Elektromagnetische Feldtheorie I (EFT I) / Electromagnetic Field Theory I (EFT I)

7th Lecture / 7. Vorlesung

Dr.-Ing. René Marklein

marklein@uni-kassel.de

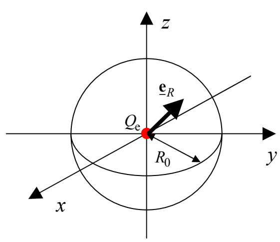
http://www.tet.e-technik.uni-kassel.de

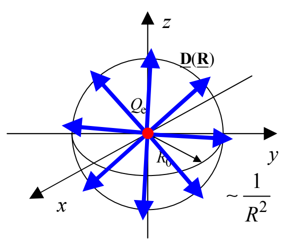
http://www.uni-kassel.de/fb16/tet/marklein/index.html

Universität Kassel
Fachbereich Elektrotechnik / Informatik
(FB 16)
Fachgebiet Theoretische Elektrotechnik
(FG TET)
Wilhelmshöher Allee 71
Büro: Raum 2113 / 2115
D-34121 Kassel

University of Kassel
Dept. Electrical Engineering / Computer Science (FB 16)
Electromagnetic Field Theory
(FG TET)
Wilhelmshöher Allee 71
Office: Room 2113 / 2115
D-34121 Kassel

Electrostatic (ES) Fields – Point Charge Concept / Elektrostatische (ES) Felder – Konzept der Punktladung





$$\iint_{\underbrace{S=\partial V}_{\text{Sphere } R_0}} \underline{\mathbf{D}}(\underline{\mathbf{R}})\cdot\underline{\mathbf{n}}\,\mathrm{d}S = \iiint_V \underbrace{\rho_\mathrm{e}(\underline{\mathbf{R}})}_{Q_\mathrm{e}\delta(\underline{\mathbf{R}})}\mathrm{d}V = Q_\mathrm{e}$$

$$\underline{\mathbf{E}}(\underline{\mathbf{R}}) = -\nabla\Phi_\mathrm{e}(\underline{\mathbf{R}}) = -\nabla\Phi_\mathrm{e}(R)$$

$$= -\frac{\partial}{\partial R}\Phi_\mathrm{e}(R)\,\underline{\mathbf{e}}_R(\varphi,\vartheta) = \alpha\frac{1}{R^2}\underline{\mathbf{e}}_R(\varphi,\vartheta)$$

$$\underline{\mathbf{D}}(\underline{\mathbf{R}}) = \underline{\mathbf{D}}(R) = \varepsilon_0\alpha\frac{1}{R^2}\underline{\mathbf{e}}_R(\varphi,\vartheta)$$

$$\iint_{\underbrace{S=\partial V}_{\text{Sphere } R_0}} \varepsilon_0\alpha\frac{1}{R_0^2}\underbrace{\underline{\mathbf{e}}_R(\varphi,\vartheta)\cdot\underline{\mathbf{e}}_R(\varphi,\vartheta)}_{=1}\mathrm{d}S = \varepsilon_0\alpha\frac{1}{R_0^2}\underbrace{\iint_{\underbrace{S=\partial V}_{\text{Sphere } R_0}}\mathrm{d}S}_{4\pi R_0^2}$$

$$= 4\pi\varepsilon_0\alpha$$

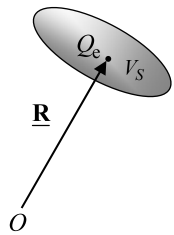
$$= Q_\mathrm{e}$$

$$\Phi_\mathrm{e}(R) = \frac{Q_\mathrm{e}}{4\pi\varepsilon_0 R} \qquad \alpha = \frac{Q_\mathrm{e}}{4\pi\varepsilon_0}$$

Electrostatic (ES) Fields – Point Charge Concept / Elektrostatische (ES) Felder – Konzept der Punktladung (...)

Point Source / Punktquelle

$$Q_\mathrm{e} \quad [\mathrm{As/m^3} = \mathrm{Coulomb}]$$



$$\rho_\mathrm{e}(\underline{\mathbf{R}}) = ?$$
$$= \text{infinite} \ / \ \text{unendlich}$$

$$\iiint_{V_S} \rho_\mathrm{e}(\underline{\mathbf{R}})\,\mathrm{d}V = Q_\mathrm{e}$$

Mathematically Nonsense / Mathematischer Unsinn

$$V_S \to 0$$

Integration Theory of Riemann / Riemannsche Integralrechnung:

$$\iiint_{V_S} \rho_\mathrm{e}(\underline{\mathbf{R}})\,\mathrm{d}V = 0$$

To Define Something New / Definiere etwas Neues

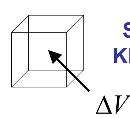
Electrostatic (ES) Fields – Point Charge Concept / Elektrostatische (ES) Felder – Konzept der Punktladung (...)

Electrostatic Charge / Elektrostatisches Ladung

$$Q_\mathrm{e} = \iiint_V \rho_\mathrm{e}(\underline{\mathbf{R}})\,\mathrm{d}V$$

Electrostatic Volume Charge Density / Elektrostatisches Raumladungsdichte

$$\rho_\mathrm{e} = \frac{\Delta Q_\mathrm{e}}{\Delta V}$$



Small Volume / Kleines Volumen

Electrostatic Charge / Elektrostatisches Ladung

$$\Delta Q_\mathrm{e} = \rho_\mathrm{e} \Delta V$$

In the Limit / Grenzübergang

$$\Delta V \to 0$$

Constant / Konstant

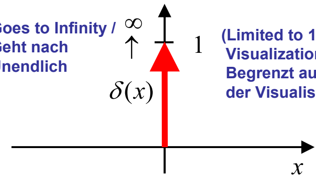
$$\Delta Q_\mathrm{e} = \lim_{\substack{\Delta V \to 0 \\ \rho_\mathrm{e} \to \infty}} \rho_\mathrm{e} \Delta V$$

Point / Punkt

ΔQ_e =Constant if ΔV Goes to Zero, than the Volume Charge Density must go to Infinity. /
ΔQ_e=konstant bleiben soll wenn ΔV nach null geht, dann muss die Raumladungsdichte nach unendlich gehen.

