Numerical Methods of Electromagnetic Field Theory I (NFT I) Numerische Methoden der Elektromagnetischen Feldtheorie I (NFT I) /

7th Lecture / 7. Vorlesung

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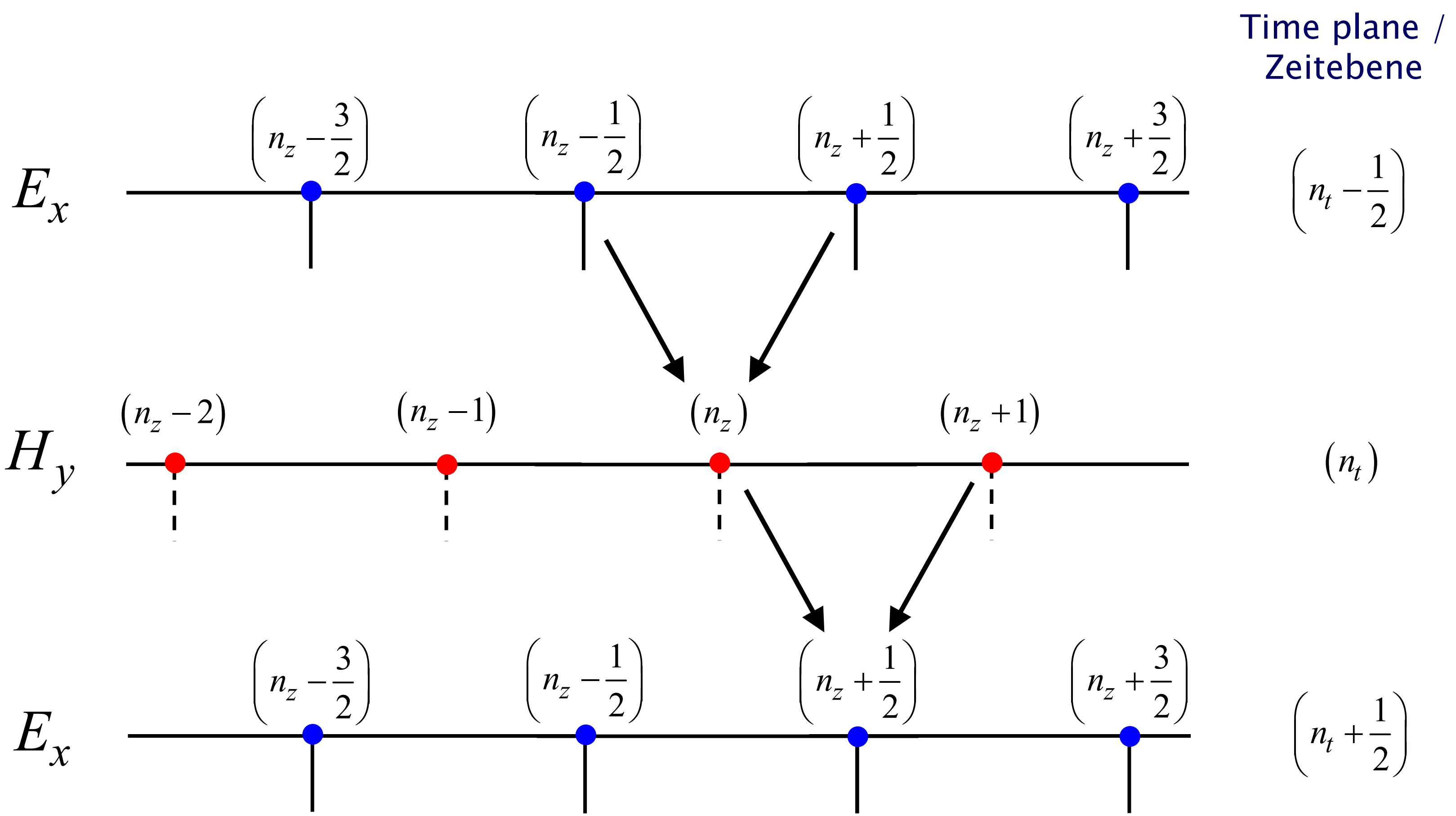
1–D EM Wave Propagation – 1–D FDTD – Staggered Grid in Space / 1D EM Wellenausbreitung – 1–D FDTD – Versetztes Gitter im Raum

Interleaving of the E_x and H_y field components in space and time in the 1–D FDTD formulation / Überlappung der E_x– und H_y–Feldkomponente in der 1D–FDTD–Formulierung im Raum und in der Zeit



1-D EM Wave Propagation – FDTD – Normalization / 1D EM Wellenausbreitung – FDTD – Normierung

$$H_y^{(n_z,n_t)} = H_y^{(n_z,n_t-1)} - \frac{\Delta t}{\mu_0 \Delta z}\left[E_x^{(n_z+1/2,n_t-1/2)} - E_x^{(n_z-1/2,n_t-1/2)}\right] - \frac{\Delta t}{\mu_0} J_{\text{my}}^{(n_z,n_t-1/2)}$$

$$E_x^{(n_z+1/2,n_t+1/2)} = E_x^{(n_z+1/2,n_t-1/2)} - \frac{\Delta t}{\varepsilon_0 \Delta z}\left[H_y^{(n_z+1,n_t)} - H_y^{(n_z,n_t)}\right] - \frac{\Delta t}{\varepsilon_0} J_{\text{ex}}^{(n_z+1/2,n_t)}$$

$$\Delta t = \Delta t_{\text{ref}}\,\widehat{\Delta t} \qquad \Delta t_{\text{ref}} = \frac{\Delta x_{\text{ref}}}{c_{\text{ref}}} \qquad \Delta t = \frac{\Delta x_{\text{ref}}}{c_{\text{ref}}}\widehat{\Delta t}$$

$$\Delta z = \Delta x_{\text{ref}}\,\widehat{\Delta z} \qquad c = c_{\text{ref}}\hat{c} \qquad \varepsilon = \varepsilon_{\text{ref}}\,\hat{\varepsilon} \qquad \mu = \mu_{\text{ref}}\,\hat{\mu} \qquad \mu_{\text{ref}} = \mu_0$$

$$E_x = E_{\text{ref}}\hat{E}_x$$

$$H_y = H_{\text{ref}}\hat{H}_y \qquad H_{\text{ref}} = \frac{E_{\text{ref}}}{c_{\text{ref}}\mu_{\text{ref}}} = \frac{\sqrt{\varepsilon_{\text{ref}}\mu_{\text{ref}}}}{\mu_{\text{ref}}}E_{\text{ref}} = \sqrt{\frac{\varepsilon_{\text{ref}}}{\mu_{\text{ref}}}}E_{\text{ref}} = \frac{E_{\text{ref}}}{Z_{\text{ref}}}$$

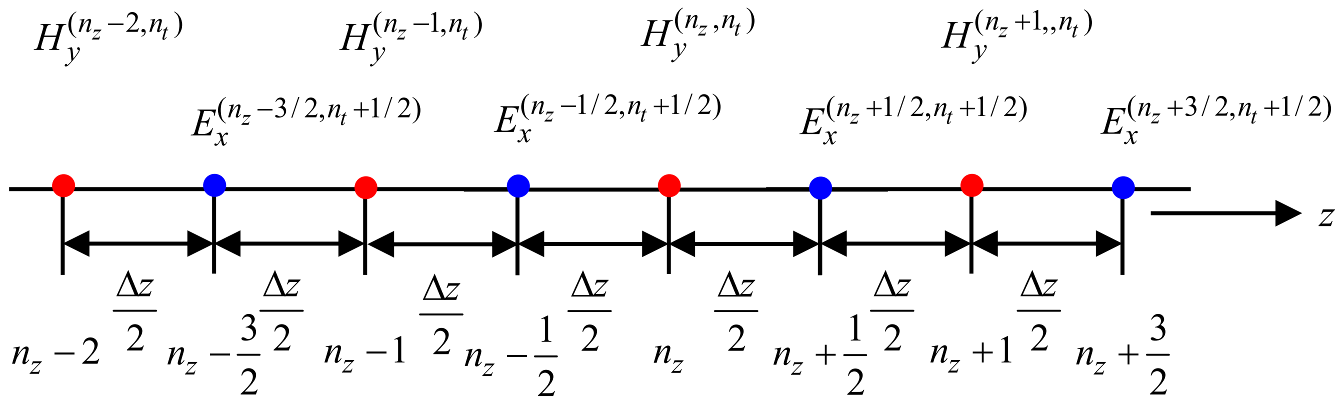
$$J_{\text{ex}} = J_{\text{e ref}}\hat{J}_{\text{ex}} \qquad J_{\text{e ref}} = \frac{\varepsilon_{\text{ref}}}{\Delta t_{\text{ref}}}E_{\text{ref}}$$

