# Numerical Methods of Electromagnetic Field Theory I (NFT I) Numerische Methoden der Elektromagnetischen Feldtheorie I (NFT I) /

## 7th Lecture / 7. Vorlesung


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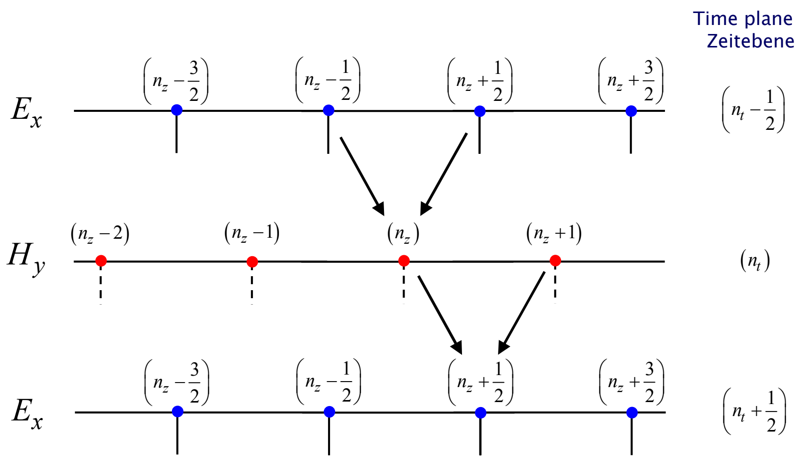
## 1-D EM Wave Propagation – 1-D FDTD – Staggered Grid in Space / 1D EM Wellenausbreitung – 1-D FDTD – Versetztes Gitter im Raum

Interleaving of the $E_x$ and $H_y$ field components in space and time in the 1-D FDTD formulation / Überlappung der $E_x$- und $H_y$-Feldkomponente in der 1D-FDTD-Formulierung im Raum und in der Zeit



## 1-D EM Wave Propagation – FDTD – Normalization / 1D EM Wellenausbreitung – FDTD – Normierung

$$H_y^{(n_z,n_t)} = H_y^{(n_z,n_t-1)} - \frac{\Delta t}{\mu_0 \Delta z}\left[E_x^{(n_z+1/2,n_t-1/2)} - E_x^{(n_z-1/2,n_t-1/2)}\right] - \frac{\Delta t}{\mu_0} J_{\mathrm{my}}^{(n_z,n_t-1/2)}$$

$$E_x^{(n_z+1/2,n_t+1/2)} = E_x^{(n_z+1/2,n_t-1/2)} - \frac{\Delta t}{\varepsilon_0 \Delta z}\left[H_y^{(n_z+1,n_t)} - H_y^{(n_z,n_t)}\right] - \frac{\Delta t}{\varepsilon_0} J_{\mathrm{ex}}^{(n_z+1/2,n_t)}$$

$$\Delta t = \Delta t_{\mathrm{ref}} \widehat{\Delta t} \qquad \Delta t_{\mathrm{ref}} = \frac{\Delta x_{\mathrm{ref}}}{c_{\mathrm{ref}}} \qquad \Delta t = \frac{\Delta x_{\mathrm{ref}}}{c_{\mathrm{ref}}} \widehat{\Delta t}$$

$$\Delta z = \Delta x_{\mathrm{ref}} \widehat{\Delta z} \qquad c = c_{\mathrm{ref}} \hat{c} \qquad \varepsilon = \varepsilon_{\mathrm{ref}} \hat{\varepsilon} \qquad \mu = \mu_{\mathrm{ref}} \hat{\mu} \qquad \mu_{\mathrm{ref}} = \mu_0$$