Electrostatic (ES) Fields – Point Charge Concept / Elektrostatische (ES) Felder – Konzept der Punktladung (...)

1-D Delta-Distribution / 1D Delta-Distribution



Delta-Function / Delta-Funktion
δ-Distribution / δ-Distribution
δ-Dirac-Pulse / δ-Dirac-Impuls

Distribution ➡ Generalized Function / Verallgemeinerte Funktion

$$\delta(x) = \begin{cases} \text{"}\infty\text{"} & \text{for/ für} \quad x = 0 \\ 0 & \text{for/ für} \quad x \neq 0 \end{cases} \qquad x\,[\mathrm{m}],\ \delta(x)\left[\frac{1}{\mathrm{m}}\right]$$

The Unit of the Delta-Distribution is the Inverse Unit of the Argument / Die Einheit der Delta-Distribution ist die inverse Einheit des Argumentes

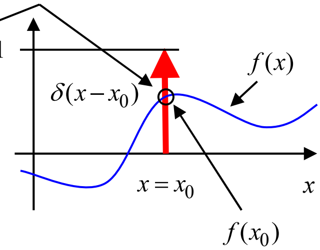
Definition of the δ –Distribution / Definition der δ –Distribution

$$\int_{x=-\infty}^{\infty} \delta(x - x_0)\,\mathrm{d}x = 1$$

$$\int_{x=-\infty}^{\infty} f(x)\,\delta(x - x_0)\,\mathrm{d}x = f(x_0)$$

$$f(x)\,\delta(x - x_0) = f(x_0)\,\delta(x - x_0)$$

Sifting Property / Siebeigenschaft



Electrostatic (ES) Fields – Point Charge Concept / Elektrostatische (ES) Felder – Konzept der Punktladung (...)

1-D Delta-Distribution / 1D Delta-Distribution

$$\int_{x=-\infty}^{\infty} f(x)\,\delta(x - x_0)\,\mathrm{d}x = f(x_0)$$

$$\langle f(x), \delta(x - x_0) \rangle = f(x_0)$$

Properties: Algebraic and Calculus Properties / Eigenschaften: Algebraische Eigenschaften und Rechenregeln

$\alpha\,\delta(x - x_0)$:

$$\int_{x=-\infty}^{\infty} \alpha\,\delta(x - x_0)\,f(x)\,\mathrm{d}x = \alpha\,f(x_0)$$

$\alpha(x)\,\delta(x - x_0)$:

$$\int_{x=-\infty}^{\infty} \alpha(x)\,\delta(x - x_0)\,f(x)\mathrm{d}x = \alpha(x_0)\,f(x_0)$$

$$\alpha(x)\,\delta(x - x_0) = \alpha(x_0)\,\delta(x - x_0)$$

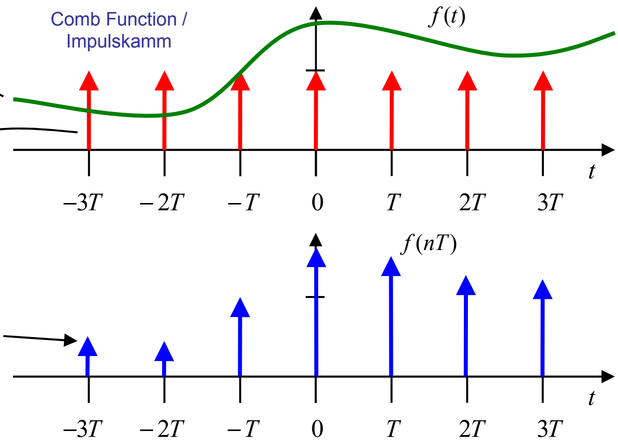
Electrostatic (ES) Fields – Point Charge Concept / Elektrostatische (ES) Felder – Konzept der Punktladung (...)

1-D Delta-Distribution – Signal Processing – Sampling / 1D Delta-Distribution – Signalverarbeitung –Abtastung

$$f(t)\sum_{n=-N}^{N}\delta(t-nT)$$

$$=\sum_{n=-N}^{N} f(t)\,\delta(t-nT)$$

$$=\sum_{n=-N}^{N} f(nT)\,\delta(t-nT)$$



Electrostatic (ES) Fields – Point Charge Concept / Elektrostatische (ES) Felder – Konzept der Punktladung (...)

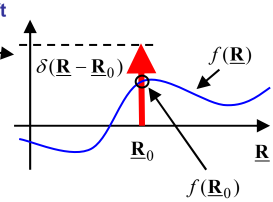
3-D Delta-Distribution / 3D Delta-Distribution

Sifting Property / Siebeigenschaft

$$\iiint_{\underline{\mathbf{R}}=-\infty}^{\infty} \delta(\underline{\mathbf{R}}-\underline{\mathbf{R}}_0)\,\mathrm{d}^3\underline{\mathbf{R}}=1$$

$$\iiint_{\underline{\mathbf{R}}=-\infty}^{\infty} f(\underline{\mathbf{R}})\,\delta(\underline{\mathbf{R}}-\underline{\mathbf{R}}_0)\,\mathrm{d}^3\underline{\mathbf{R}}=f(\underline{\mathbf{R}}_0)$$

$$f(\underline{\mathbf{R}})\,\delta(\underline{\mathbf{R}}-\underline{\mathbf{R}}_0)=f(\underline{\mathbf{R}}_0)\,\delta(\underline{\mathbf{R}}-\underline{\mathbf{R}}_0)$$