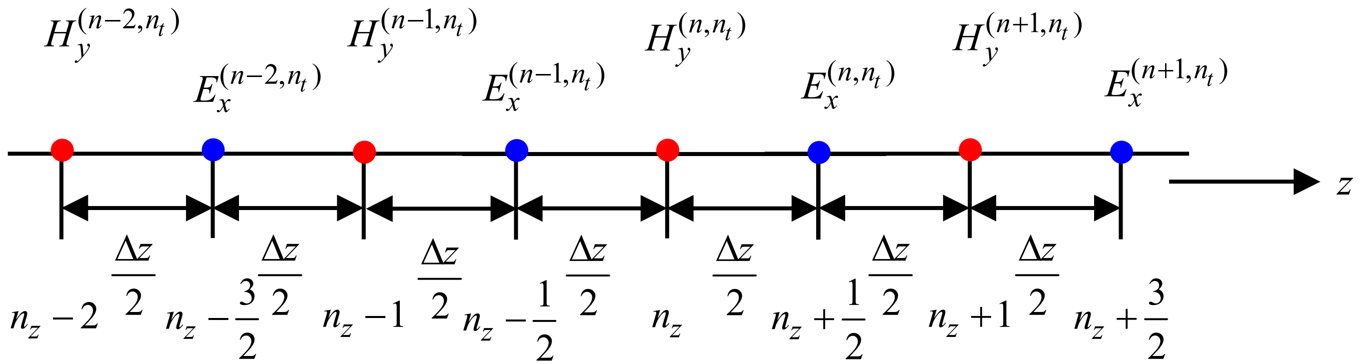
$$J_{\text{mx}} = J_{\text{m ref}}\hat{J}_{\text{mx}} \qquad J_{\text{m ref}} = \frac{\mu_{\text{ref}}}{\Delta t_{\text{ref}}}H_{\text{ref}} = \frac{E_{\text{ref}}}{\Delta t_{\text{ref}}c_{\text{ref}}}$$

$$\hat{H}_y^{(n_z,n_t)} = \hat{H}_y^{(n_z,n_t-1)} - \widehat{\Delta t}\left[\hat{E}_x^{(n_z+1/2,n_t-1/2)} - \hat{E}_x^{(n_z-1/2,n_t-1/2)}\right] - \widehat{\Delta t}\hat{J}_{\text{my}}^{(n_z,n_t-1/2)}$$

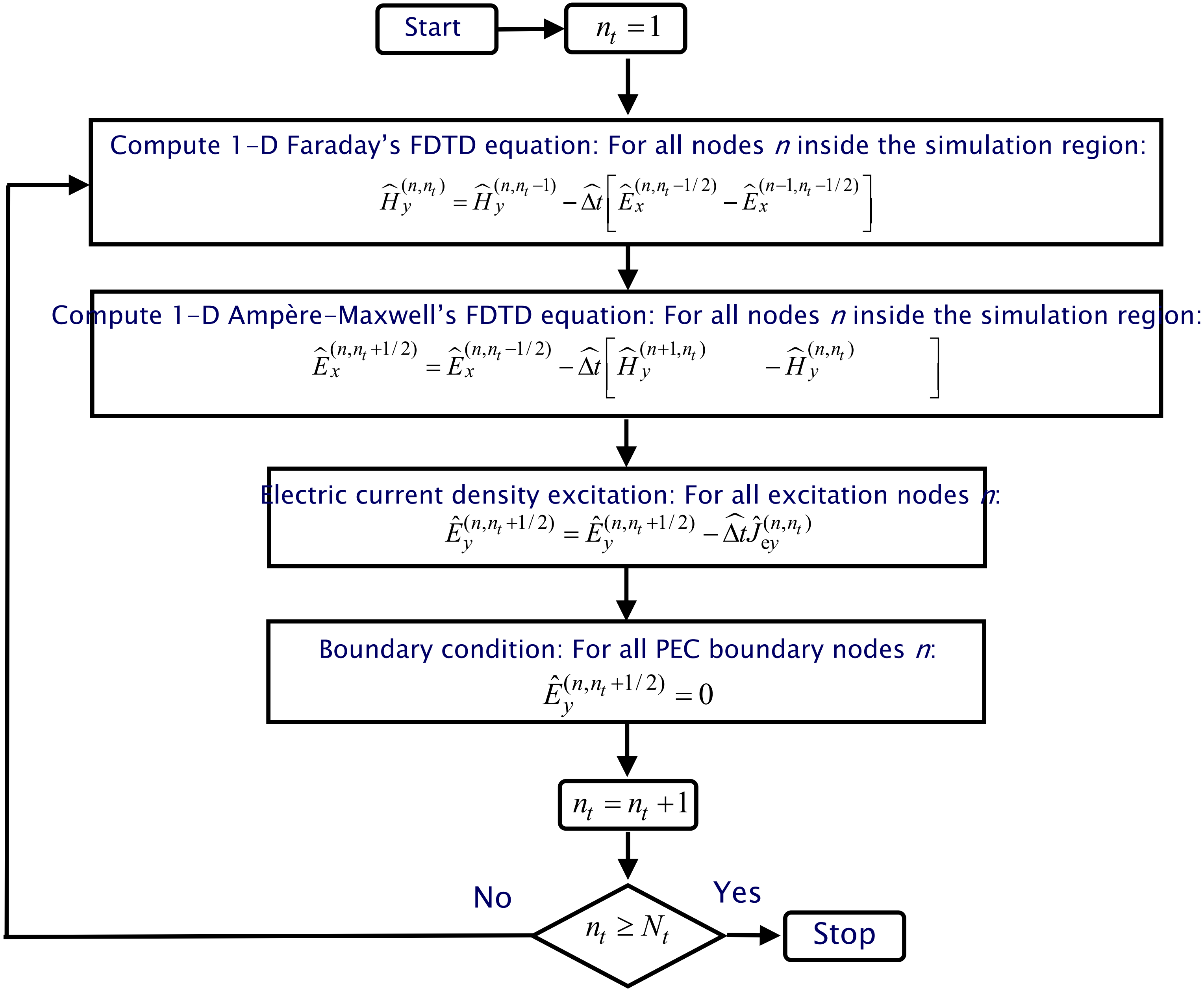
$$\hat{E}_x^{(n_z+1/2,n_t+1/2)} = \hat{E}_x^{(n_z+1/2,n_t-1/2)} - \widehat{\Delta t}\left[\hat{H}_y^{(n_z+1,n_t)} - \hat{H}_y^{(n_z,n_t)}\right] - \widehat{\Delta t}\,\hat{J}_{\text{ex}}^{(n_z+1/2,n_t)}$$

1–D FDTD – Staggered Grid in Space – Global Node Numbering / 1D–FDTD – Versetztes Gitter im Raum – Globale Knotennummerierung