$$E_x = E_{\mathrm{ref}} \hat{E}_x$$

$$H_y = H_{\mathrm{ref}} \hat{H}_y \qquad H_{\mathrm{ref}} = \frac{E_{\mathrm{ref}}}{c_{\mathrm{ref}} \mu_{\mathrm{ref}}} = \frac{\sqrt{\varepsilon_{\mathrm{ref}} \mu_{\mathrm{ref}}}}{\mu_{\mathrm{ref}}} E_{\mathrm{ref}} = \sqrt{\frac{\varepsilon_{\mathrm{ref}}}{\mu_{\mathrm{ref}}}} E_{\mathrm{ref}} = \frac{E_{\mathrm{ref}}}{Z_{\mathrm{ref}}}$$

$$J_{\mathrm{ex}} = J_{\mathrm{e\,ref}} \hat{J}_{\mathrm{ex}} \qquad J_{\mathrm{e\,ref}} = \frac{\varepsilon_{\mathrm{ref}}}{\Delta t_{\mathrm{ref}}} E_{\mathrm{ref}}$$

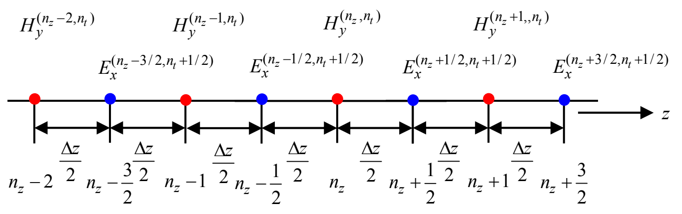
$$J_{\mathrm{mx}} = J_{\mathrm{m\,ref}} \hat{J}_{\mathrm{mx}} \qquad J_{\mathrm{m\,ref}} = \frac{\mu_{\mathrm{ref}}}{\Delta t_{\mathrm{ref}}} H_{\mathrm{ref}} = \frac{E_{\mathrm{ref}}}{\Delta t_{\mathrm{ref}} c_{\mathrm{ref}}}$$

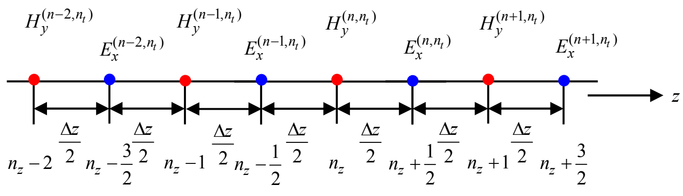
$$\widehat{H}_y^{(n_z,n_t)} = \widehat{H}_y^{(n_z,n_t-1)} - \widehat{\Delta t}\left[\widehat{E}_x^{(n_z+1/2,n_t-1/2)} - \widehat{E}_x^{(n_z-1/2,n_t-1/2)}\right] - \widehat{\Delta t}\,\widehat{J}_{\mathrm{my}}^{(n_z,n_t-1/2)}$$

$$\widehat{E}_x^{(n_z+1/2,n_t+1/2)} = \widehat{E}_x^{(n_z+1/2,n_t-1/2)} - \widehat{\Delta t}\left[\widehat{H}_y^{(n_z+1,n_t)} - \widehat{H}_y^{(n_z,n_t)}\right] - \widehat{\Delta t}\,\widehat{J}_{\mathrm{ex}}^{(n_z+1/2,n_t)}$$