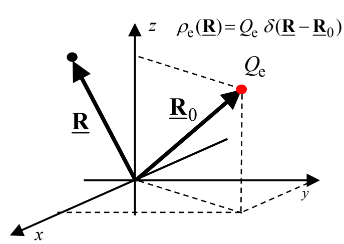


Distribution ➨ Generalized Function / Verallgemeinerte Funktion

$$\iiint_{\underline{\mathbf{R}}=-\infty}^{\infty} \rho_\mathrm{e}(\underline{\mathbf{R}})\,\mathrm{d}^3\underline{\mathbf{R}}=\iiint_{\underline{\mathbf{R}}=-\infty}^{\infty} Q_\mathrm{e}\,\delta(\underline{\mathbf{R}}-\underline{\mathbf{R}}_0)\,\mathrm{d}^3\underline{\mathbf{R}}$$

$$=Q_\mathrm{e}\underbrace{\iiint_{\underline{\mathbf{R}}=-\infty}^{\infty} \delta(\underline{\mathbf{R}}-\underline{\mathbf{R}}_0)\,\mathrm{d}^3\underline{\mathbf{R}}}_{=1}$$

$$=Q_\mathrm{e}$$



Electrostatic (ES) Fields – Point Charge Concept / Elektrostatische (ES) Felder – Konzept der Punktladung (...)

3-D Delta-Distribution / 3D Delta-Distribution

$$\delta(\underline{\mathbf{R}}-\underline{\mathbf{R}}_0)=\delta(x-x_0)\delta(y-y_0)\delta(z-z_0)$$

Cartesian Coordinate System / Kartesisches Koordinatensystem

$$=\delta(r-r_0)\frac{\delta(\varphi-\varphi_0)}{r}\delta(z-z_0)=\frac{\delta(r-r_0)\delta(\varphi-\varphi_0)\delta(z-z_0)}{r}$$

Cylindrical Coordinate System / Zylinderkoordinatensystem

$$=\delta(R-R_0)\frac{\delta(\vartheta-\vartheta_0)}{R}\frac{\delta(\varphi-\varphi_0)}{R\sin\vartheta}=\frac{\delta(R-R_0)\delta(\vartheta-\vartheta_0)\delta(\varphi-\varphi_0)}{R^2\sin\vartheta}$$

Spherical Coordinate System / Kugelkoordinatensystem

$$\iiint\limits_{\underline{\mathbf{R}}=-\infty}^{\infty}\delta(\underline{\mathbf{R}}-\underline{\mathbf{R}}_0)\,\mathrm{d}^3\underline{\mathbf{R}}=\iiint\limits_{\underline{\mathbf{R}}=-\infty}^{\infty}\delta(x-x_0)\delta(y-y_0)\delta(z-z_0)\,\mathrm{d}^3\underline{\mathbf{R}}=\int\limits_{z=-\infty}^{\infty}\int\limits_{y=-\infty}^{\infty}\int\limits_{x=-\infty}^{\infty}\delta(x-x_0)\delta(y-y_0)\delta(z-z_0)\,\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z$$

$$=\underbrace{\left[\int\limits_{z=-\infty}^{\infty}\underbrace{\left[\int\limits_{y=-\infty}^{\infty}\underbrace{\left[\int\limits_{x=-\infty}^{\infty}\delta(x-x_0)\,\mathrm{d}x\right]}_{=1}\delta(y-y_0)\mathrm{d}y\right]}_{=1}\delta(z-z_0)\,\mathrm{d}z\right]}_{=1}=1$$

Electrostatic (ES) Fields – Point Charge Concept / Elektrostatische (ES) Felder – Konzept der Punktladung (...)

3-D Delta-Distribution / 3D Delta-Distribution

$$\iiint\limits_{\underline{\mathbf{R}}=-\infty}^{\infty}\delta(\underline{\mathbf{R}}-\underline{\mathbf{R}}_0)\,\mathrm{d}^3\underline{\mathbf{R}}=\iiint\limits_{\underline{\mathbf{R}}=-\infty}^{\infty}\frac{\delta(r-r_0)\delta(\varphi-\varphi_0)\delta(z-z_0)}{r}\,\mathrm{d}^3\underline{\mathbf{R}}=\int\limits_{z=-\infty}^{\infty}\int\limits_{\varphi=0}^{2\pi}\int\limits_{r=0}^{\infty}\frac{\delta(r-r_0)\delta(\varphi-\varphi_0)\delta(z-z_0)}{r}r\,\mathrm{d}r\,\mathrm{d}\varphi\,\mathrm{d}z$$

$$=\underbrace{\left[\int\limits_{z=-\infty}^{\infty}\underbrace{\left[\int\limits_{\varphi=0}^{2\pi}\underbrace{\left[\int\limits_{r=0}^{\infty}\delta(r-r_0)\,\mathrm{d}r\right]}_{=1}\frac{\delta(\varphi-\varphi_0)}{r}r\mathrm{d}\varphi\right]}_{=1}\delta(z-z_0)\,\mathrm{d}z\right]}_{=1}=1$$

$$\iiint\limits_{\underline{\mathbf{R}}=-\infty}^{\infty}\delta(\underline{\mathbf{R}}-\underline{\mathbf{R}}_0)\,\mathrm{d}^3\underline{\mathbf{R}}=\iiint\limits_{\underline{\mathbf{R}}=-\infty}^{\infty}\frac{\delta(R-R_0)\delta(\vartheta-\vartheta_0)\delta(\varphi-\varphi_0)}{R^2\sin\vartheta}\,\mathrm{d}^3\underline{\mathbf{R}}=\int\limits_{\varphi=0}^{2\pi}\int\limits_{\vartheta=0}^{\pi}\int\limits_{R=0}^{\infty}\frac{\delta(R-R_0)\delta(\vartheta-\vartheta_0)\delta(\varphi-\varphi_0)}{R^2\sin\vartheta}R^2\sin\vartheta\,\mathrm{d}R\,\mathrm{d}\vartheta\,\mathrm{d}\varphi$$

$$=\underbrace{\left[\int\limits_{\varphi=0}^{2\pi}\underbrace{\left[\int\limits_{\vartheta=0}^{\pi}\underbrace{\left[\int\limits_{R=0}^{\infty}\delta(R-R_0)\,\mathrm{d}R\right]}_{=1}\frac{\delta(\vartheta-\vartheta_0)}{R}R\,\mathrm{d}\vartheta\right]}_{=1}\frac{\delta(\varphi-\varphi_0)}{R\sin\vartheta}R\,\sin\vartheta\,\mathrm{d}\varphi\right]}_{=1}=1$$

Electrostatic (ES) Fields – Point Charge Concept / Elektrostatische (ES) Felder – Konzept der Punktladung (...)

