischen Stromes**

$$\left|\underline{\mathbf{J}}_{\mathrm{m}t}(\underline{\mathbf{R}},t)\right|\to\infty\quad ?$$

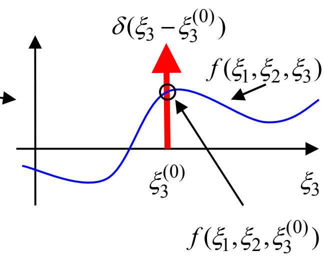
We Make use of the One-Dimensional Delta-Distribution / Mache Gebrauch von der eindimensionalen Delta-Distribution

$$\int_{\xi_3=-\infty}^{\infty}\delta(\xi_3-\xi_3^{(0)})\,h_{\xi_3}\,d\xi_3=1$$

$$\int_{\xi_3=-\infty}^{\infty}f(\xi_1,\xi_2,\xi_3)\frac{\delta(\xi_3-\xi_3^{(0)})}{h_{\xi_3}}h_{\xi_3}\,d\xi_3=f(\xi_1,\xi_2,\xi_3^{(0)})$$

$$f(\xi_1,\xi_2,\xi_3)\frac{\delta(\xi_3-\xi_3^{(0)})}{h_{\xi_3}}=f(\xi_1,\xi_2,\xi_3^{(0)})\frac{\delta(\xi_3-\xi_3^{(0)})}{h_{\xi_3}}$$

Sifting Property / Siebeigenschaft



Distribution ➡ Generalized Function / Distribution ➡ Verallgemeinerte Funktion

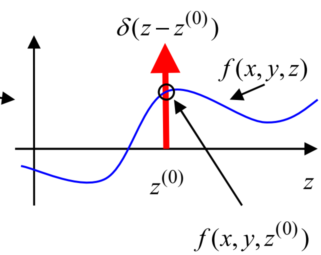
$$(\xi_1,\xi_2,\xi_3)\to(x,y,z)$$

$$\int_{z=-\infty}^{\infty}\delta(z-z^{(0)})\,dz=1$$

$$\int_{z=-\infty}^{\infty}f(x,y,z)\,\delta(z-z^{(0)})\,dz=f(x,y,z^{(0)})$$

$$f(x,y,z)\,\delta(z-z^{(0)})=f(x,y,z-z^{(0)})\,\delta(z-z^{(0)})$$

Sifting Property / Siebeigenschaft



Transition and Boundary Conditions / Übergangs- und Randbedingungen (...)

$$\underline{\mathbf{J}}_{\mathrm{m}t}(\xi_1,\xi_2,\xi_3,t) = \underline{\mathbf{K}}_{\mathrm{m}}(\xi_1,\xi_2,\xi_3,t)\frac{\delta\left(\xi_3-\xi_3^{(0)}\right)}{h_{\xi_3}}$$

$$= \underline{\mathbf{K}}_{\mathrm{m}}(\xi_1,\xi_2,\xi_3^{(0)},t)\frac{\delta\left(\xi_3-\xi_3^{(0)}\right)}{h_{\xi_3}}$$

$$(\xi_1,\xi_2,\xi_3) \rightarrow (x,y,z)$$

$$\underline{\mathbf{J}}_{\mathrm{m}t}(x,y,z,t) = \underline{\mathbf{K}}_{\mathrm{m}}(x,y,z,t)\,\delta\left(z-z_0\right)$$

$$\underline{\mathbf{n}} \times \left[\underline{\mathbf{E}}^{(2)}(\underline{\mathbf{R}},t) - \underline{\mathbf{E}}^{(1)}(\underline{\mathbf{R}},t)\right] = -\lim_{\Delta h \to 0}\left[\underline{\mathbf{J}}_{\mathrm{m}t}(\underline{\mathbf{R}},t)\Delta h\right]$$

$$\lim_{\substack{\Delta h \to 0 \\ \left|\underline{\mathbf{J}}_{\mathrm{m}t}(\underline{\mathbf{R}},t)\right| \to \infty}}\left[\underline{\mathbf{J}}_{\mathrm{m}t}(\underline{\mathbf{R}},t)\Delta h\right] = \underline{\mathbf{K}}_{\mathrm{m}}(\underline{\mathbf{R}},t)$$

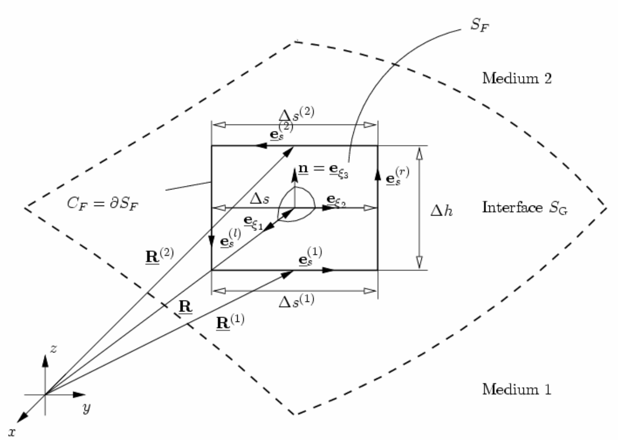
Source Term / Quellterm

$$\underline{\mathbf{n}} \times \left[\underline{\mathbf{E}}^{(2)}(\underline{\mathbf{R}},t) - \underline{\mathbf{E}}^{(1)}(\underline{\mathbf{R}},t)\right] = \begin{cases} -\underline{\mathbf{K}}_{\mathrm{m}}(\underline{\mathbf{R}},t) & \text{ws / mq} \\ \underline{\mathbf{0}} & \text{sf / qf} \end{cases}$$

ws: with sources; sf = source-free /
mq = mit Quellen; qf = quellenfrei

Transition and Boundary Conditions / Übergangs- und Randbedingungen (...)

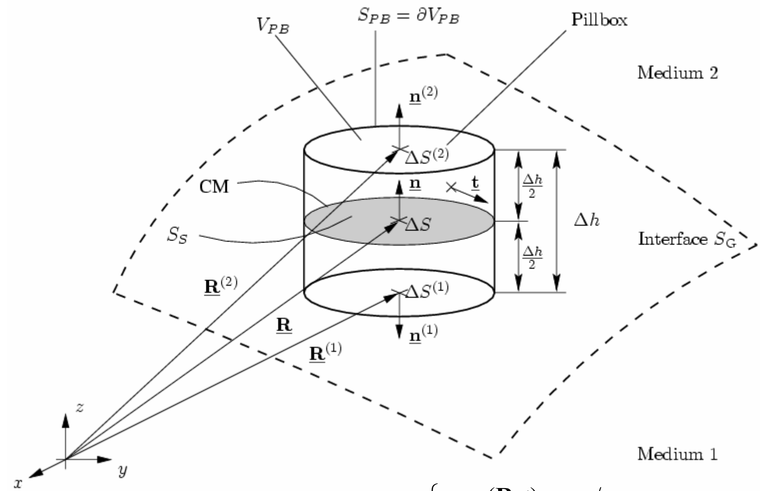
$$\oint_{C_F=\partial S_F} \underline{\mathbf{H}}(\underline{\mathbf{R}},t)\bullet \underline{\mathbf{dR}} = \iint_{S_F} \underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}},t)\bullet \underline{\mathbf{dS}} + \iint_{S_F} \frac{\partial}{\partial t}\underline{\mathbf{D}}(\underline{\mathbf{R}},t)\bullet \underline{\mathbf{dS}}$$



$$\underline{\mathbf{n}} \times \left[\underline{\mathbf{H}}^{(2)}(\underline{\mathbf{R}},t) - \underline{\mathbf{H}}^{(1)}(\underline{\mathbf{R}},t)\right] = \begin{cases} \underline{\mathbf{K}}_{\mathrm{e}}(\underline{\mathbf{R}},t) & \text{ws / mq} \\ \underline{\mathbf{0}} & \text{sf / qf} \end{cases}$$

Transition and Boundary Conditions / Übergangs- und Randbedingungen (...)

$$\oint\!\!\oint_{S_{PB}=\partial V_{PB}} \underline{\mathbf{D}}(\underline{\mathbf{R}},t)\bullet\underline{\mathbf{dS}} = \iiint_{V_{PB}} \rho_e(\underline{\mathbf{R}},t)\,\mathrm{d}V$$



$$\underline{\mathbf{n}}\bullet\left[\underline{\mathbf{D}}^{(2)}(\underline{\mathbf{R}},t)-\underline{\mathbf{D}}^{(1)}(\underline{\mathbf{R}},t)\right]=\begin{cases}\eta_e(\underline{\mathbf{R}},t) & \text{ws / mq}\\ 0 & \text{sf / qf}\end{cases}$$