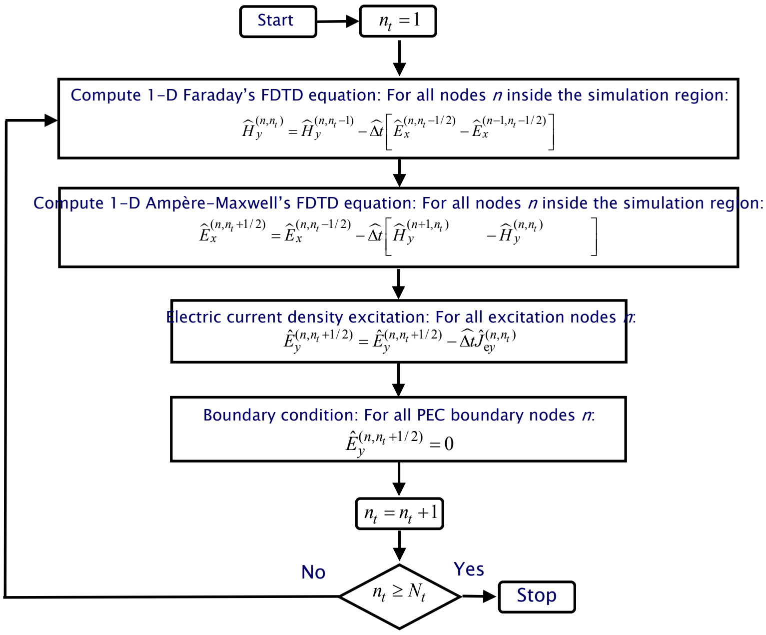


## 1-D FDTD – Staggered Grid in Space – Global Node Numbering / 1D-FDTD – Versetztes Gitter im Raum – Globale Knotennummerierung



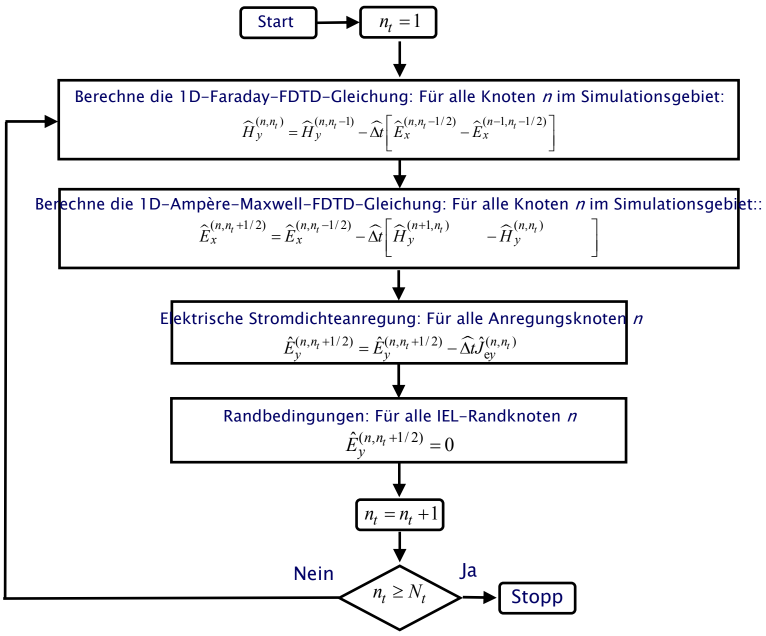


# 1-D FDTD Algorithm – Flow Chart / 1D-FDTD-Algorithmus – Flussdiagramm






# 1-D FDTD Algorithm – Flow Chart / 1D-FDTD-Algorithmus – Flussdiagramm



## FDTD Solution of the First Two 1-D Scalar Maxwell's Equations / FDTD-Lösung der ersten beiden 1D skalaren Maxwell-Gleichungen

**Maxwell's equations / Maxwellsche Gleichungen**

$$\frac{\partial}{\partial t} H_y(z,t) = -\frac{1}{\mu_0}\frac{\partial}{\partial z} E_x(z,t) - \frac{1}{\mu_0} J_{my}(z,t) \quad \text{for / für} \quad \begin{cases} 0 \le z \le Z \\ 0 \le t \le T \end{cases}$$

$$\frac{\partial}{\partial t} E_x(z,t) = -\frac{1}{\varepsilon_0}\frac{\partial}{\partial z} H_y(z,t) - \frac{1}{\varepsilon_0} J_{ex}(z,t)$$