Electrostatic Point Charge Density / Elektrostatische Punktladung

$$Q_\mathrm{e} = Q_\mathrm{e}(x_0, y_0, z_0)\ [\mathrm{As}]$$

Electrostatic Volume Charge Density / Elektrostatische Raumladungsdichte

$$\rho_\mathrm{e}(x,y,z) = Q_\mathrm{e}\,\delta(x-x_0)\delta(y-y_0)\delta(z-z_0)$$

Q_e

Electrostatic Line Charge Density / Elektrostatische Linienladungsdichte

$$\varsigma_\mathrm{e}(z) = \varsigma_\mathrm{e}(x_0, y_0, z)[\mathrm{As/m}]$$

Electrostatic Line Charge Density / Elektrostatische Linienladungsdichte

$$\rho_\mathrm{e}(x,y,z) = \varsigma_\mathrm{e}(z)\,\delta(x-x_0)\delta(y-y_0)$$

$\varsigma_\mathrm{e}(z)$

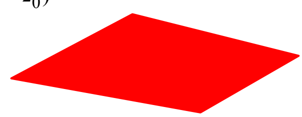
Electrostatic Surface Charge Density / Elektrostatische Flächenladungsdichte

$$\eta_\mathrm{e}(x,y) = \eta_\mathrm{e}(x,y,z_0)\left[\mathrm{As/m^2}\right]$$

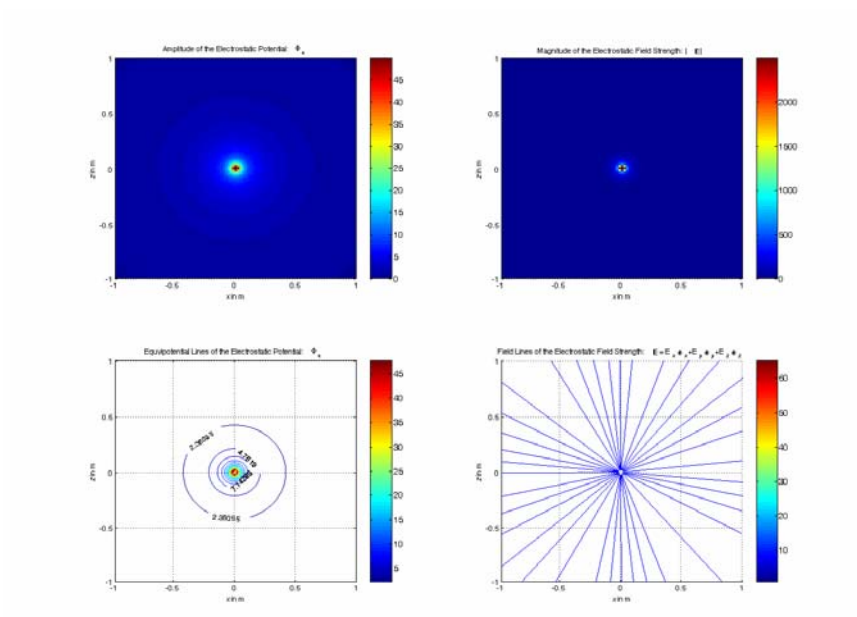
Electrostatic Charge Density / Elektrostatische Ladungsdichte

$$\rho_\mathrm{e}(x,y,z) = \eta_\mathrm{e}(x,y)\,\delta(z-z_0)$$

$\eta_\mathrm{e}(x,y,z_0)$



Electrostatic (ES) Fields – Point Charge Concept / Elektrostatische (ES) Felder – Konzept der Punktladung (...)



ES Fields – Point Charge Concept / ES Felder – Konzept der Punktladung (...)

Electrostatic Charge Density / Elektrostatische Ladungsdichte

$$\rho_\mathrm{e}(\underline{\mathbf{R}}) = Q_\mathrm{e}\,\delta(\underline{\mathbf{R}} - \underline{\mathbf{R}}_0)$$

Electrostatic Potential / Elektrostatisches Potential

$$\Phi_\mathrm{e}(\underline{\mathbf{R}}) = \frac{1}{4\pi\varepsilon_0}\frac{Q_\mathrm{e}}{|\underline{\mathbf{R}} - \underline{\mathbf{R}}_0|}$$

$$\frac{1}{|\underline{\mathbf{R}} - \underline{\mathbf{R}}_0|} = \frac{1}{\sqrt{(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2}}$$

Electrostatic Field Strength / Elektrostatische Feldstärke

$$\underline{\mathbf{E}}(\underline{\mathbf{R}}) = -\nabla\Phi_\mathrm{e}(\underline{\mathbf{R}}) = \frac{Q_\mathrm{e}}{4\pi\varepsilon_0}\frac{\underline{\mathbf{R}} - \underline{\mathbf{R}}_0}{|\underline{\mathbf{R}} - \underline{\mathbf{R}}_0|^3}$$

$$\frac{1}{|\underline{\mathbf{R}} - \underline{\mathbf{R}}_0|^3} = \frac{1}{\left(\sqrt{(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2}\right)^3} = \frac{1}{\left[(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2\right]^{3/2}}$$