Transition and Boundary Conditions / Übergangs- und Randbedingungen (...)

$$\oint\!\!\oint_{S_{PB}=\partial V_{PB}} \underline{\mathbf{D}}(\underline{\mathbf{R}},t)\bullet\underline{\mathbf{dS}} = \iiint_{V_{PB}} \rho_e(\underline{\mathbf{R}},t)\,\mathrm{d}V$$

$$\underline{\mathbf{n}}\bullet\left[\underline{\mathbf{D}}^{(2)}(\underline{\mathbf{R}},t)-\underline{\mathbf{D}}^{(1)}(\underline{\mathbf{R}},t)\right]=\begin{cases}\eta_e(\underline{\mathbf{R}},t) & \text{ws / mq}\\ 0 & \text{sf / qf}\end{cases}$$

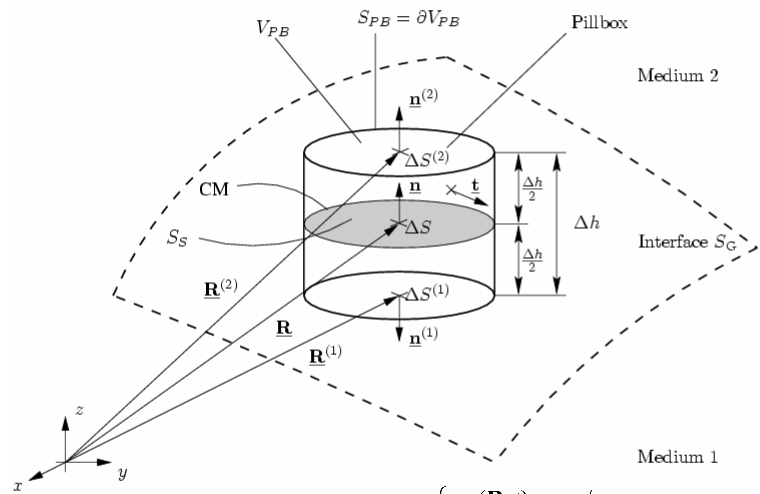
$$\lim_{\substack{\Delta h\to 0\\ |\rho_e(\underline{\mathbf{R}},t)|\to\infty}} \left[\rho_e(\underline{\mathbf{R}},t)\Delta h\right] = \eta_e(\underline{\mathbf{R}},t)$$

$\rho_e(\underline{\mathbf{R}},t)$ [As/m^3] : **Volume Density of Electric Charge / Volumendichte der elektrischen Ladung**

$\eta_e(\underline{\mathbf{R}},t)$ [As/m^2] : **Surface Density of Electric Charge / Flächendichte der elektrischen Ladung**

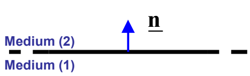
Transition and Boundary Conditions / Übergangs- und Randbedingungen (...)

$$\oiint_{S_{PB}=\partial V_{PB}} \underline{\mathbf{B}}(\underline{\mathbf{R}},t)\bullet \underline{\mathbf{dS}} = \iiint_{V_{PB}} \eta_{\mathrm{m}}(\underline{\mathbf{R}},t)\,\mathrm{d}V$$



$$\underline{\mathbf{n}}\bullet\left[\underline{\mathbf{B}}^{(2)}(\underline{\mathbf{R}},t)-\underline{\mathbf{B}}^{(1)}(\underline{\mathbf{R}},t)\right]=\begin{cases}\eta_{\mathrm{m}}(\underline{\mathbf{R}},t) & \text{ws / mq}\\ 0 & \text{sf / qf}\end{cases}$$

Transition Conditions / Übergangsbedingungen (...)



For / Für $\underline{\mathbf{R}} \in S_{\mathrm{I(nterface)}}$

$$\underline{\mathbf{n}}\times\left[\underline{\mathbf{E}}^{(2)}(\underline{\mathbf{R}},t)-\underline{\mathbf{E}}^{(1)}(\underline{\mathbf{R}},t)\right]=\begin{cases}-\underline{\mathbf{K}}_{\mathrm{m}}(\underline{\mathbf{R}},t) & \text{ws / mq}\\ \underline{\mathbf{0}} & \text{sf / qf}\end{cases}$$

$$\underline{\mathbf{n}}\times\left[\underline{\mathbf{H}}^{(2)}(\underline{\mathbf{R}},t)-\underline{\mathbf{H}}^{(1)}(\underline{\mathbf{R}},t)\right]=\begin{cases}\underline{\mathbf{K}}_{\mathrm{e}}(\underline{\mathbf{R}},t) & \text{ws / mq}\\ \underline{\mathbf{0}} & \text{sf / qf}\end{cases}$$

$$\underline{\mathbf{n}}\bullet\left[\underline{\mathbf{D}}^{(2)}(\underline{\mathbf{R}},t)-\underline{\mathbf{D}}^{(1)}(\underline{\mathbf{R}},t)\right]=\begin{cases}\eta_{\mathrm{e}}(\underline{\mathbf{R}},t) & \text{ws / mq}\\ 0 & \text{sf / qf}\end{cases}$$

$$\underline{\mathbf{n}}\bullet\left[\underline{\mathbf{B}}^{(2)}(\underline{\mathbf{R}},t)-\underline{\mathbf{B}}^{(1)}(\underline{\mathbf{R}},t)\right]=\begin{cases}\eta_{\mathrm{m}}(\underline{\mathbf{R}},t) & \text{ws / mq}\\ 0 & \text{sf / qf}\end{cases}$$

ε_r μ_r-Medium:

$$\underline{\mathbf{D}}^{(1)}(\underline{\mathbf{R}},t)=\varepsilon_0\varepsilon_{\mathrm{r}}^{(i)}\underline{\mathbf{E}}^{(i)}(\underline{\mathbf{R}},t)$$

$$\underline{\mathbf{B}}^{(1)}(\underline{\mathbf{R}},t)=\mu_0\mu_{\mathrm{r}}^{(i)}\underline{\mathbf{H}}^{(i)}(\underline{\mathbf{R}},t)$$

for $i=1,2$

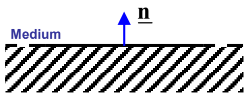
$$\underline{\mathbf{n}}\times\left[\underline{\mathbf{D}}^{(2)}(\underline{\mathbf{R}},t)-\frac{\varepsilon_{\mathrm{r}}^{(2)}}{\varepsilon_{\mathrm{r}}^{(1)}}\underline{\mathbf{D}}^{(1)}(\underline{\mathbf{R}},t)\right]=\begin{cases}-\varepsilon_0\varepsilon_{\mathrm{r}}^{(2)}\underline{\mathbf{K}}_{\mathrm{m}}(\underline{\mathbf{R}},t) & \text{ws / mq}\\ \underline{\mathbf{0}} & \text{sf / qf}\end{cases}$$

$$\underline{\mathbf{n}}\times\left[\underline{\mathbf{B}}^{(2)}(\underline{\mathbf{R}},t)-\frac{\mu_{\mathrm{r}}^{(2)}}{\mu_{\mathrm{r}}^{(1)}}\underline{\mathbf{B}}^{(1)}(\underline{\mathbf{R}},t)\right]=\begin{cases}\mu_0\mu_{\mathrm{r}}^{(2)}\underline{\mathbf{K}}_{\mathrm{e}}(\underline{\mathbf{R}},t) & \text{ws / mq}\\ \underline{\mathbf{0}} & \text{sf / qf}\end{cases}$$

$$\underline{\mathbf{n}}\bullet\left[\underline{\mathbf{E}}^{(2)}(\underline{\mathbf{R}},t)-\frac{\varepsilon_{\mathrm{r}}^{(1)}}{\varepsilon_{\mathrm{r}}^{(2)}}\underline{\mathbf{E}}^{(1)}(\underline{\mathbf{R}},t)\right]=\begin{cases}\dfrac{1}{\varepsilon_0\varepsilon_{\mathrm{r}}^{(2)}}\eta_{\mathrm{e}}(\underline{\mathbf{R}},t) & \text{ws / mq}\\ 0 & \text{sf / qf}\end{cases}$$

$$\underline{\mathbf{n}}\bullet\left[\underline{\mathbf{H}}^{(2)}(\underline{\mathbf{R}},t)-\frac{\mu_{\mathrm{r}}^{(1)}}{\mu_{\mathrm{r}}^{(2)}}\underline{\mathbf{H}}^{(1)}(\underline{\mathbf{R}},t)\right]=\begin{cases}\dfrac{1}{\mu_0\mu_{\mathrm{r}}^{(2)}}\eta_{\mathrm{m}}(\underline{\mathbf{R}},t) & \text{ws / mq}\\ 0 & \text{sf / qf}\end{cases}$$

Boundary Conditions / Randbedingungen (...)