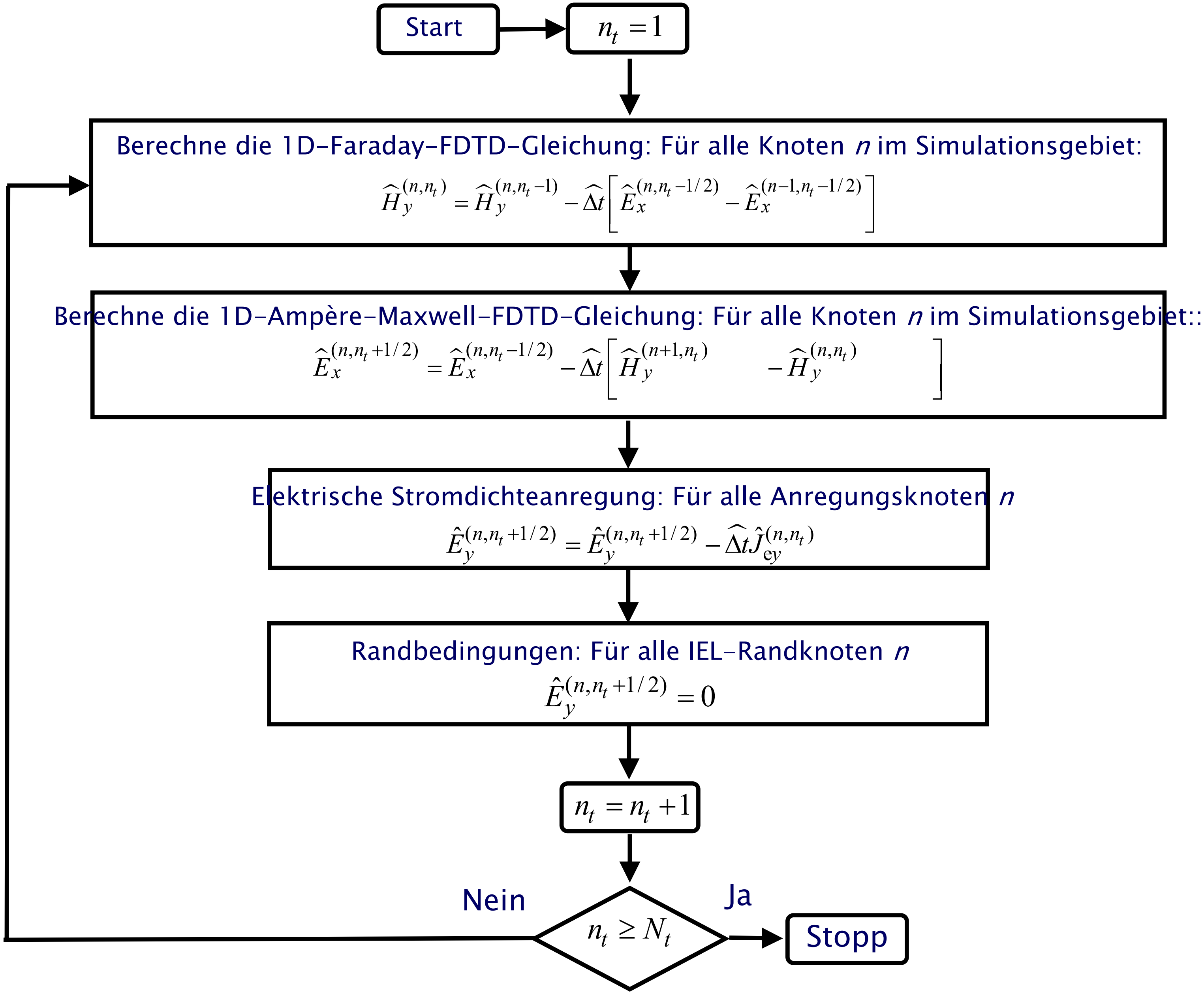




1-D FDTD Algorithm – Flow Chart /
1D-FDTD-Algorithmus – Flussdiagramm



1-D FDTD Algorithm – Flow Chart / 1D-FDTD-Algorithmus – Flussdiagramm



FDTD Solution of the First Two 1-D Scalar Maxwell's Equations / FDTD-Lösung der ersten beiden 1D skalaren Maxwell-Gleichungen

Maxwell's equations / Maxwellsche Gleichungen

$$\frac{\partial}{\partial t} H_y(z,t) = -\frac{1}{\mu_0}\frac{\partial}{\partial z} E_x(z,t) - \frac{1}{\mu_0} J_{\mathrm{my}}(z,t) \quad \text{for / für} \quad \begin{cases} 0 \le z \le Z \\ 0 \le t \le T \end{cases}$$

$$\frac{\partial}{\partial t} E_x(z,t) = -\frac{1}{\varepsilon_0}\frac{\partial}{\partial z} H_y(z,t) - \frac{1}{\varepsilon_0} J_{\mathrm{ex}}(z,t)$$

Initial condition / Anfangsbedingung

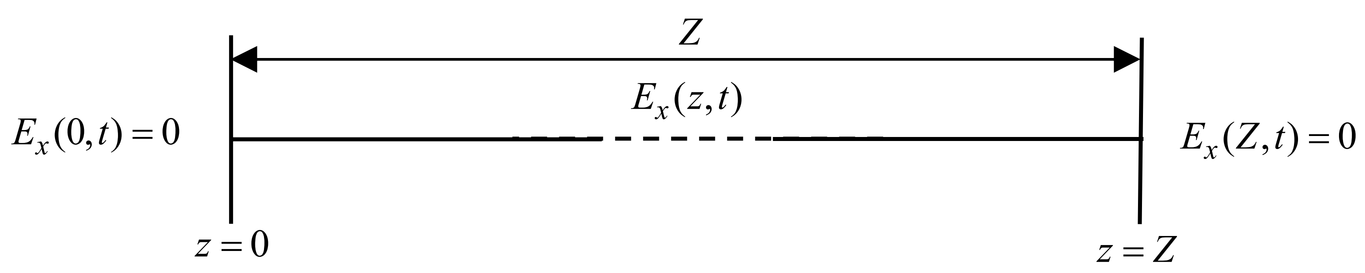
$$\begin{aligned} H_y(z,t) &= J_{\mathrm{my}}(z,t) = 0 & t \le 0 \\ E_x(z,t) &= J_{\mathrm{ex}}(z,t) = 0 & t \le 0 \\ J_{\mathrm{ex}}(z,t) &= K_{\mathrm{e0}}(z_0)\,\delta(z-z_0)\,f(t) & t > 0 \end{aligned}$$

Causality / Kausalität

Boundary condition for a perfectly electrically conducting (PEC) material / Randbedingung für ein ideal elektrisch leitendes Material

$$\left.\begin{aligned} E_x(0,t) &= 0 \\ E_x(Z,t) &= 0 \end{aligned}\right\} \quad \forall t$$

Hyperbolic initial-boundary-value problem / Hyperbolisches Anfangs-Randwert-Problem



FDTD Solution of the First Two 1-D Scalar Maxwell's Equations / FDTD-Lösung der ersten beiden 1D skalaren Maxwell-Gleichungen

Discrete 1-D FDTD equations / Diskrete 1D-FDTD-Gleichungen

$$\widehat{H}_y^{(n_z,n_t)} = \widehat{H}_y^{(n_z,n_t-1)} - \widehat{\Delta t}\left[\widehat{E}_x^{(n_z+1/2,n_t-1/2)} - \widehat{E}_x^{(n_z-1/2,n_t-1/2)}\right] - \widehat{\Delta t}\widehat{J}_{\mathrm{my}}^{(n_z,n_t-1/2)} \quad \text{for / für} \quad \begin{cases} 1 \le n_z \le N_z \\ 1 \le n_t \le N_t \end{cases}$$

$$\widehat{E}_x^{(n_z+1/2,n_t+1/2)} = \widehat{E}_x^{(n_z+1/2,n_t-1/2)} - \widehat{\Delta t}\left[\widehat{H}_y^{(n_z+1,n_t)} - \widehat{H}_y^{(n_z,n_t)}\right] - \widehat{\Delta t}\ \widehat{J}_{\mathrm{ex}}^{(n_z+1/2,n_t)}$$