ES Fields – Coulomb Integral / ES Felder – Coulomb-Integral

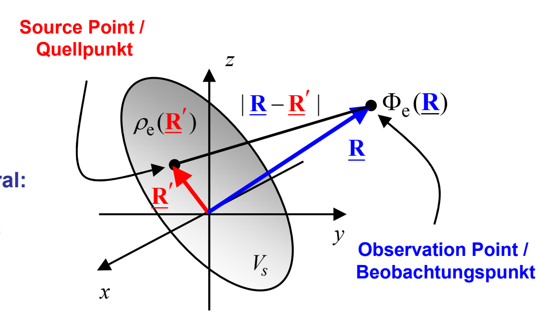
Poisson and Laplace Equation / Poisson- und Laplace-Gleichung

$$\Delta\Phi_\mathrm{e}(\underline{\mathbf{R}}) = \begin{cases} -\dfrac{\rho_\mathrm{e}(\underline{\mathbf{R}})}{\varepsilon_0} & \text{for / für } \rho_\mathrm{e}(\underline{\mathbf{R}}) \neq 0 \quad \text{Poisson Equation / Poisson-Gleichung} \\ 0 & \text{for / für } \rho_\mathrm{e}(\underline{\mathbf{R}}) = 0 \quad \text{Laplace Equation / Laplace-Gleichung} \end{cases}$$

$\Delta = \nabla^2 = \nabla \bullet \nabla$: Laplace Operator / Laplace-Operator

Limited Source Volume / Begrenztes Quellvolumen

$$\rho_\mathrm{e}(\underline{\mathbf{R}}) \begin{cases} \neq 0 & \underline{\mathbf{R}} \in V_s \\ 0 & \underline{\mathbf{R}} \notin V_s \end{cases}$$



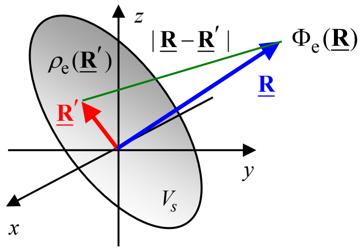
Coulomb Integral / Coulomb-Integral:

$$\Phi_\mathrm{e}(\underline{\mathbf{R}}) = \frac{1}{4\pi\varepsilon_0}\iiint_{V_s}\frac{\rho_\mathrm{e}(\underline{\mathbf{R}}')}{|\underline{\mathbf{R}} - \underline{\mathbf{R}}'|}\,d^3\underline{\mathbf{R}}'$$

$\rho_\mathrm{e}(\underline{\mathbf{R}}')$: known / bekannt

$\Phi_\mathrm{e}(\underline{\mathbf{R}})$: unknown / unbekannt

ES Fields – Coulomb Integral / ES Felder – Coulomb-Integral (...)



Coulomb Integral / Coulomb-Integral:

$$\Phi_e(\mathbf{R}) = \frac{1}{4\pi\varepsilon_0} \iiint_{V_s} \frac{\rho_e(\mathbf{R}')}{|\mathbf{R}-\mathbf{R}'|} \mathrm{d}^3\mathbf{R}'$$

$\rho_e(\mathbf{R}')$: known / bekannt

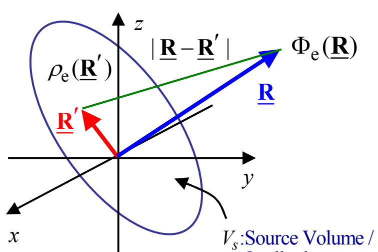
$\Phi_e(\mathbf{R})$: unknown / unbekannt

$$\Delta\Phi_e(\mathbf{R}) = \frac{1}{4\pi\varepsilon_0} \Delta \iiint_{V_s} \frac{1}{|\mathbf{R}-\mathbf{R}'|} \rho_e(\mathbf{R}') \mathrm{d}^3\mathbf{R}'$$

$$= \frac{1}{4\pi\varepsilon_0} \iiint_{V_s} \underbrace{\left[\Delta \frac{1}{|\mathbf{R}-\mathbf{R}'|}\right]}_{=-4\pi\delta(\mathbf{R}-\mathbf{R}')} \rho_e(\mathbf{R}') \mathrm{d}^3\mathbf{R}' \qquad \text{with} \quad \Delta \frac{1}{4\pi\left|\mathbf{R}-\mathbf{R}'\right|} = -\delta\left(\mathbf{R}-\mathbf{R}'\right)$$

$$= -\frac{1}{4\pi\varepsilon_0} \underbrace{\iiint_{V_s} 4\pi\delta(\mathbf{R}-\mathbf{R}')\rho_e(\mathbf{R}') \mathrm{d}^3\mathbf{R}'}_{=\rho_e(\mathbf{R})} = -\frac{1}{\varepsilon_0}\rho_e(\mathbf{R})$$

ES Fields – Green's Function / ES Felder – Greensche Funktion



$$\Phi_e(\mathbf{R}) = \frac{1}{4\pi\varepsilon_0} \iiint_{V_s} \frac{\rho_e(\mathbf{R}')}{|\mathbf{R}-\mathbf{R}'|} \mathrm{d}^3\mathbf{R}'$$

$$= \frac{1}{\varepsilon_0} \iiint_{V_s} \underbrace{\frac{1}{4\pi} \frac{1}{|\mathbf{R}-\mathbf{R}'|}}_{=G_e^{ES}(\mathbf{R}-\mathbf{R}')} \rho_e(\mathbf{R}') \mathrm{d}^3\mathbf{R}'$$

Electrostatic Green's Function / Elektrostatische Greensche Funktion

$$G_e^{ES}(\mathbf{R}-\mathbf{R}') = \frac{1}{4\pi} \frac{1}{|\mathbf{R}-\mathbf{R}'|} \qquad \text{for / für} \quad \mathbf{R} \neq \mathbf{R}'$$

$$\text{with} \quad \Delta G_e^{ES}(\mathbf{R}-\mathbf{R}') = -\delta\left(\mathbf{R}-\mathbf{R}'\right)$$

Normalized Potential of a Point Charge / Normiertes Potential einer Punktladung

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Electrostatic Potential of an Electrostatic Point Charge / Elektrostatisches Potential einer elektrostatischen Punktladung