ε_r μ_r-Medium:

$$\underline{\mathbf{D}}^{(1)}(\underline{\mathbf{R}},t)=\varepsilon_0\varepsilon_r^{(i)}\underline{\mathbf{E}}^{(i)}(\underline{\mathbf{R}},t)$$

$$\underline{\mathbf{B}}^{(1)}(\underline{\mathbf{R}},t)=\mu_0\mu_r^{(i)}\underline{\mathbf{H}}^{(i)}(\underline{\mathbf{R}},t)$$

$$\text{for} \quad i=1,2$$

For / Für $\underline{\mathbf{R}} \in S_{\mathrm{B(oundary)}}$

$$\underline{\mathbf{n}}\times\underline{\mathbf{E}}(\underline{\mathbf{R}},t)=\begin{cases}\underline{\mathbf{0}} & \text{pec / iel}\\ -\underline{\mathbf{K}}_\mathrm{m}(\underline{\mathbf{R}},t) & \text{pem / iml}\end{cases}$$

$$\underline{\mathbf{n}}\times\underline{\mathbf{H}}(\underline{\mathbf{R}},t)=\begin{cases}\underline{\mathbf{K}}_\mathrm{e}(\underline{\mathbf{R}},t) & \text{pec / iel}\\ \underline{\mathbf{0}} & \text{pem / iml}\end{cases}$$

$$\underline{\mathbf{n}}\bullet\underline{\mathbf{D}}(\underline{\mathbf{R}},t)=\begin{cases}\eta_\mathrm{e}(\underline{\mathbf{R}},t) & \text{pec / iel}\\ 0 & \text{pem / iml}\end{cases}$$

$$\underline{\mathbf{n}}\bullet\underline{\mathbf{B}}(\underline{\mathbf{R}},t)=\begin{cases}0 & \text{pec / iel}\\ \eta_\mathrm{m}(\underline{\mathbf{R}},t) & \text{pem / iml}\end{cases}$$

$$\underline{\mathbf{n}}\times\underline{\mathbf{D}}(\underline{\mathbf{R}},t)=\begin{cases}\underline{\mathbf{0}} & \text{pec / iel}\\ -\varepsilon_0\varepsilon_r\underline{\mathbf{K}}_\mathrm{m}(\underline{\mathbf{R}},t) & \text{pem / iml}\end{cases}$$

$$\underline{\mathbf{n}}\times\underline{\mathbf{H}}(\underline{\mathbf{R}},t)=\begin{cases}\mu_0\mu_r\underline{\mathbf{K}}_\mathrm{e}(\underline{\mathbf{R}},t) & \text{pec / iel}\\ \underline{\mathbf{0}} & \text{pem / iml}\end{cases}$$

$$\underline{\mathbf{n}}\bullet\underline{\mathbf{E}}(\underline{\mathbf{R}},t)=\begin{cases}\dfrac{1}{\varepsilon_0\varepsilon_r}\eta_\mathrm{e}(\underline{\mathbf{R}},t) & \text{pec / iel}\\ 0 & \text{pem / iml}\end{cases}$$

$$\underline{\mathbf{n}}\bullet\underline{\mathbf{H}}(\underline{\mathbf{R}},t)=\begin{cases}0 & \text{pec / iel}\\ \dfrac{1}{\mu_0\mu_r}\eta_\mathrm{m}(\underline{\mathbf{R}},t) & \text{pem / iml}\end{cases}$$

Transition and Boundary Conditions / Übergangs- und Randbedingungen (...)

Continuity Equations / Kontinuitätsgleichungen

$$\oiint_{S=\partial V}\underline{\mathbf{J}}_\mathrm{e}(\underline{\mathbf{R}},t)\bullet\underline{\mathbf{dS}}=-\iiint_V\frac{\partial}{\partial t}\rho_\mathrm{e}(\underline{\mathbf{R}},t)\mathrm{d}V$$

$$\oiint_{S=\partial V}\underline{\mathbf{J}}_\mathrm{m}(\underline{\mathbf{R}},t)\bullet\underline{\mathbf{dS}}=-\iiint_V\frac{\partial}{\partial t}\rho_\mathrm{m}(\underline{\mathbf{R}},t)\mathrm{d}V$$

$$\nabla\bullet\underline{\mathbf{J}}_\mathrm{e}(\underline{\mathbf{R}},t)=-\frac{\partial}{\partial t}\rho_\mathrm{e}(\underline{\mathbf{R}},t)$$

$$\nabla\bullet\underline{\mathbf{J}}_\mathrm{m}(\underline{\mathbf{R}},t)=-\frac{\partial}{\partial t}\rho_\mathrm{m}(\underline{\mathbf{R}},t)$$

Transition Conditions / Übergangsbedingungen

$$\underline{\mathbf{n}}\bullet\left[\underline{\mathbf{K}}_\mathrm{e}^{(1)}(\underline{\mathbf{R}},t)-\underline{\mathbf{K}}_\mathrm{e}^{(2)}(\underline{\mathbf{R}},t)\right]=\begin{cases}-\dfrac{\partial}{\partial t}\eta_\mathrm{e}(\underline{\mathbf{R}},t) & \text{ws / mq}\\ 0 & \text{sf / qf}\end{cases}$$

$$\underline{\mathbf{n}}\bullet\left[\underline{\mathbf{K}}_\mathrm{m}^{(1)}(\underline{\mathbf{R}},t)-\underline{\mathbf{K}}_\mathrm{m}^{(2)}(\underline{\mathbf{R}},t)\right]=\begin{cases}-\dfrac{\partial}{\partial t}\eta_\mathrm{m}(\underline{\mathbf{R}},t) & \text{ws / mq}\\ 0 & \text{sf / qf}\end{cases}$$

Boundary Conditions / Randbedingungen

$$\underline{\mathbf{n}}\bullet\underline{\mathbf{K}}_\mathrm{e}(\underline{\mathbf{R}},t)=\begin{cases}-\dfrac{\partial}{\partial t}\eta_\mathrm{e}(\underline{\mathbf{R}},t) & \text{pec / iel}\\ 0 & \text{pem / iml}\end{cases}$$

$$\underline{\mathbf{n}}\bullet\underline{\mathbf{K}}_\mathrm{m}(\underline{\mathbf{R}},t)=\begin{cases}0 & \text{pec / iel}\\ -\dfrac{\partial}{\partial t}\eta_\mathrm{m}(\underline{\mathbf{R}},t) & \text{pem / iml}\end{cases}$$

Transition and Boundary Conditions / Übergangs- und Randbedingungen (...)

Governing Equations in Differential Form / Grundgleichungen in Differentialform

$$\nabla \times \underline{\mathbf{E}}(\underline{\mathbf{R}},t) = -\underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},t) - \frac{\partial}{\partial t}\underline{\mathbf{B}}(\underline{\mathbf{R}},t)$$

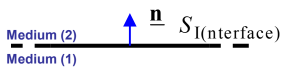
$$\nabla \times \underline{\mathbf{H}}(\underline{\mathbf{R}},t) = \underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}},t) + \frac{\partial}{\partial t}\underline{\mathbf{D}}(\underline{\mathbf{R}},t)$$

$$\nabla \cdot \underline{\mathbf{D}}(\underline{\mathbf{R}},t) = \rho_{\mathrm{e}}(\underline{\mathbf{R}},t)$$

$$\nabla \cdot \underline{\mathbf{B}}(\underline{\mathbf{R}},t) = \underbrace{\rho_{\mathrm{m}}(\underline{\mathbf{R}},t)}$$

Field Independent Sources / Feldunabhängige Quellen

Transition Conditions / Übergangsbedingungen



For / Für $\underline{\mathbf{R}} \in S_{\mathrm{I}}$

$$\underline{\mathbf{n}} \times \left[\underline{\mathbf{E}}^{(2)}(\underline{\mathbf{R}},t) - \underline{\mathbf{E}}^{(1)}(\underline{\mathbf{R}},t)\right] = \begin{cases} -\underline{\mathbf{K}}_{\mathrm{m}}(\underline{\mathbf{R}},t) & \text{ws / mq} \\ \underline{\mathbf{0}} & \text{sf / qf} \end{cases}$$