**Initial condition / Anfangsbedingung**

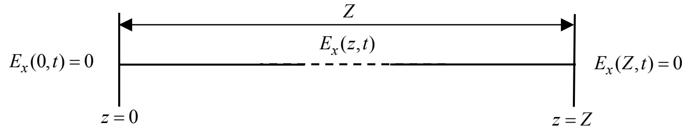
$$\begin{aligned} &H_y(z,t) = J_{my}(z,t) = 0 && t \le 0 \\ &E_x(z,t) = J_{ex}(z,t) = 0 && t \le 0 \\ &J_{ex}(z,t) = K_{e0}(z_0)\,\delta(z - z_0)\, f(t) && t > 0 \end{aligned}$$

**Causality / Kausalität**

**Boundary condition for a perfectly electrically conducting (PEC) material / Randbedingung für ein ideal elektrisch leitendes Material**

$$\left.\begin{aligned} E_x(0,t) = 0 \\ E_x(Z,t) = 0 \end{aligned}\right\} \forall t$$

**Hyperbolic initial-boundary-value problem / Hyperbolisches Anfangs-Randwert-Problem**






## FDTD Solution of the First Two 1-D Scalar Maxwell's Equations / FDTD-Lösung der ersten beiden 1D skalaren Maxwell-Gleichungen

**Discrete 1-D FDTD equations / Diskrete 1D-FDTD-Gleichungen**

$$\widehat{H}_y^{(n_z,n_t)} = \widehat{H}_y^{(n_z,n_t-1)} - \widehat{\Delta t}\left[\widehat{E}_x^{(n_z+1/2,n_t-1/2)} - \widehat{E}_x^{(n_z-1/2,n_t-1/2)}\right] - \widehat{\Delta t}\,\widehat{J}_{my}^{(n_z,n_t-1/2)} \quad \text{for / für} \quad \begin{cases} 1 \le n_z \le N_z \\ 1 \le n_t \le N_t \end{cases}$$

$$\widehat{E}_x^{(n_z+1/2,n_t+1/2)} = \widehat{E}_x^{(n_z+1/2,n_t-1/2)} - \widehat{\Delta t}\left[\widehat{H}_y^{(n_z+1,n_t)} - \widehat{H}_y^{(n_z,n_t)}\right] - \widehat{\Delta t}\,\widehat{J}_{ex}^{(n_z+1/2,n_t)}$$

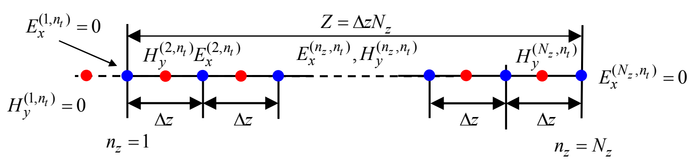
**Initial condition / Anfangsbedingung**

$$\begin{aligned} &H_y^{(n_z,n_t)} = J_{my}^{(n_z,n_t)} = 0 && n_t \le 1 \\ &E_x^{(n_z,n_t)} = J_{ex}^{(n_z,n_t)} = 0 && n_t \le 1 \\ &J_{ex}^{(n_z,n_t)} = K_{ex}^{(n_{z0})}\,\delta^{(n_z - n_{z0})}\, f^{(n_t)} && n_t > 1 \end{aligned}$$