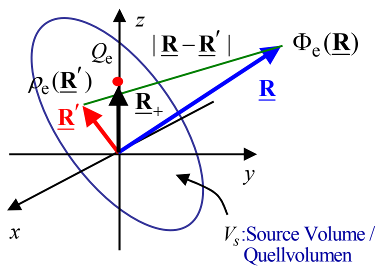
$$\Phi_e(\mathbf{R}) = \frac{Q_e}{4\pi} \frac{1}{|\mathbf{R}-\mathbf{R}_+|} \qquad \text{for / für} \quad \rho_e(\mathbf{R}) = Q_e\delta(\mathbf{R}-\mathbf{R}_+)$$

ES Fields – Potential of a Point Charge / ES Felder – Potential einer Punktladung

Electrostatic Volume Charge Density / Elektrostatisches Raumladungsdichte

$$\rho_e(\mathbf{R}) = Q_e \delta(\mathbf{R} - \mathbf{R}_+)$$

with / mit $\mathbf{R} = x\mathbf{e}_x + y\mathbf{e}_y + z\mathbf{e}_z$ $\quad \mathbf{R}_+ = x_+\mathbf{e}_x + y_+\mathbf{e}_y + z_+\mathbf{e}_z$



$$\Phi_e(\mathbf{R}) = \frac{1}{4\pi\varepsilon_0} \iiint_{V_s} \frac{Q_e \delta(\mathbf{R}' - \mathbf{R}_+)}{|\mathbf{R} - \mathbf{R}'|} d^3\mathbf{R}'$$

$$= \frac{Q_e}{4\pi\varepsilon_0} \iiint_{V_s} \frac{\delta(\mathbf{R}' - \mathbf{R}_+)}{|\mathbf{R} - \mathbf{R}'|} d^3\mathbf{R}'$$

$$= \frac{Q_e}{4\pi\varepsilon_0} \frac{1}{|\mathbf{R} - \mathbf{R}_+|}$$

$$\Phi_e(\mathbf{R}) = \frac{Q_e}{4\pi\varepsilon_0} \frac{1}{|\mathbf{R} - \mathbf{R}_+|}$$

ES Fields – Potential of a Point Charge / ES Felder – Potential einer Punktladung (...)

Electrostatic Potential of a Point Charge / Elektrostatisches Potential einer Punktladung

$$\Phi_e(\mathbf{R}) = \frac{Q_e}{4\pi\varepsilon_0} \frac{1}{|\mathbf{R} - \mathbf{R}_+|}$$

with $\mathbf{R} = x\mathbf{e}_x + y\mathbf{e}_y + z\mathbf{e}_z$

$\mathbf{R}_+ = x_+\mathbf{e}_x + y_+\mathbf{e}_y + z_+\mathbf{e}_z$

$$\frac{1}{|\mathbf{R} - \mathbf{R}_+|} = \frac{1}{\sqrt{(x-x_+)^2 + (y-y_+)^2 + (z-z_+)^2}} = \frac{1}{\left[(x-x_+)^2 + (y-y_+)^2 + (z-z_+)^2\right]^{1/2}}$$

Electrostatic Field Strength of a Point Charge / Elektrostatische Feldstärke einer Punktladung

$$\mathbf{E}(\mathbf{R}) = -\nabla\Phi_e(\mathbf{R}) = \frac{Q_e}{4\pi\varepsilon_0} \frac{\mathbf{R} - \mathbf{R}_+}{|\mathbf{R} - \mathbf{R}_+|^3}$$

with $\mathbf{R} = x\mathbf{e}_x + y\mathbf{e}_y + z\mathbf{e}_z$

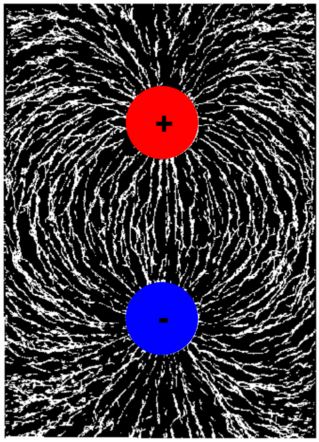
$\mathbf{R}_+ = x_+\mathbf{e}_x + y_+\mathbf{e}_y + z_+\mathbf{e}_z$

$$\frac{\mathbf{R} - \mathbf{R}_+}{|\mathbf{R} - \mathbf{R}_+|^3} = \frac{(x-x_+)\mathbf{e}_x + (y-y_+)\mathbf{e}_y + (z-z_+)\mathbf{e}_z}{\left[\sqrt{(x-x_+)^2 + (y-y_+)^2 + (z-z_+)^2}\right]^3} = \frac{(x-x_+)\mathbf{e}_x + (y-y_+)\mathbf{e}_y + (z-z_+)\mathbf{e}_z}{\left[(x-x_+)^2 + (y-y_+)^2 + (z-z_+)^2\right]^{3/2}}$$

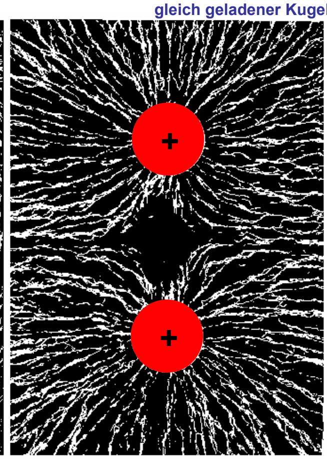
ES Fields – Potential of Two Point Charges / ES Felder – Potential zweier Punktladung

Electrostatic Dipole / Elektrostatischer Dipol

Field Lines of the Electric Field Strength of Two Spheres Carrying Charges of Opposite Sign / Feldlinien der elektrischen Feldstärke zweier ungleich geladener Kugeln



Electric Field Lines of Two Spheres Carrying Charges of the Same Sign / Feldlinien der elektrischen Feldstärke zweier gleich geladener Kugeln



ES Fields – Potential of Two Point Charges / ES Felder – Potential zweier Punktladung (...)