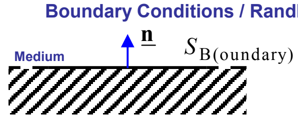
$$\underline{\mathbf{n}} \times \left[\underline{\mathbf{H}}^{(2)}(\underline{\mathbf{R}},t) - \underline{\mathbf{H}}^{(1)}(\underline{\mathbf{R}},t)\right] = \begin{cases} \underline{\mathbf{K}}_{\mathrm{e}}(\underline{\mathbf{R}},t) & \text{ws / mq} \\ \underline{\mathbf{0}} & \text{sf / qf} \end{cases}$$

$$\underline{\mathbf{n}} \cdot \left[\underline{\mathbf{D}}^{(2)}(\underline{\mathbf{R}},t) - \underline{\mathbf{D}}^{(1)}(\underline{\mathbf{R}},t)\right] = \begin{cases} \eta_{\mathrm{e}}(\underline{\mathbf{R}},t) & \text{ws / mq} \\ 0 & \text{sf / qf} \end{cases}$$

$$\underline{\mathbf{n}} \cdot \left[\underline{\mathbf{B}}^{(2)}(\underline{\mathbf{R}},t) - \underline{\mathbf{B}}^{(1)}(\underline{\mathbf{R}},t)\right] = \begin{cases} \eta_{\mathrm{m}}(\underline{\mathbf{R}},t) & \text{ws / mq} \\ 0 & \text{sf / qf} \end{cases}$$

ws: with sources; sf = source-free /
mq = mit Quellen; qf = quellenfrei

Boundary Conditions / Randbedingungen



For / Für $\underline{\mathbf{R}} \in S_{\mathrm{B}}$

$$\underline{\mathbf{n}} \times \underline{\mathbf{E}}(\underline{\mathbf{R}},t) = \begin{cases} \underline{\mathbf{0}} & \text{pec / iel} \\ -\underline{\mathbf{K}}_{\mathrm{m}}(\underline{\mathbf{R}},t) & \text{pem / iml} \end{cases}$$

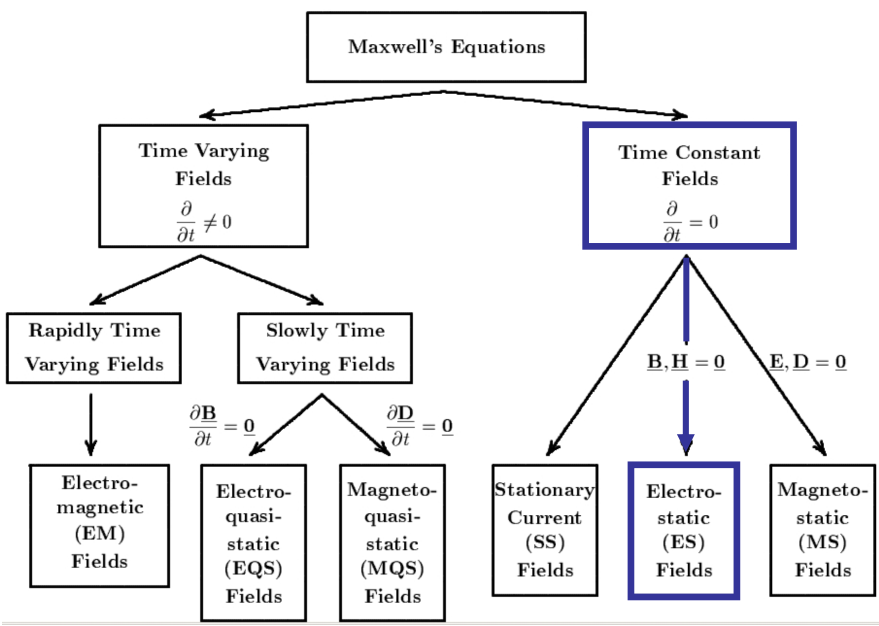
$$\underline{\mathbf{n}} \times \underline{\mathbf{H}}(\underline{\mathbf{R}},t) = \begin{cases} \underline{\mathbf{K}}_{\mathrm{e}}(\underline{\mathbf{R}},t) & \text{pec / iel} \\ \underline{\mathbf{0}} & \text{pem / iml} \end{cases}$$

$$\underline{\mathbf{n}} \cdot \underline{\mathbf{D}}(\underline{\mathbf{R}},t) = \begin{cases} \eta_{\mathrm{e}}(\underline{\mathbf{R}},t) & \text{pec / iel} \\ 0 & \text{pem / iml} \end{cases}$$

$$\underline{\mathbf{n}} \cdot \underline{\mathbf{B}}(\underline{\mathbf{R}},t) = \begin{cases} 0 & \text{pec / iel} \\ \eta_{\mathrm{m}}(\underline{\mathbf{R}},t) & \text{pem / iml} \end{cases}$$

pec = perfectly electric conducting; pmc = perfectly magnetic conducting / iel = ideal elektrisch leitend; iml = ideal magnetisch leitend

Electrostatic (ES) Fields / Elektrostatische (ES) Felder



Electrostatic (ES) Fields – Governing Equations / Elektrostatische (ES) Felder – Grundgleichungen

Electrostatic / Elektrostatik $\quad \dfrac{\partial}{\partial t} \equiv 0 \quad$ No Time Dependence and No Magnetic Field Quantities / Keine Zeitabhängigkeit und keine magnetischen Feldgrößen

$\underline{\mathbf{E}}(\underline{\mathbf{R}})$: Electric Field Strength / Elektrische Feldstärke
$\underline{\mathbf{D}}(\underline{\mathbf{R}})$: Electric Flux Density / Elektrische Flussdichte
$\rho_\mathrm{e}(\underline{\mathbf{R}})$: Electric Charge Density / Elektrische Raumladungsdichte

Integral Form / Integralform	Differential Form / Differentialform
$\oint_{C=\partial S} \underline{\mathbf{E}}(\underline{\mathbf{R}}) \bullet \mathbf{d}\underline{\mathbf{R}} = 0$	$\nabla \times \underline{\mathbf{E}}(\underline{\mathbf{R}}) = \underline{\mathbf{0}}$
$\oiint_{S=\partial V} \underline{\mathbf{D}}(\underline{\mathbf{R}}) \bullet \mathbf{d}\underline{\mathbf{S}} = \iiint_V \rho_\mathrm{e}(\underline{\mathbf{R}})\,\mathrm{d}V$	$\nabla \bullet \underline{\mathbf{D}}(\underline{\mathbf{R}}) = \rho_\mathrm{e}(\underline{\mathbf{R}})$

Curl-Free $\underline{\mathrm{E}}$-Field / Rotationsfreies $\underline{\mathrm{E}}$-Feld

Divergence of $\underline{\mathrm{D}}$ Represents Electric Charge Density / Quellstärke von $\underline{\mathrm{D}}$ entspricht der elektrischen Raumladungsdichte

⬇

Method of Gauss' Electric Law / Methode des Gaußschen elektrischen Gesetzes

Electrostatic (ES) Fields – Governing Equations / Elektrostatische (ES) Felder – Grundgleichungen

Integral Form / Integralform

$$\oint_{C=\partial S} \underline{\mathbf{E}}(\underline{\mathbf{R}}) \bullet \mathbf{d}\underline{\mathbf{R}} = 0$$

$$\oiint_{S=\partial V} \underline{\mathbf{D}}(\underline{\mathbf{R}}) \bullet \mathbf{d}\underline{\mathbf{S}} = \iiint_V \rho_\mathrm{e}(\underline{\mathbf{R}})\,\mathrm{d}V = Q_\mathrm{e}$$

$\underline{\mathbf{E}}(\underline{\mathbf{R}})$ [V/m = Newton/Coulomb = N/C]
$\underline{\mathbf{D}}(\underline{\mathbf{R}})$ [As/m^2]
$\rho_\mathrm{e}(\underline{\mathbf{R}})$ [As/m^3]

Vacuum / Vakuum

$$\underline{\mathbf{D}}(\underline{\mathbf{R}}) = \varepsilon_0 \underline{\mathbf{E}}(\underline{\mathbf{R}})$$

Electric Field Constant / Elektrische Feldkonstante (IEEE, VDE)

Differential Form / Differentialform

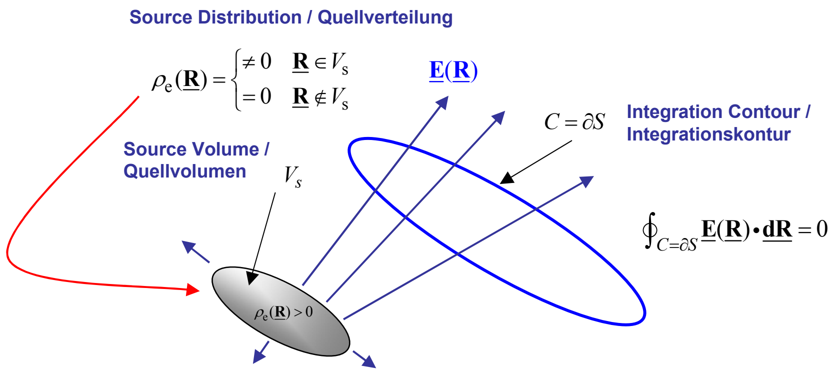
$$\nabla \times \underline{\mathbf{E}}(\underline{\mathbf{R}}) = \underline{\mathbf{0}}$$
$$\nabla \bullet \underline{\mathbf{D}}(\underline{\mathbf{R}}) = \rho_\mathrm{e}(\underline{\mathbf{R}})$$