**Causality / Kausalität**

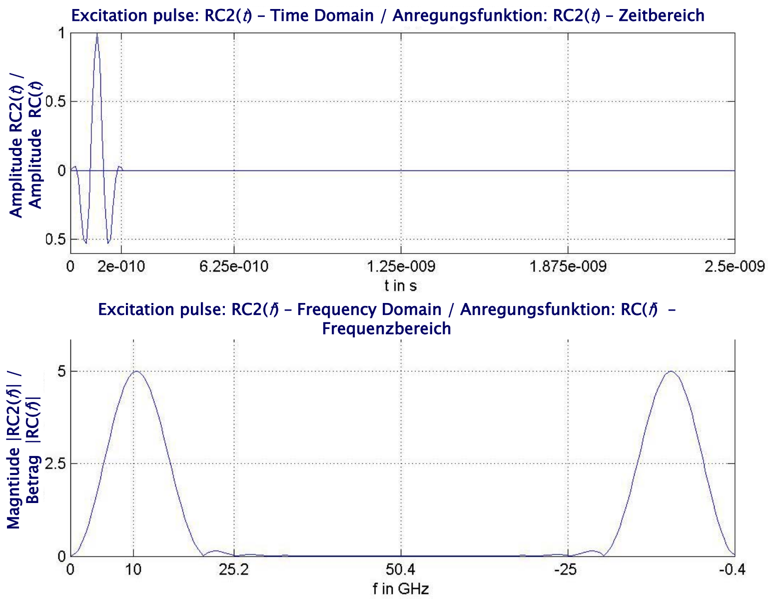
**Boundary condition for a perfectly electrically conducting (PEC) material / Randbedingung für ein ideal elektrisch leitendes Material**

$$\left.\begin{aligned} E_x^{(1,n_t)} = 0 \\ E_x^{(N_z,n_t)} = 0 \end{aligned}\right\} \quad 1 \le n_t \le N_t$$




**Discrete hyperbolic initial-boundary-value problem / Diskretes hyperbolisches Anfangs-Randwert-Problem**

## FDTD Solution of the First Two 1-D Scalar Maxwell's Equations / FDTD-Lösung der ersten beiden 1D skalaren Maxwell-Gleichungen



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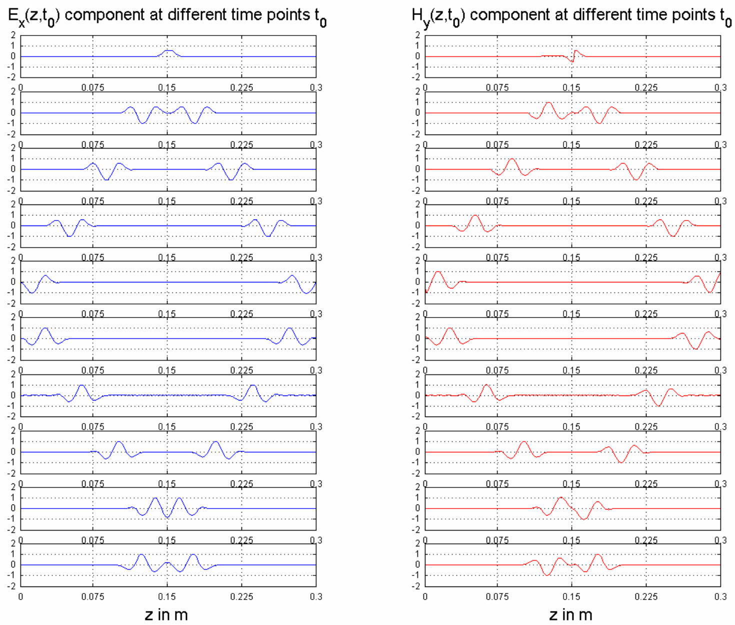
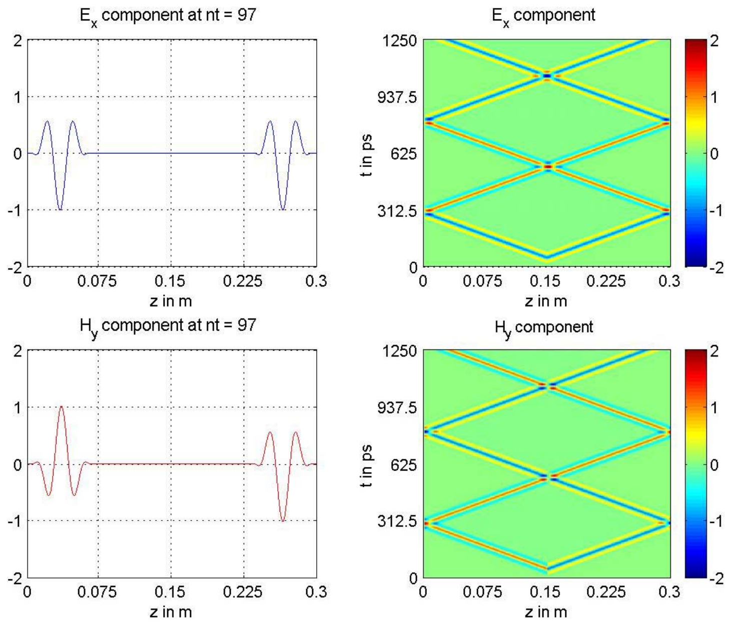