Initial condition / Anfangsbedingung

$$H_y^{(n_z,n_t)} = J_{\mathrm{my}}^{(n_z,n_t)} = 0 \qquad n_t \le 1$$

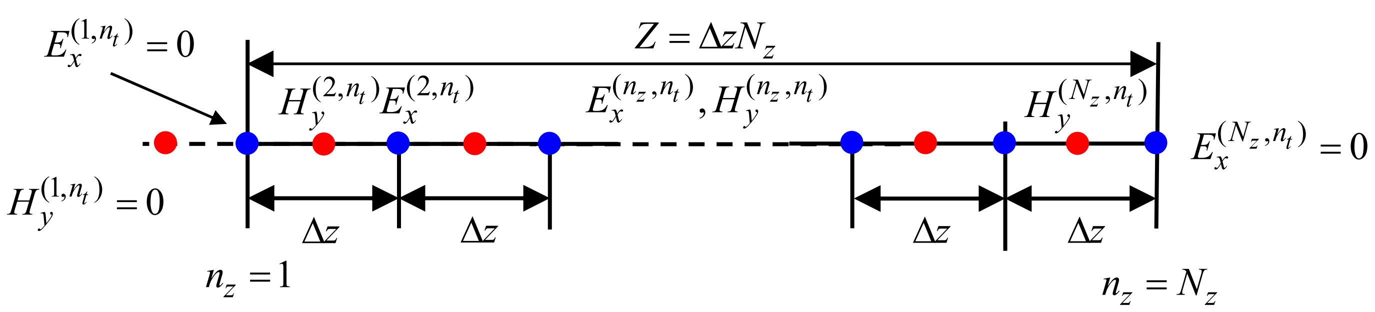
$$E_x^{(n_z,n_t)} = J_{\mathrm{ex}}^{(n_z,n_t)} = 0 \qquad n_t \le 1$$

$$J_{\mathrm{ex}}^{(n_z,n_t)} = K_{\mathrm{ex}}^{(n_{z0})}\ \delta^{(n_z-n_{z0})}\ f^{(n_t)} \qquad n_t > 1$$

Causality / Kausalität

Boundary condition for a perfectly electrically conducting (PEC) material / Randbedingung für ein ideal elektrisch leitendes Material

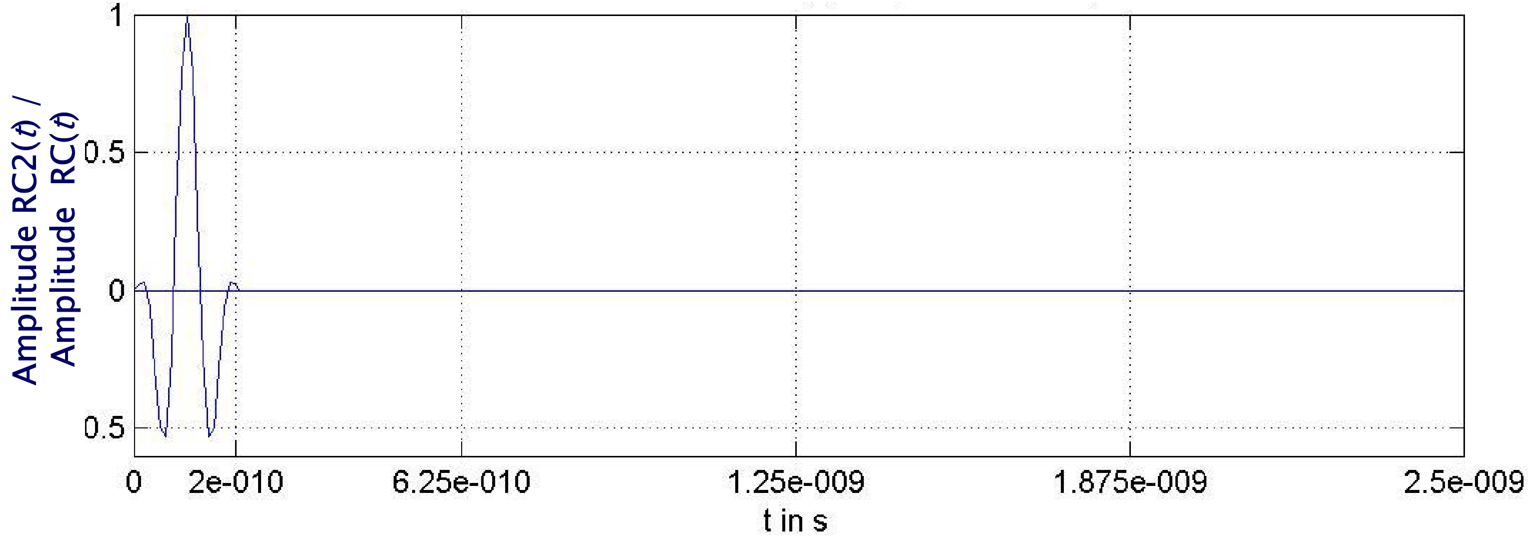
$$\left.\begin{array}{l} E_x^{(1,n_t)} = 0 \\ E_x^{(N_z,n_t)} = 0 \end{array}\right\} \quad 1 \le n_t \le N_t$$



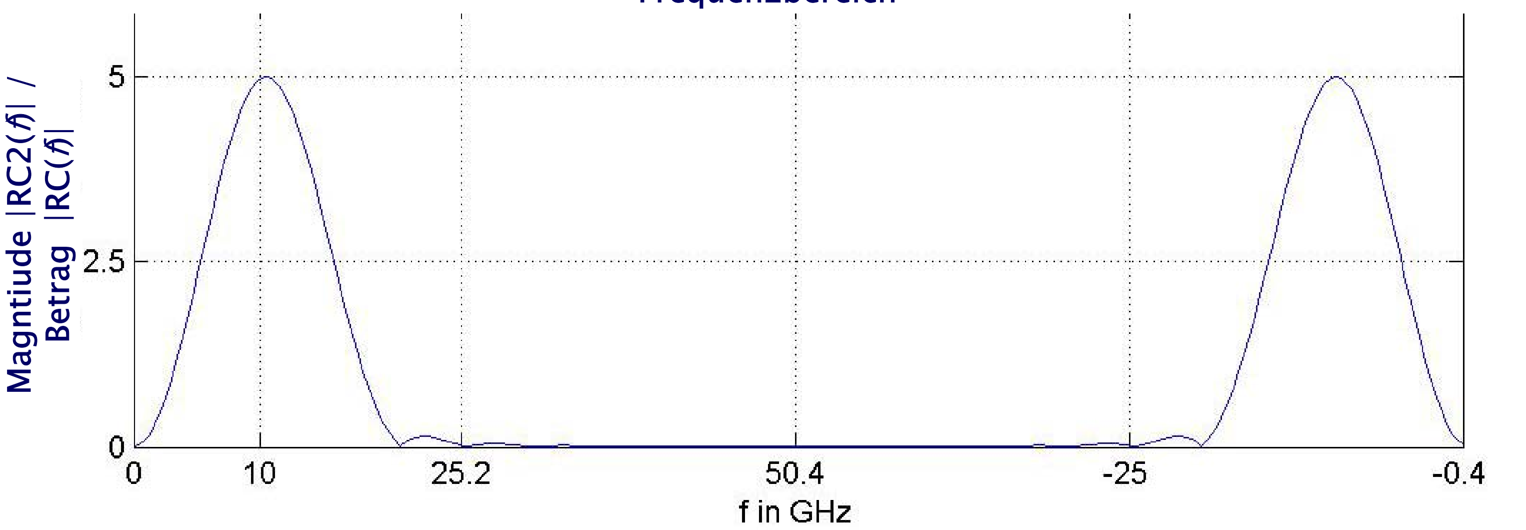
Discrete hyperbolic initial-boundary-value problem / Diskretes hyperbolisches Anfangs-Randwert-Problem