Electrostatic Dipole / Elektrostatischer Dipol

Electrostatic Potential / Elektrostatisches Potential

$$\Phi_{\mathrm{e}}(\underline{\mathbf{R}}) = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q_{\mathrm{e}+}}{|\underline{\mathbf{R}}-\underline{\mathbf{R}}_+|} + \frac{Q_{\mathrm{e}-}}{|\underline{\mathbf{R}}-\underline{\mathbf{R}}_-|}\right)$$

with/mit $\underline{\mathbf{R}} = x\underline{\mathbf{e}}_x + y\underline{\mathbf{e}}_y + z\underline{\mathbf{e}}_z$

$\underline{\mathbf{R}}_\pm = x_\pm\underline{\mathbf{e}}_x + y_\pm\underline{\mathbf{e}}_y + z_\pm\underline{\mathbf{e}}_z$

$$\frac{1}{|\underline{\mathbf{R}}-\underline{\mathbf{R}}_\pm|} = \frac{1}{\sqrt{(x-x_\pm)^2+(y-y_\pm)^2+(z-z_\pm)^2}} = \frac{1}{\left[(x-x_\pm)^2+(y-y_\pm)^2+(z-z_\pm)^2\right]^{1/2}}$$

Electrostatic Field Strength / Elektrostatische Feldstärke

$$\underline{\mathbf{E}}(\underline{\mathbf{R}}) = -\nabla\Phi_{\mathrm{e}}(\underline{\mathbf{R}}) = \frac{1}{4\pi\varepsilon_0}\left(Q_{\mathrm{e}+}\frac{\underline{\mathbf{R}}-\underline{\mathbf{R}}_+}{|\underline{\mathbf{R}}-\underline{\mathbf{R}}_+|^3} + Q_{\mathrm{e}-}\frac{\underline{\mathbf{R}}-\underline{\mathbf{R}}_-}{|\underline{\mathbf{R}}-\underline{\mathbf{R}}_-|^3}\right)$$

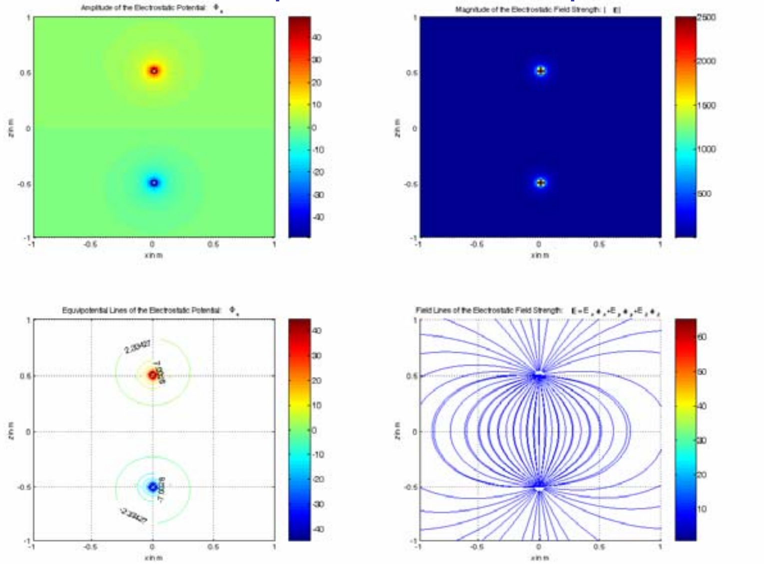
with/mit $\underline{\mathbf{R}} = x\underline{\mathbf{e}}_x + y\underline{\mathbf{e}}_y + z\underline{\mathbf{e}}_z$

$\underline{\mathbf{R}}_\pm = x_\pm\underline{\mathbf{e}}_x + y_\pm\underline{\mathbf{e}}_y + z_\pm\underline{\mathbf{e}}_z$

$$\frac{\underline{\mathbf{R}}-\underline{\mathbf{R}}_\pm}{|\underline{\mathbf{R}}-\underline{\mathbf{R}}_\pm|^3} = \frac{(x-x_\pm)\underline{\mathbf{e}}_x+(y-y_\pm)\underline{\mathbf{e}}_y+(z-z_\pm)\underline{\mathbf{e}}_z}{\left[\sqrt{(x-x_\pm)^2+(y-y_\pm)^2+(z-z_\pm)^2}\right]^3} = \frac{(x-x_\pm)\underline{\mathbf{e}}_x+(y-y_\pm)\underline{\mathbf{e}}_y+(z-z_\pm)\underline{\mathbf{e}}_z}{\left[(x-x_\pm)^2+(y-y_\pm)^2+(z-z_\pm)^2\right]^{3/2}}$$

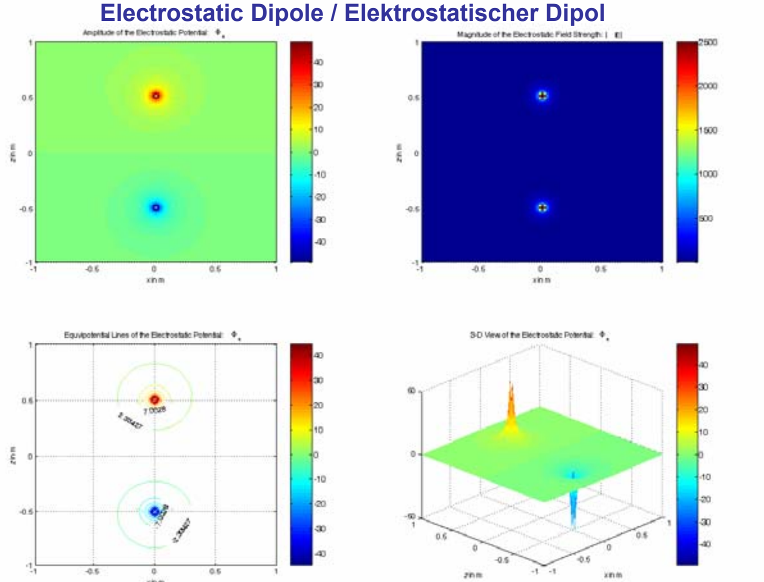
ES Fields – Potential of Two Point Charges / ES Felder – Potential zweier Punktladung (...)