Side Remark: In some Cases / Nebenbemerkung: In einigen Fällen

$$\underline{\mathbf{D}}(\underline{\mathbf{R}}) = \varepsilon_0 \varepsilon_\mathrm{r} \underline{\mathbf{E}}(\underline{\mathbf{R}})$$

Permittivity / Permittivität

ES Fields – Method of Electric Gauss' Law / ES-Felder – Methode des elektrischen Gaußschen Gesetzes



Electrostatic (ES) Fields / Elektrostatische (ES) Felder

Method of Electric Gauss' Law / Methode des elektrischen Gaußschen Gesetzes

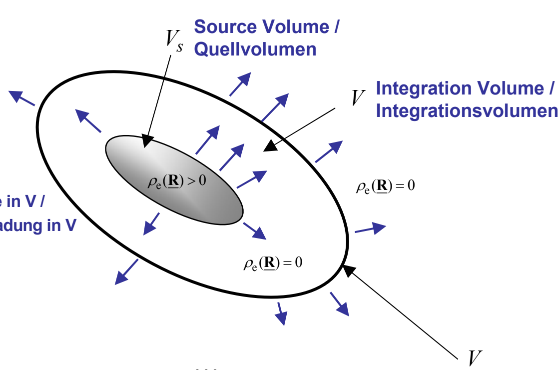
Source Distribution / Quellverteilung

$$\rho_e(\underline{\mathbf{R}}) = \begin{cases} \neq 0 & \underline{\mathbf{R}} \in V_s \\ = 0 & \underline{\mathbf{R}} \notin V_s \end{cases}$$

$$\oiint_{S=\partial V} \underline{\mathbf{D}}(\underline{\mathbf{R}}) \cdot \underline{\mathbf{dS}} = \iiint_V \rho_e(\underline{\mathbf{R}})\, dV$$

$$= Q_e$$ Total Electric Charge in V / Elektrische Gesamtladung in V

$$\underline{\mathbf{D}}(\underline{\mathbf{R}}) \cdot \underline{\mathbf{dS}} = \underbrace{\underline{\mathbf{D}}(\underline{\mathbf{R}}) \cdot \underline{\mathbf{n}}}_{D_n(\underline{\mathbf{R}})}\, dS$$



$$\underbrace{\underbrace{\oiint_{S=\partial V} \underline{\mathbf{D}}(\underline{\mathbf{R}}) \cdot \underline{\mathbf{dS}}}_{\oiint_{S=\partial V} \underbrace{\underline{\mathbf{D}}(\underline{\mathbf{R}}) \cdot \underline{\mathbf{n}}}_{D_n(\underline{\mathbf{R}})} dS}} = \underbrace{\underbrace{\iiint_V \rho_e(\underline{\mathbf{R}})\, dV}_{Q_e}}$$

Summation of all $D_n = \underline{\mathbf{n}} \cdot \underline{\mathbf{D}}$ Contributions / Summation aller $D_n = \underline{\mathbf{n}} \cdot \underline{\mathbf{D}}$-Beiträge

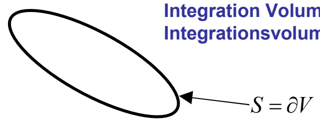
Total electric charge inside the volume V with the closed surface $S=\partial V$ / Gesamte elektrische Ladung im Volumen V mit der geschlossenen Oberfläche $S=\partial V$

Flux of $\underline{\mathbf{D}}$ through $S = Q_e$ in V / Fluss von $\underline{\mathbf{D}}$ durch $S = Q_e$ in V

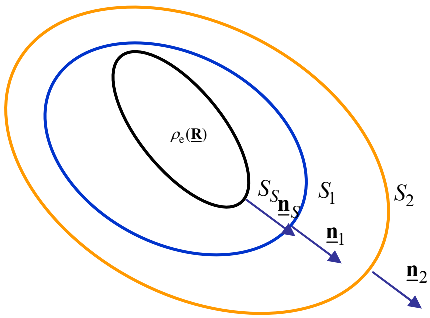
Electrostatic (ES) Fields / Elektrostatische (ES) Felder

Method of Electric Gauss' Law / Methode des elektrischen Gaußschen Gesetzes

$$\oiint_{S=\partial V} \underline{\mathbf{D}}(\underline{\mathbf{R}}) \bullet \underline{\mathbf{dS}} = \iiint_V \rho_e(\underline{\mathbf{R}}) \mathbf{d}V$$
$$= Q_e$$

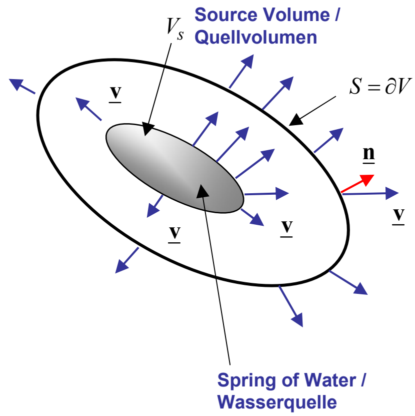


$$\oiint_{S=\partial V} \underline{\mathbf{D}}(\underline{\mathbf{R}}) \bullet \underline{\mathbf{dS}} \begin{cases} = 0 & \text{source-free / quellenfrei} \\ > 0 & \text{Source / Quelle} \\ < 0 & \text{Sink / Senke} \end{cases}$$



$$\oiint_{S_S=\partial V_S} \underline{\mathbf{D}}(\underline{\mathbf{R}}) \bullet \underline{\mathbf{n}}_S \, \mathrm{d}S$$
$$= \oiint_{S_1=\partial V_1} \underline{\mathbf{D}}(\underline{\mathbf{R}}) \bullet \underline{\mathbf{n}}_1 \, \mathrm{d}S$$
$$= \oiint_{S_2=\partial V_2} \underline{\mathbf{D}}(\underline{\mathbf{R}}) \bullet \underline{\mathbf{n}}_2 \, \mathrm{d}S$$
$$= Q_e$$

Example: Fluid Mechanics – Spring of Water / Beispiel: Strömungsmechanik – Wasserquelle



Total Flux through the Closed Surface /
Gesamtfluss durch die geschlossene Oberfläche

$$\oiint_{S=\partial V} \underline{\mathbf{v}}(\underline{\mathbf{R}}) \bullet \underline{\mathbf{dS}} = \oiint_{S=\partial V} \underbrace{\underline{\mathbf{v}}(\underline{\mathbf{R}}) \bullet \underline{\mathbf{n}}}_{=\mathrm{v}_n(\underline{\mathbf{R}})} \, \mathrm{d}S$$
$$= \oiint_{S=\partial V} \mathrm{v}_n(\underline{\mathbf{R}}) \, \mathrm{d}S$$
$$= \Phi_\mathrm{v}$$

Example: Electric Field Due to Spherically Symmetric Charge Distribution / Beispiel: Elektrisches Feld einer kugelsymmetrischen Raumladungsdichte

Consider the Electrostatic (ES) Case / Betrachte den elektrostatischen (ES) Fall

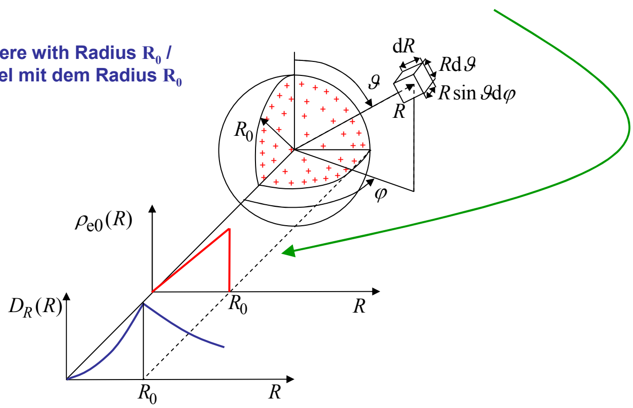
$$\oiint_{S=\partial V} \underbrace{\underline{\mathbf{D}}(\underline{\mathbf{R}})\bullet\underline{\mathbf{n}}}_{=D_n(\underline{\mathbf{R}})}\, \mathrm{d}S = \iiint_V \rho_\mathrm{e}(\underline{\mathbf{R}})\mathrm{d}V = Q_\mathrm{e}$$

Prescribed: Electric Charge Density / Vorgegeben: Elektrische Raumladungsdichte

$$\rho_\mathrm{e}(\underline{\mathbf{R}}) = \rho_\mathrm{e}(R) = \begin{cases} \rho_{\mathrm{e}0}\dfrac{R}{R_0} & R < R_0 \\ 0 & R > R_0 \end{cases}$$