FDTD Solution of the First Two 1-D Scalar Maxwell's Equations / FDTD-Lösung der ersten beiden 1D skalaren Maxwell-Gleichungen

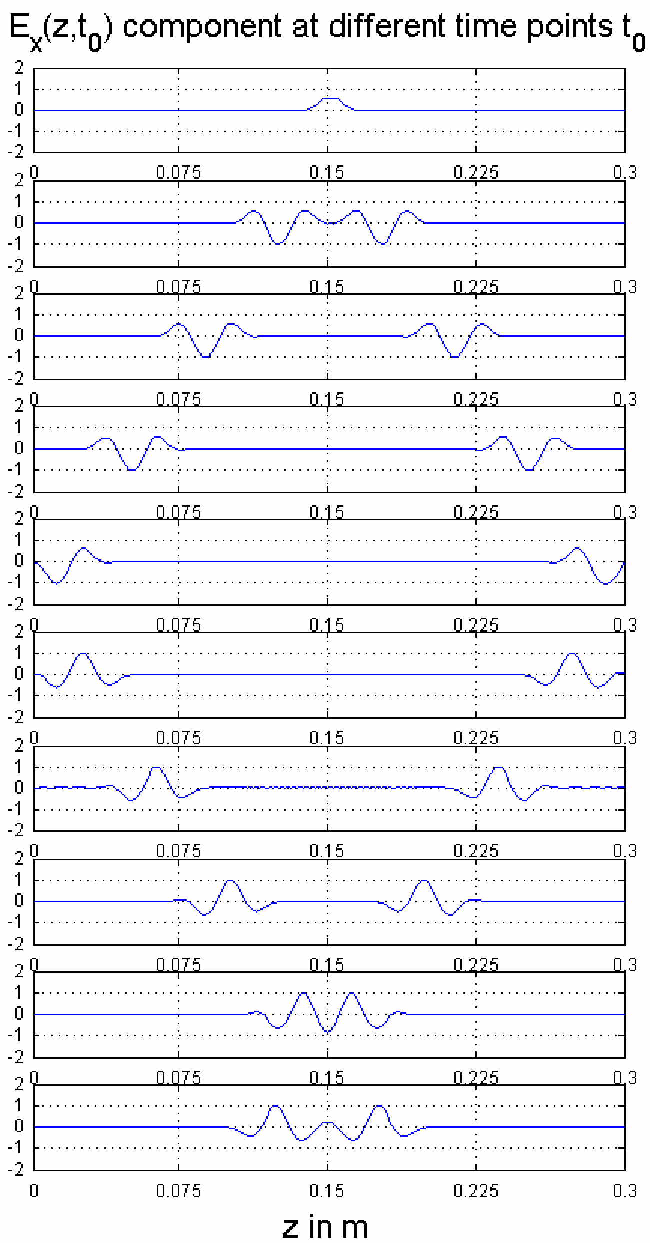
Excitation pulse: RC2(*t*) – Time Domain / Anregungsfunktion: RC2(*t*) – Zeitbereich

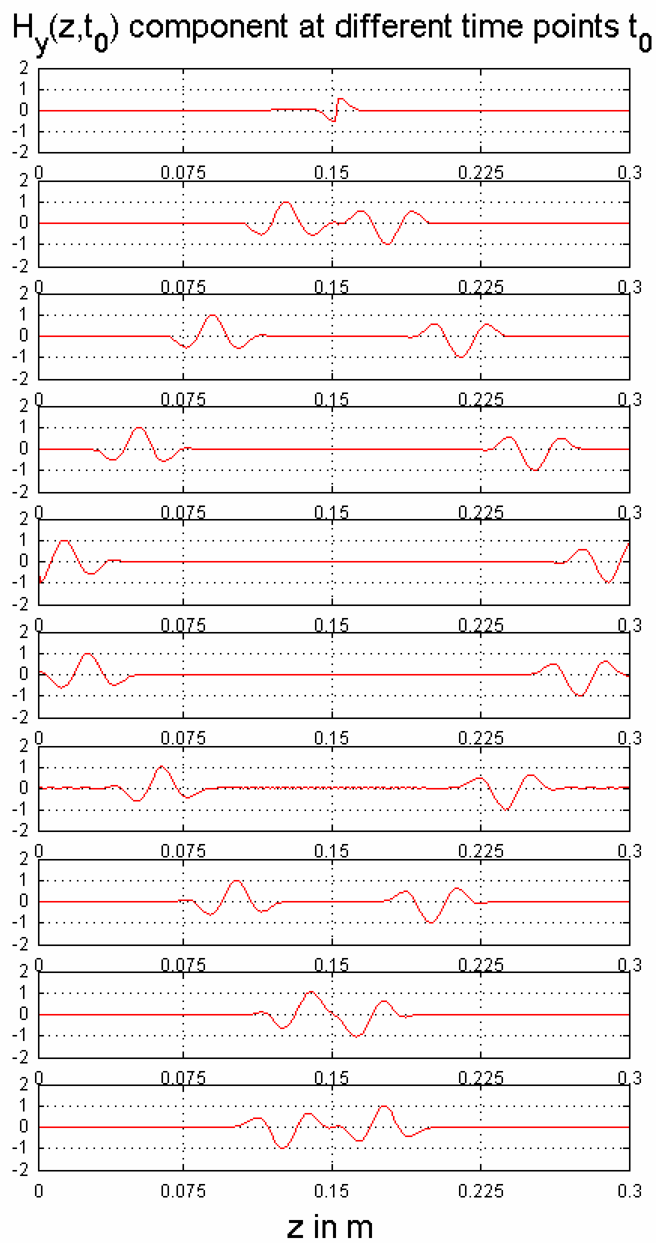


Excitation pulse: RC2(*f*) – Frequency Domain / Anregungsfunktion: RC(*f*) – Frequenzbereich



FDTD Solution of the First Two 1-D Scalar Maxwell's Equations / FDTD-Lösung der ersten beiden 1D skalaren Maxwell-Gleichungen