Electrostatic Dipole / Elektrostatischer Dipol

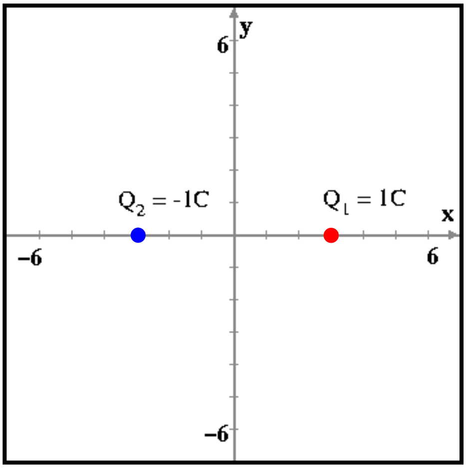


ES Fields – Potential of Two Point Charges / ES Felder – Potential zweier Punktladung (...)

Electrostatic Dipole / Elektrostatischer Dipol



Electrostatic Field Due To Two Point Charges / Elektrostatische Feld von zwei Punktladungen



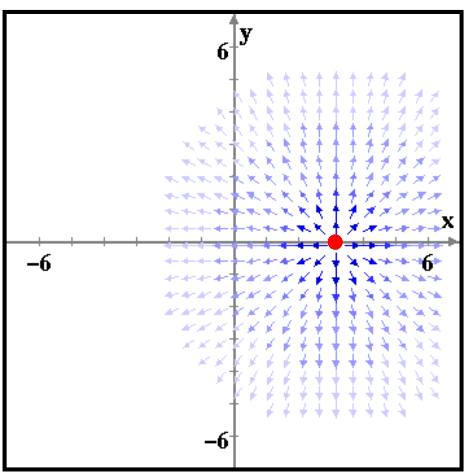
Q_1 = 1 As located at $P(x,y,z) = (3,0,0)$ and
Q_2 = -1 As located at $P(x,y,z) = (-3,0,0)$

In this demo, arrows are used to sketch the electric field pattern in the *x-y* plane due to:

Press	Q_1 alone / Q_1 alleine
Press	Q_2 alone
Press	Q_1 and Q_2 / Q_1 and Q_2

Note:
Color Intensity is Proportional to the Magnitude of the Electric Field Strength. /
Die Farbintensität ist proportional zur Magnitude der elektrischen Feldstärke.

Electrostatic Field… / Elektrostatische Feld… Q_1 alone / Q_1 alleine



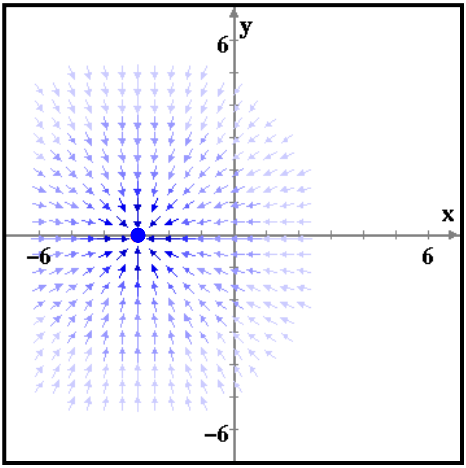
Q_1 = 1 As located at $P(x,y,z) = (3,0,0)$ and
Q_2 = -1 As located at $P(x,y,z) = (-3,0,0)$

In this demo, arrows are used to sketch the electric field pattern in the xy plane due to:

Press	Geometry / Geometrie
Press	Q_1 alone / Q_1 alleine
Press	Q_2 alone / Q_2 alleine
Press	Q_1 and Q_2 / Q_1 und Q_2

Note:
Color Intensity is Proportional to the Magnitude of the Electric Field Strength. /
Die Farbintensität ist proportional zur Magnitude der elektrischen Feldstärke.

Electrostatic Field… / Elektrostatische Feld…
Q_2 alone / Q_2 alleine



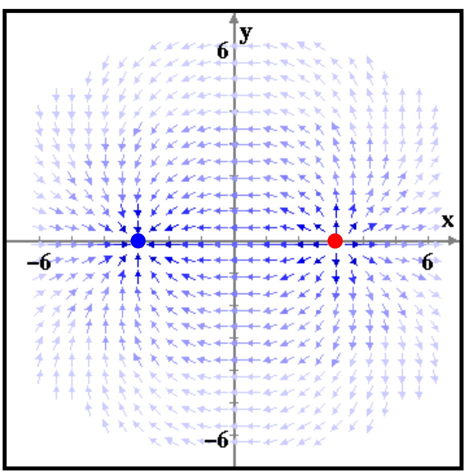
Q_1 = 1 As located at $P(x,y,z) = (3,0,0)$ and
Q_2 = -1 As located at $P(x,y,z) = (-3,0,0)$

In this demo, arrows are used to sketch the electric field pattern in the *x-y* plane due to:

Press	Geometry / Geometrie
Press	Q_1 alone / Q_1 alleine
Press	Q_2 alone / Q_2 alleine
Press	Q_1 and Q_2 / Q_1 und Q_2

Note:
Color Intensity is Proportional to the Magnitude of the Electric Field Strength. /
Die Farbintensität ist proportional zur Magnitude der elektrischen Feldstärke.

Electrostatic Field… / Elektrostatische Feld…
Q_1 and Q_2 / Q_1 und Q_2



Q_1 = 1 As located at $P(x,y,z) = (3,0,0)$ and
Q_2 = -1 As located at $P(x,y,z) = (-3,0,0)$

In this demo, arrows are used to sketch the electric field pattern in the *x-y* plane due to:

Press	Geometry / Geometrie
Press	Q_1 alone / Q_1 alleine
Press	Q_2 alone / Q_2 alleine
Press	Q_1 and Q_2 / Q_1 und Q_2

Note:
Color Intensity is Proportional to the Magnitude of the Electric Field Strength. /
Die Farbintensität ist proportional zur Magnitude der elektrischen Feldstärke.

End of 7th Lecture /
Ende der 7. Vorlesung