Radial Symmetry / Radialsymmetrie **!**

Charged Sphere with Radius R_0 / Geladene Kugel mit dem Radius R_0



Solution for $\underline{\mathbf{D}}(\underline{\mathbf{R}})$ / Lösung für $\underline{\mathbf{D}}(\underline{\mathbf{R}})$

Electrostatic (ES) Fields / Elektrostatische (ES) Felder

Electrostatic Potential / Elektrostatisches Potential

Integral Form / Integralform

$$\oint_{C=\partial S} \underline{\mathbf{E}}(\underline{\mathbf{R}})\bullet \mathrm{d}\underline{\mathbf{R}} = 0$$

$$\oiint_{S=\partial V} \underline{\mathbf{D}}(\underline{\mathbf{R}})\bullet \mathrm{d}\underline{\mathbf{S}} = \iiint \rho_\mathrm{e}(\underline{\mathbf{R}})\mathrm{d}V$$

Differential Form / Differentialform

$$\nabla\times\underline{\mathbf{E}}(\underline{\mathbf{R}}) = \underline{\mathbf{0}}$$

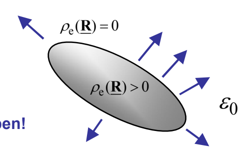
$$\nabla\bullet\underline{\mathbf{D}}(\underline{\mathbf{R}}) = \rho_\mathrm{e}(\underline{\mathbf{R}})$$

Unknown! / Unbekannt! $\underline{\mathbf{E}}(\underline{\mathbf{R}}), \underline{\mathbf{D}}(\underline{\mathbf{R}}) = ?$

Given, Prescribed! / Gegeben, vorgeschrieben!

Vacuum / Vakuum

$$\underline{\mathbf{D}}(\underline{\mathbf{R}}) = \varepsilon_0\, \underline{\mathbf{E}}(\underline{\mathbf{R}})$$



$$\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} = 0, \quad \frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} = 0, \quad \frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = 0$$

$$\frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z} = \rho_\mathrm{e}$$

Standard Way: Method of Potentials / Standard Weg: Methode der Potentiale

Electrostatics / Elektrostatik: $\Phi_\mathrm{e}(\underline{\mathbf{R}})$ [V] Scalar Electrostatic Potential / Skalares elektrostatisches Potential

Electrostatic (ES) Fields / Elektrostatische (ES) Felder

Electrostatic Potential / Elektrostatisches Potential

Integral Form / Integralform

$$\oint_{C=\partial S} \underline{\mathbf{E}}(\underline{\mathbf{R}}) \bullet \underline{\mathbf{dR}} = 0$$

Differential Form / Differentialform

$$\nabla \times \underline{\mathbf{E}}(\underline{\mathbf{R}}) = \underline{\mathbf{0}}$$

Irrotational Field can be always Represented by a Gradient Field / Rotationsfreies Feld kann immer als Gradientenfeld dargestellt werden

$$\underline{\mathbf{E}}(\underline{\mathbf{R}}) = -\nabla \Phi_e(\underline{\mathbf{R}})$$

Electrostatic Potential / Elektrostatisches Potential

$$\Phi_e(\underline{\mathbf{R}}) \; [\mathrm{V}]$$

because / weil

$$\nabla \times \underline{\mathbf{E}}(\underline{\mathbf{R}}) = \nabla \times \left[-\nabla \Phi_e(\underline{\mathbf{R}})\right] = -\nabla \times \nabla \Phi_e(\underline{\mathbf{R}}) = \underline{\mathbf{0}}$$

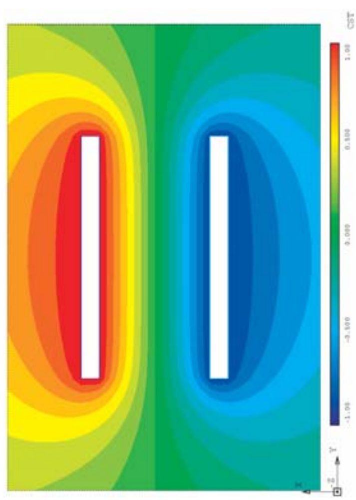
In General / Im allgemeinen

$$\nabla \times \nabla \equiv \underline{\mathbf{0}}$$

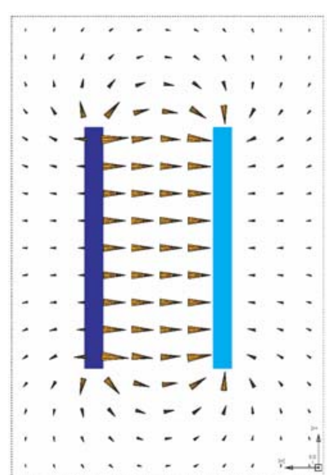
General Vector Analytic Property / Allgemeine Vektoridentität

Electrostatic Field Problem – Example: Parallel Plate Capacitor / Elektrostatisches Feldproblem – Beispiel: Paralleler Plattenkondensator

Scalar Field: Electrostatic Potential / Skalarfeld: Elektrostatisches Potenzial

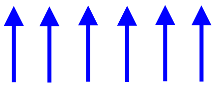


Vector Field: Electrostatic Field Strength / Vektorfeld: Elektrostatische Feldstärke

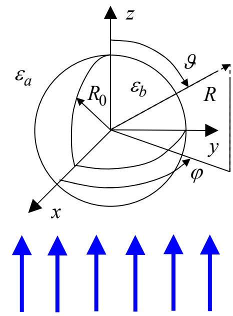


Example: Dielectric Sphere in a Homogeneous Electrostatic Field / Beispiel: Dielektrische Kugel im homogenen elektrostatischen Feld (1)

$\underline{\mathbf{E}}_0 = E_0 \underline{\mathbf{e}}_z$



$\underline{\mathbf{E}}_0 = E_0 \underline{\mathbf{e}}_z$



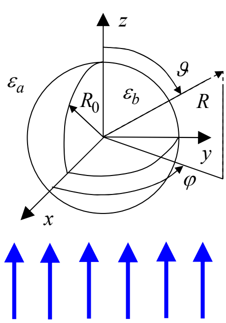
$$\varepsilon(\underline{\mathbf{R}}) = \varepsilon_a$$

$$\underline{\mathbf{E}}(\underline{\mathbf{R}}) = E_0 \hat{\underline{\mathbf{E}}}_0$$

$$\varepsilon(\underline{\mathbf{R}}) = \begin{cases} \varepsilon_b & 0 < R \le R_0 \\ \varepsilon_a & R > R_0 \end{cases}$$

$$\underline{\mathbf{E}}(\underline{\mathbf{R}}) = E_0 \underline{\mathbf{e}}_z + \underline{\mathbf{E}}_b(\underline{\mathbf{R}})$$

Example: Dielectric Sphere in a Homogeneous Electrostatic Field / Beispiel: Dielektrische Kugel im homogenen elektrostatischen Feld (2)



$\underline{\mathbf{E}}_0 = E_0 \underline{\mathbf{e}}_z$

$$\varepsilon(\underline{\mathbf{R}}) = \begin{cases} \varepsilon_b & 0 < R \le R_0 \\ \varepsilon_a & R > R_0 \end{cases}$$

$$\underline{\mathbf{E}}(\underline{\mathbf{R}}) = E_0 \underline{\mathbf{e}}_z + \underline{\mathbf{E}}_b(\underline{\mathbf{R}})$$

$$\Phi_\mathrm{e}(\underline{\mathbf{R}}) = \begin{cases} -E_0 \beta R \cos\vartheta & 0 < R \le R_0 \\ -E_0 \left[1 - \dfrac{\alpha}{R^3}\right] R \cos\vartheta & R > R_0 \end{cases}$$

$$\alpha = \frac{\varepsilon_b - \varepsilon_a}{\varepsilon_b + 2\varepsilon_a} R_0^3$$