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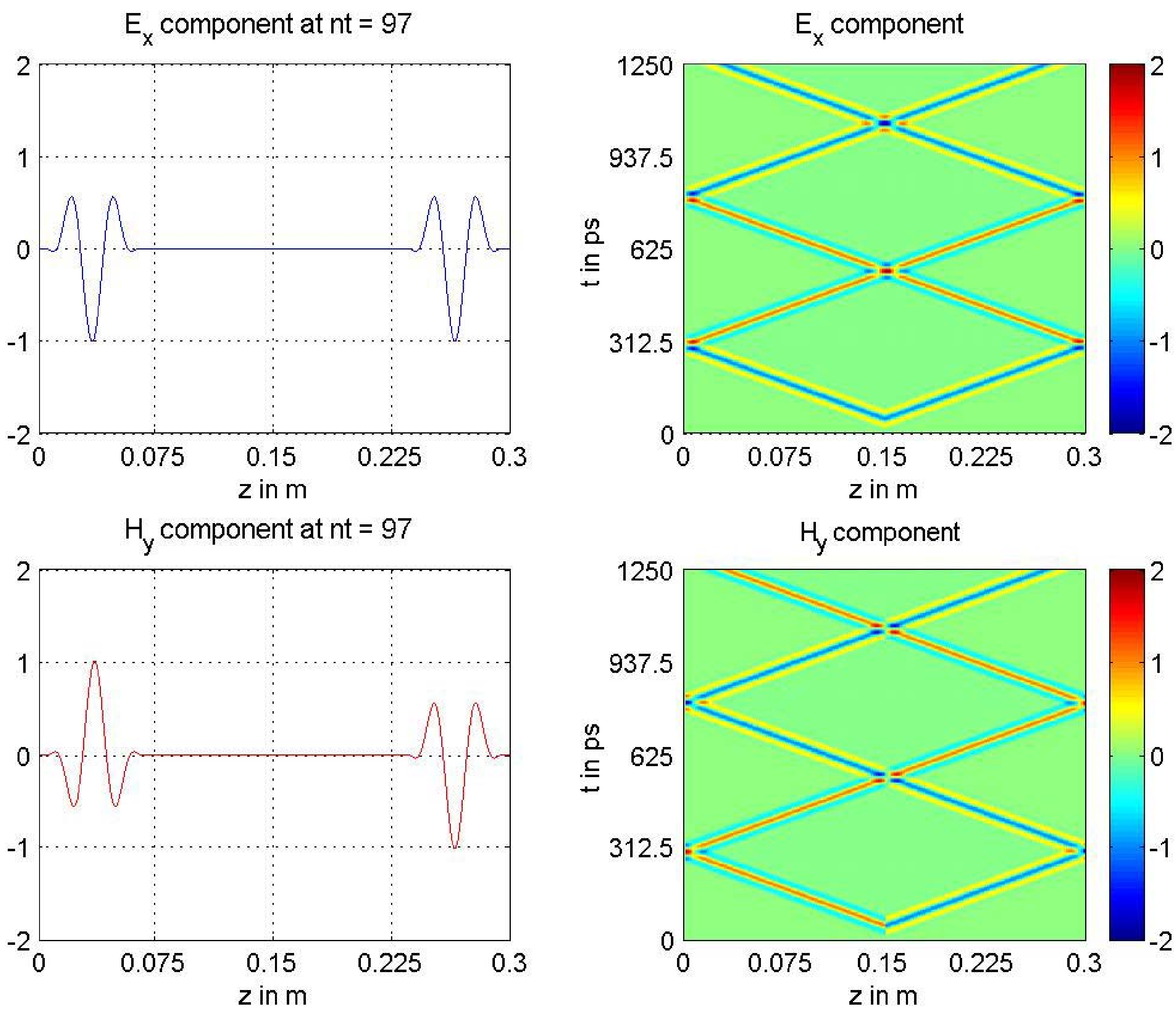


Implementation of Boundary Conditions / Implementierung von Randbedingungen

Boundary condition for a perfectly electrically conducting (PEC) material / Randbedingung für ein ideal elektrisch leitendes Material

$$\left.\begin{aligned} E_x^{(1,n_t)} &= 0 \\ E_x^{(N_z,n_t)} &= 0 \end{aligned}\right\} \quad 1 \le n_t \le N_t$$

Absorbing/open boundary condition / Absorbierende/offene Randbedingung

Space-time-extrapolation of the first order / Raum-Zeit-Extrapolation der ersten Ordnung

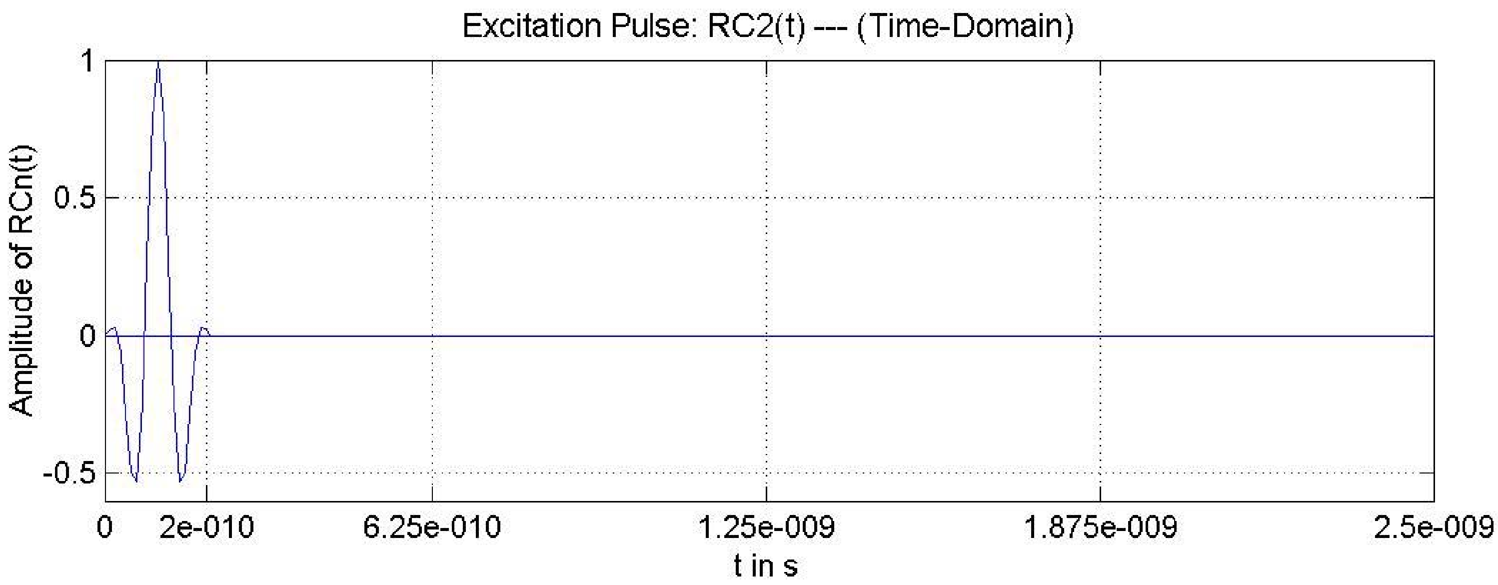
For / Für $\widehat{\Delta t} = 0.5$

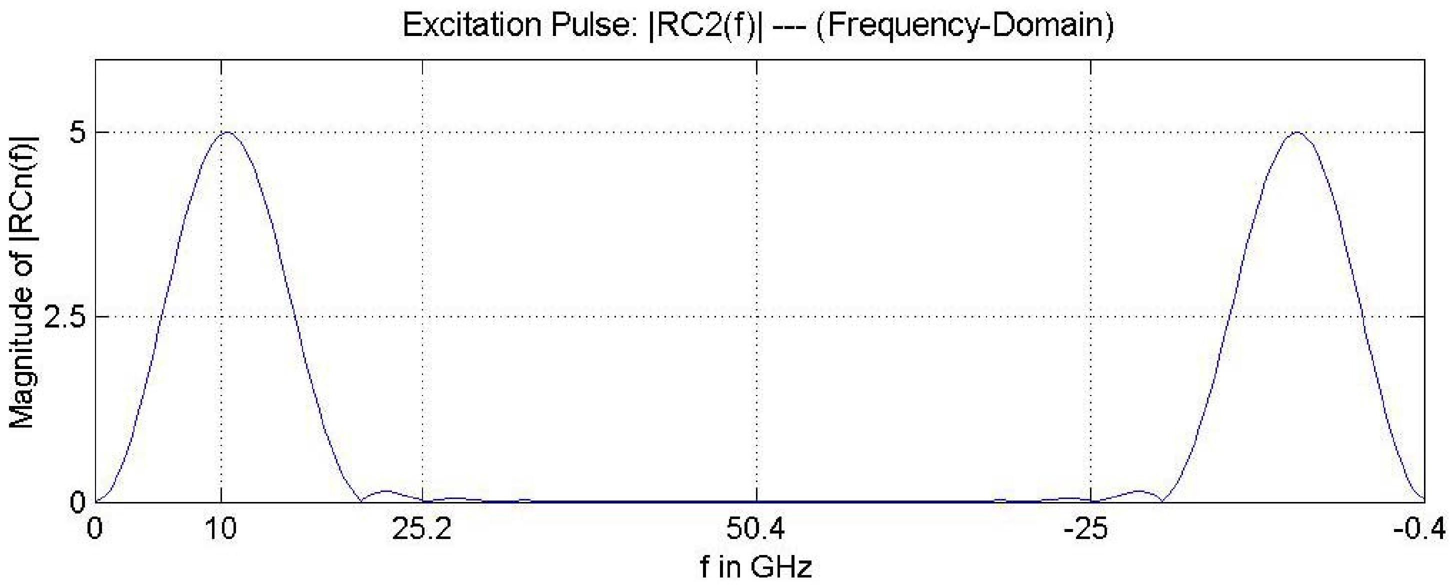
a plane wave needs two time steps, 2 n_t, to travel over one grid cell with the size Δz / braucht eine ebene Welle zwei Zeitschritte, 2 n_t, um sich über eine Gitterzelle der Größe Δz auszubreiten

$$\left.\begin{aligned} E_x^{(1,n_t)} &= E_x^{(2,n_t-2)} \\ E_x^{(N_z,n_t)} &= E_x^{(N_z-1,n_t-2)} \end{aligned}\right\} \quad 1 \le n_t \le N_t$$

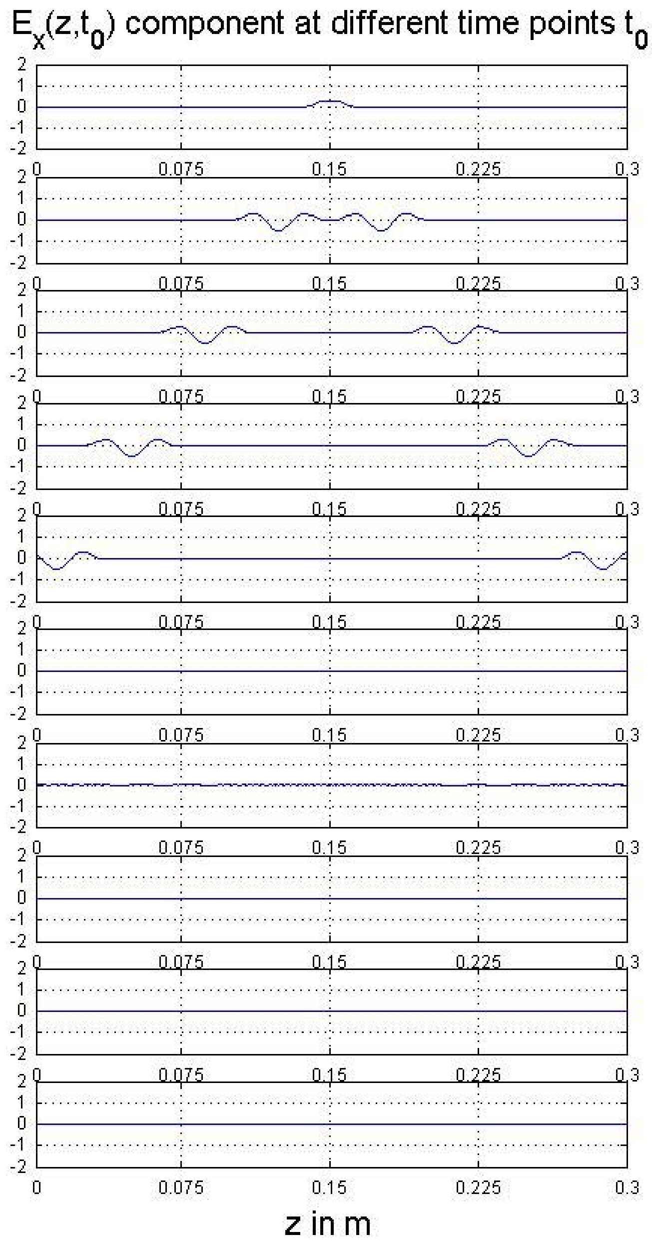
Space-time-extrapolation of the first order / Raum-Zeit-Extrapolation der ersten Ordnung

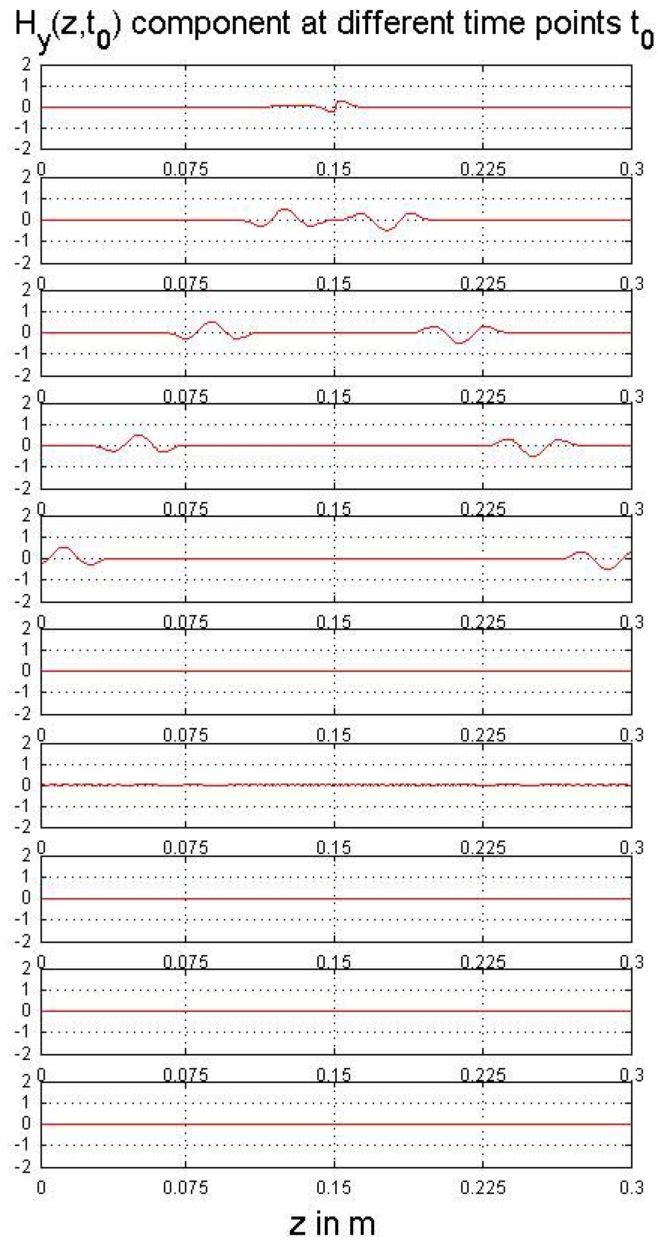
FDTD Solution of the First Two 1-D Scalar Maxwell's Equations / FDTD–Lösung der ersten beiden 1D skalaren Maxwell–Gleichungen