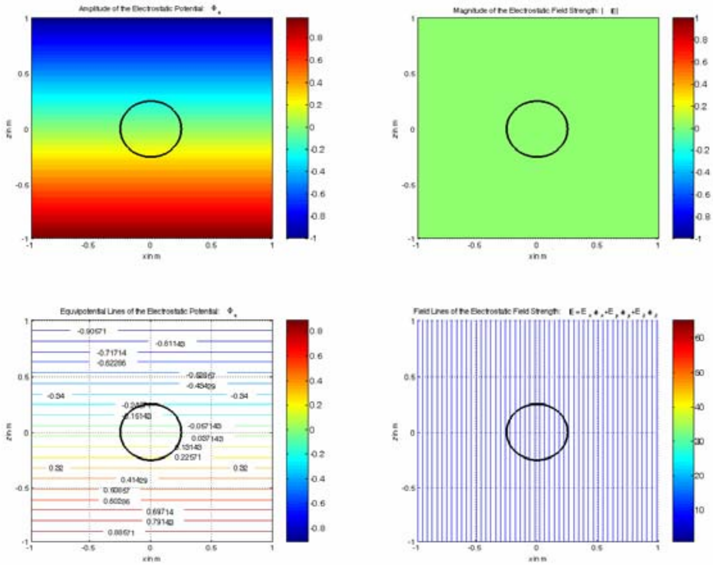
$$\beta = \frac{3\varepsilon_a}{\varepsilon_b + 2\varepsilon_a}$$

$$\underline{\mathbf{E}}(\underline{\mathbf{R}}) = \begin{cases} E_0 \beta \left[\cos\vartheta\, \underline{\mathbf{e}}_R - \cos\vartheta\, \underline{\mathbf{e}}_\vartheta\right] & 0 < R \le R_0 \\ E_0 \left[\left(1 - \dfrac{2\alpha}{R^3}\right) \cos\vartheta\, \underline{\mathbf{e}}_R - \left(1 - \dfrac{\alpha}{R^3}\right) \sin\vartheta\, \underline{\mathbf{e}}_\vartheta\right] & R > R_0 \end{cases}$$

Example: Dielectric Sphere in a Homogeneous Electrostatic Field / Beispiel: Dielektrische Kugel im homogenen elektrostatischen Feld (3)

$\varepsilon_a = \varepsilon_0$

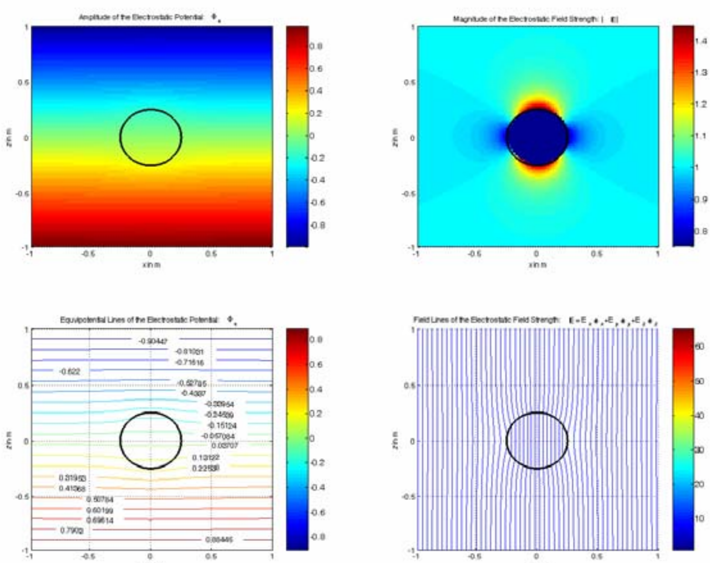
$\varepsilon_b = \varepsilon_0$



Example: Dielectric Sphere in a Homogeneous Electrostatic Field / Beispiel: Dielektrische Kugel im homogenen elektrostatischen Feld (4)

$\varepsilon_a = \varepsilon_0$

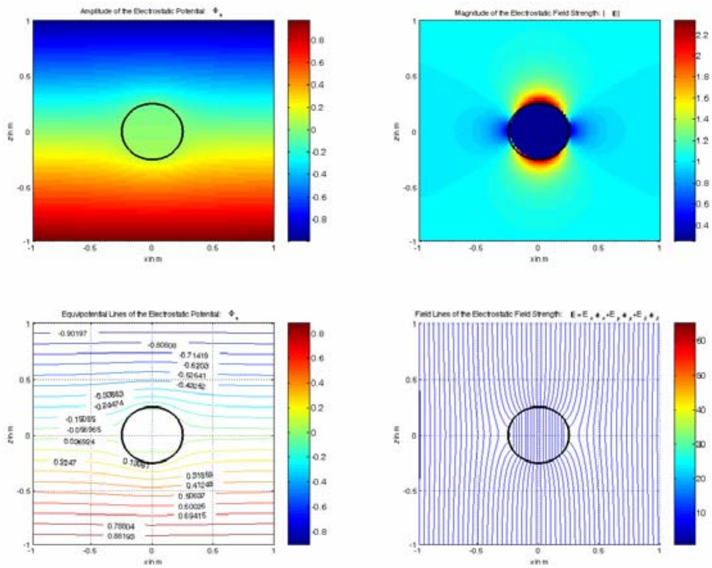
$\varepsilon_b = 2\varepsilon_0$



Example: Dielectric Sphere in a Homogeneous Electrostatic Field / Beispiel: Dielektrische Kugel im homogenen elektrostatischen Feld (5)

$\varepsilon_a = \varepsilon_0$

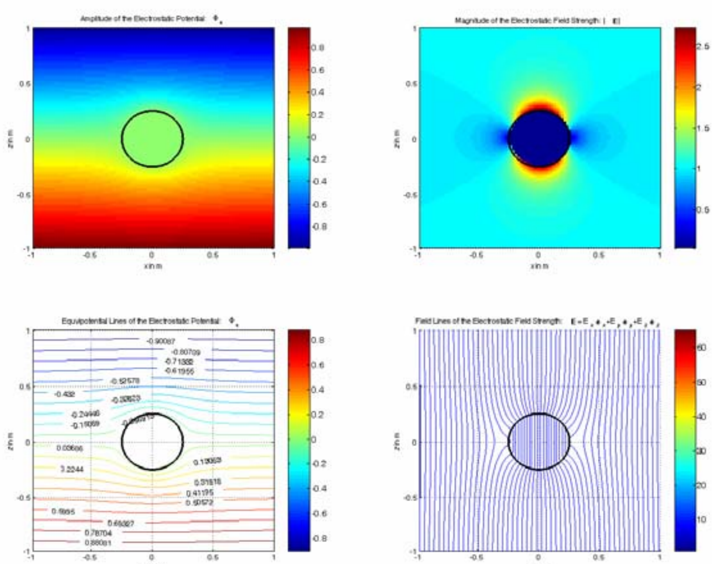
$\varepsilon_b = 10\varepsilon_0$



Example: Dielectric Sphere in a Homogeneous Electrostatic Field / Beispiel: Dielektrische Kugel im homogenen elektrostatischen Feld (6)

$\varepsilon_a = \varepsilon_0$

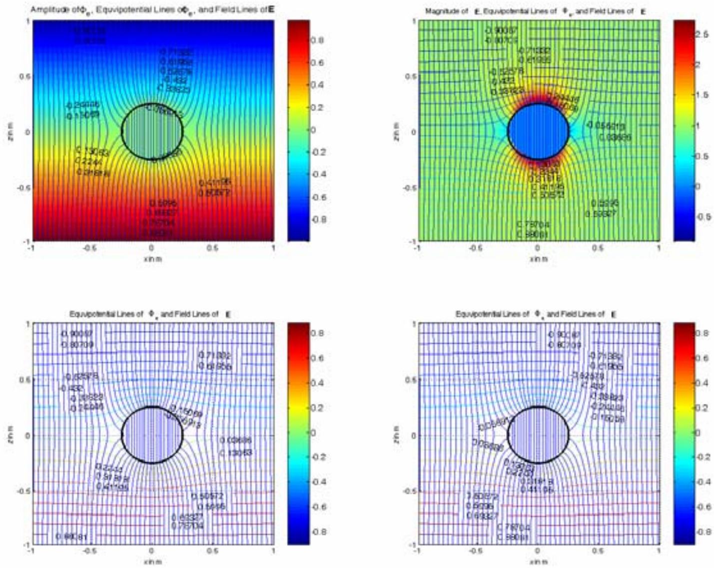
$\varepsilon_b = 100\varepsilon_0$



**Example: Dielectric Sphere in a Homogeneous Electrostatic Field /
Beispiel: Dielektrische Kugel im homogenen elektrostatischen Feld (6/2)**

$\varepsilon_a = \varepsilon_0$

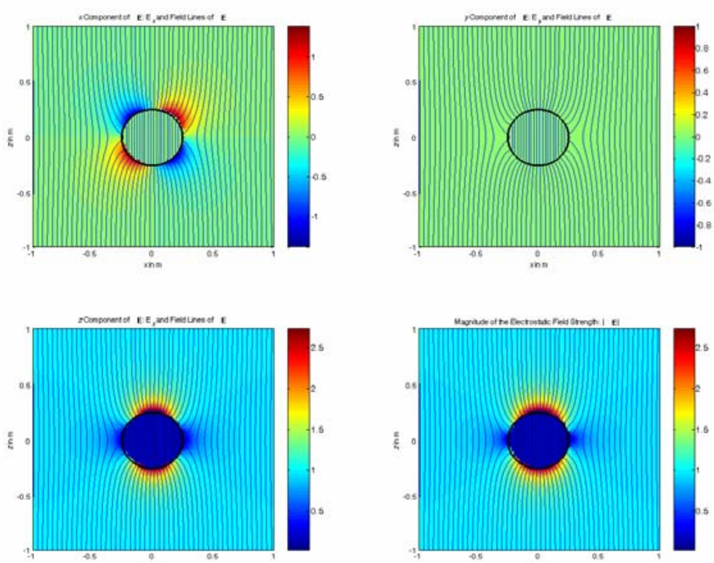
$\varepsilon_b = 100\varepsilon_0$



**Example: Dielectric Sphere in a Homogeneous Electrostatic Field /
Beispiel: Dielektrische Kugel im homogenen elektrostatischen Feld (6/3)**

$\varepsilon_a = \varepsilon_0$

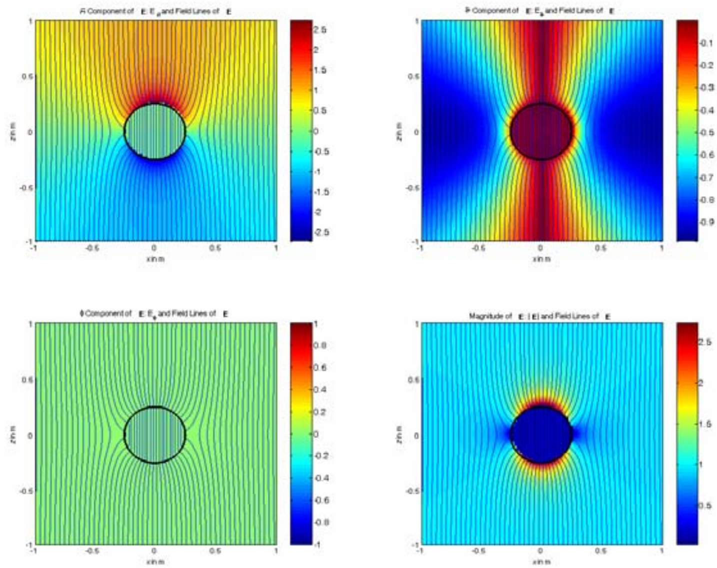
$\varepsilon_b = 100\varepsilon_0$



Example: Dielectric Sphere in a Homogeneous Electrostatic Field / Beispiel: Dielektrische Kugel im homogenen elektrostatischen Feld (6/2)

$\varepsilon_a = \varepsilon_0$

$\varepsilon_b = 100\varepsilon_0$



Electrostatic (ES) Fields – Poisson and Laplace Equation / Elektrostatische (ES) Felder – Poisson- und Laplace-Gleichung (1)

Differential Form / Differentialform

$$\nabla \times \underline{\mathbf{E}}(\underline{\mathbf{R}}) = \underline{\mathbf{0}}$$

$$\underline{\mathbf{E}}(\underline{\mathbf{R}}) = -\nabla \Phi_{\mathrm{e}}(\underline{\mathbf{R}})$$

$$\nabla \bullet \underline{\mathbf{D}}(\underline{\mathbf{R}}) = \rho_{\mathrm{e}}(\underline{\mathbf{R}})$$

Vacuum / Vakuum

$$\underline{\mathbf{D}}(\underline{\mathbf{R}}) = \varepsilon_0\, \underline{\mathbf{E}}(\underline{\mathbf{R}}) = -\varepsilon_0 \nabla \Phi_{\mathrm{e}}(\underline{\mathbf{R}})$$

because / weil

$$\nabla \bullet \underline{\mathbf{D}}(\underline{\mathbf{R}}) = \varepsilon_0 \nabla \bullet \underline{\mathbf{E}}(\underline{\mathbf{R}}) = -\varepsilon_0 \nabla \bullet \nabla \Phi_{\mathrm{e}}(\underline{\mathbf{R}}) = \rho_{\mathrm{e}}(\underline{\mathbf{R}})$$

or / oder

$$\underbrace{\nabla \bullet \nabla}_{\nabla^2 = \Delta} \Phi_{\mathrm{e}}(\underline{\mathbf{R}}) = \begin{cases} -\dfrac{\rho_{\mathrm{e}}(\underline{\mathbf{R}})}{\varepsilon_0} & \text{for / für } \rho_{\mathrm{e}}(\underline{\mathbf{R}}) \neq 0 \quad \text{Poisson Equation / Poisson-Gleichung} \\ 0 & \text{for / für } \rho_{\mathrm{e}}(\underline{\mathbf{R}}) = 0 \quad \text{Laplace Equation / Laplace-Gleichung} \end{cases}$$

Laplace Operator / Laplace-Operator

$$\nabla \bullet \nabla = \nabla^2 = \Delta$$

Electrostatic (ES) Fields – Poisson and Laplace Equation / Elektrostatische (ES) Felder – Poisson- und Laplace-Gleichung (2)

$$\underbrace{\nabla\bullet\nabla}_{\nabla^2=\Delta}\Phi_e(\underline{\mathbf{R}})=\begin{cases}-\dfrac{\rho_e(\underline{\mathbf{R}})}{\varepsilon_0} & \text{for / für } \rho_e(\underline{\mathbf{R}})\neq 0 \quad \text{Poisson Equation / Poisson-Gleichung}\\ 0 & \text{for / für } \rho_e(\underline{\mathbf{R}})=0 \quad \text{Laplace Equation / Laplace-Gleichung}\end{cases}$$

Laplace Operator / Laplace-Operator $\nabla\bullet\nabla=\nabla^2=\Delta$

Laplace Operator in Cartesian Coordinates / Laplace-Operator in Kartesischen Koordinaten

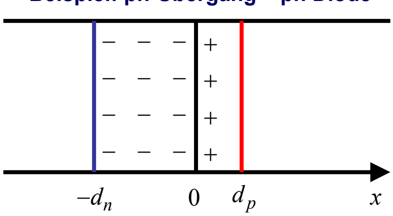
$$\begin{aligned}\nabla\bullet\nabla&=\left(\underline{\mathbf{e}}_x\frac{\partial}{\partial x}+\underline{\mathbf{e}}_y\frac{\partial}{\partial y}+\underline{\mathbf{e}}_z\frac{\partial}{\partial z}\right)\bullet\left(\underline{\mathbf{e}}_x\frac{\partial}{\partial x}+\underline{\mathbf{e}}_y\frac{\partial}{\partial y}+\underline{\mathbf{e}}_z\frac{\partial}{\partial z}\right)\\&=\underline{\mathbf{e}}_{x_i}\frac{\partial}{\partial x_i}\bullet\underline{\mathbf{e}}_{x_j}\frac{\partial}{\partial x_j}\\&=\frac{\partial}{\partial x_i}\frac{\partial}{\partial x_j}\underbrace{\underline{\mathbf{e}}_{x_i}\bullet\underline{\mathbf{e}}_{x_j}}_{\delta_{ij}}\\&=\frac{\partial}{\partial x_i}\frac{\partial}{\partial x_i}\\&=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}=\nabla^2=\Delta\end{aligned}$$

Electrostatic (ES) Fields – Poisson and Laplace Equation / Elektrostatische (ES) Felder – Poisson- und Laplace-Gleichung (3)

Laplace Operator in Cartesian Coordinates / Laplace-Operator in Kartesischen Koordinaten

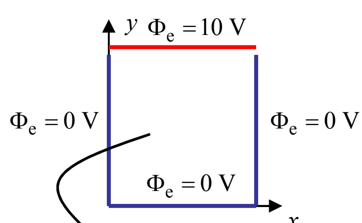
$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)\Phi_e(x,y,z)=\begin{cases}-\dfrac{\rho_e(x,y,z)}{\varepsilon_0} & \text{for / für } \rho_e(x,y,z)\neq 0 \quad \text{Poisson Equation / Poisson-Gleichung}\\ 0 & \text{for / für } \rho_e(x,y,z)=0 \quad \text{Laplace Equation / Laplace-Gleichung}\end{cases}$$

Example: pn Junction – pn Diode / Beispiel: pn-Übergang – pn Diode



$$\frac{d^2}{dx^2}\Phi_e(x)=\frac{e}{\varepsilon}\begin{cases}-n_e & \text{for / für } -d_n\le x\le 0\\ n_e & \text{for / für } 0\le x\le d_p\end{cases}$$

Example: / Beispiel:



$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\Phi_e(x,y)=0$$

➡ **Separation of Variables / Separation der Variablen !**

End of 6th Lecture /
Ende der 6. Vorlesung