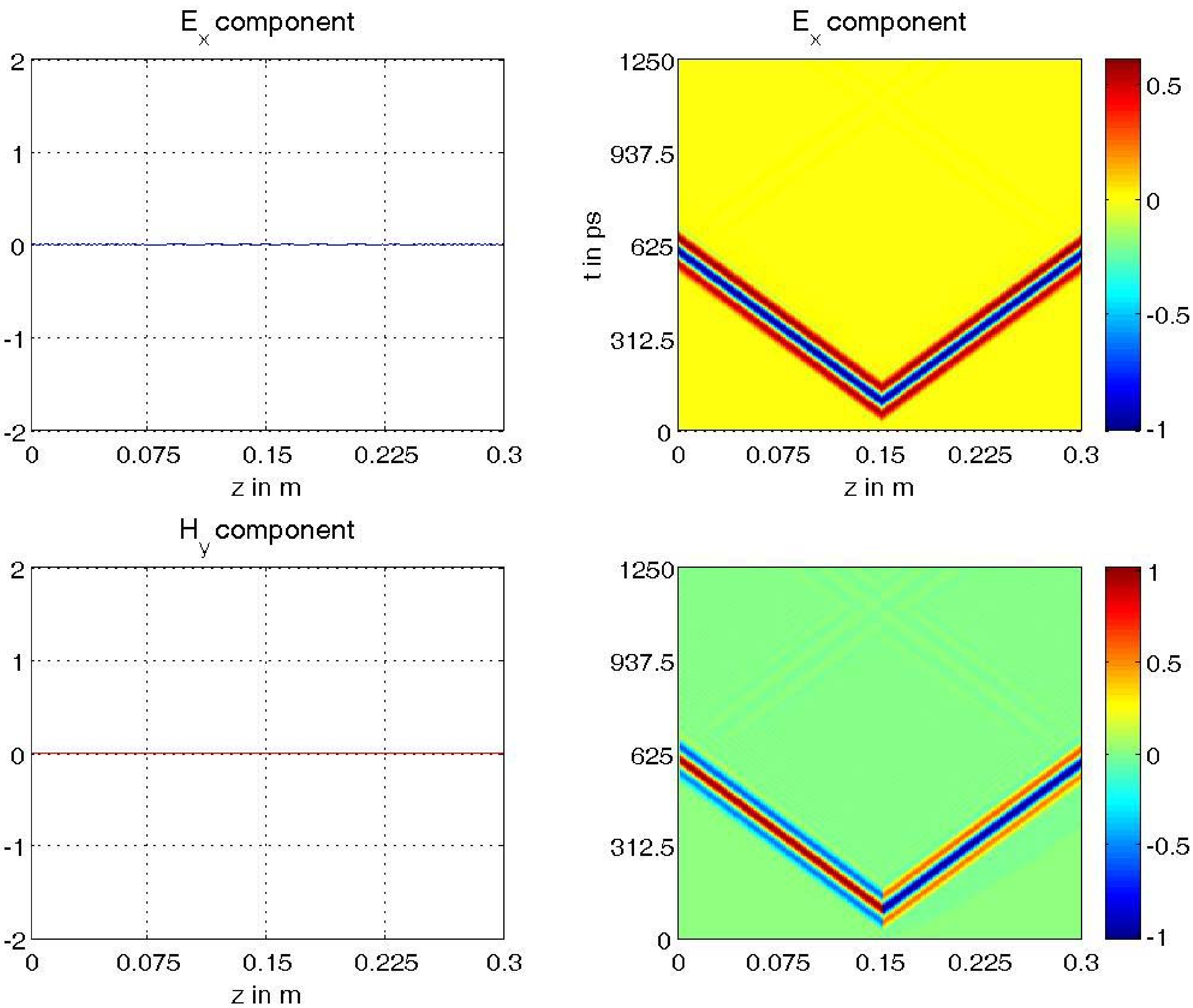


FDTD Solution of the First Two 1-D Scalar Maxwell's Equations / FDTD-Lösung der ersten beiden 1D skalaren Maxwell-Gleichungen

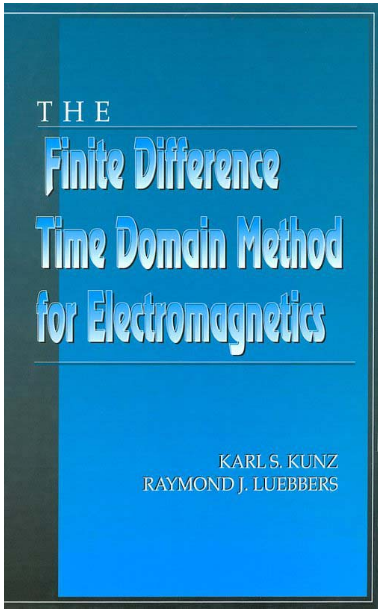




FDTD Solution of the First Two 1-D Scalar Maxwell's Equations / FDTD-Lösung der ersten beiden 1D skalaren Maxwell-Gleichungen



FDTD Books / FDTD-Bücher



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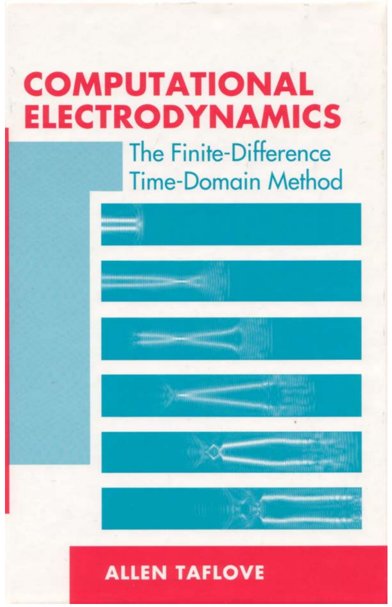

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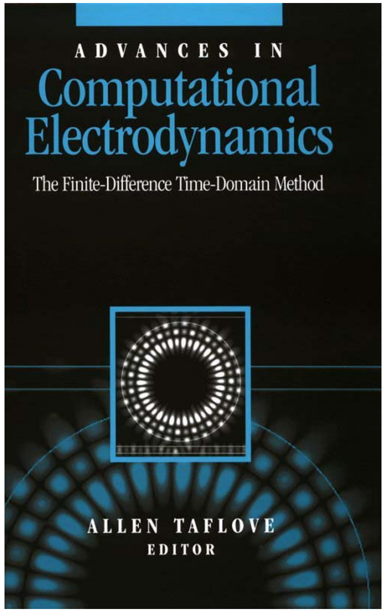

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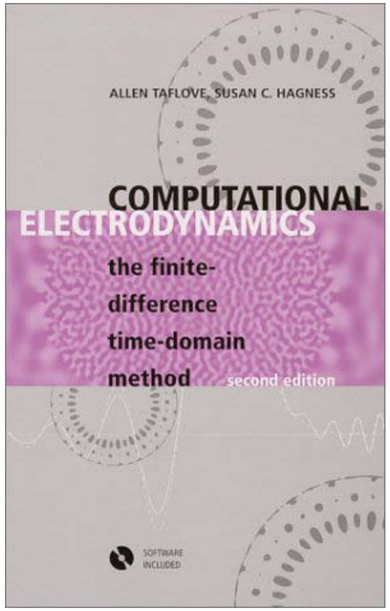

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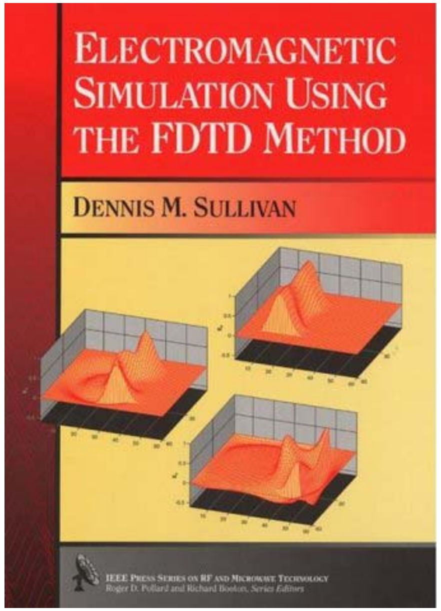
FDTD Books / FDTD-Bücher



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End of Lecture 7 /
Ende der 7. Vorlesung