## FDTD Solution of the First Two 1-D Scalar Maxwell's Equations / FDTD-Lösung der ersten beiden 1D skalaren Maxwell-Gleichungen






## Implementation of Boundary Conditions / Implementierung von Randbedingungen

**Boundary condition for a perfectly electrically conducting (PEC) material / Randbedingung für ein ideal elektrisch leitendes Material**

$$\left.\begin{aligned} E_x^{(1,n_t)} &= 0 \\ E_x^{(N_z,n_t)} &= 0 \end{aligned}\right\} \quad 1 \le n_t \le N_t$$

**Absorbing/open boundary condition / Absorbierende/offene Randbedingung**

Space-time-extrapolation of the first order / Raum-Zeit-Extrapolation der ersten Ordnung

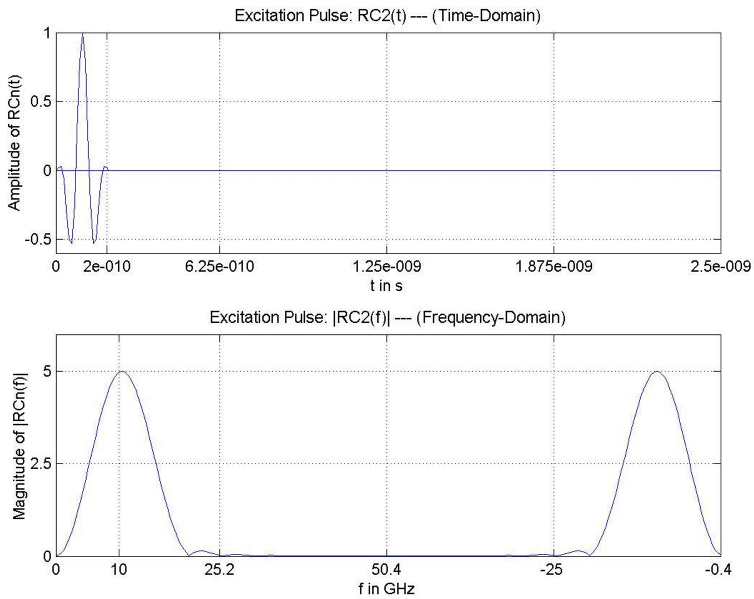
For / Für $\widehat{\Delta t} = 0.5$

a plane wave needs two time steps, 2 $n_t$, to travel over one grid cell with the size $\Delta z$ / braucht eine ebene Welle zwei Zeitschritte, 2 $n_t$, um sich über eine Gitterzelle der Größe $\Delta z$ auszubreiten

$$\left.\begin{aligned} E_x^{(1,n_t)} &= E_x^{(2,n_t-2)} \\ E_x^{(N_z,n_t)} &= E_x^{(N_z-1,n_t-2)} \end{aligned}\right\} \quad 1 \le n_t \le N_t$$

Space-time-extrapolation of the first order / Raum-Zeit-Extrapolation der ersten Ordnung

## FDTD Solution of the First Two 1-D Scalar Maxwell's Equations / FDTD-Lösung der ersten beiden 1D skalaren Maxwell-Gleichungen



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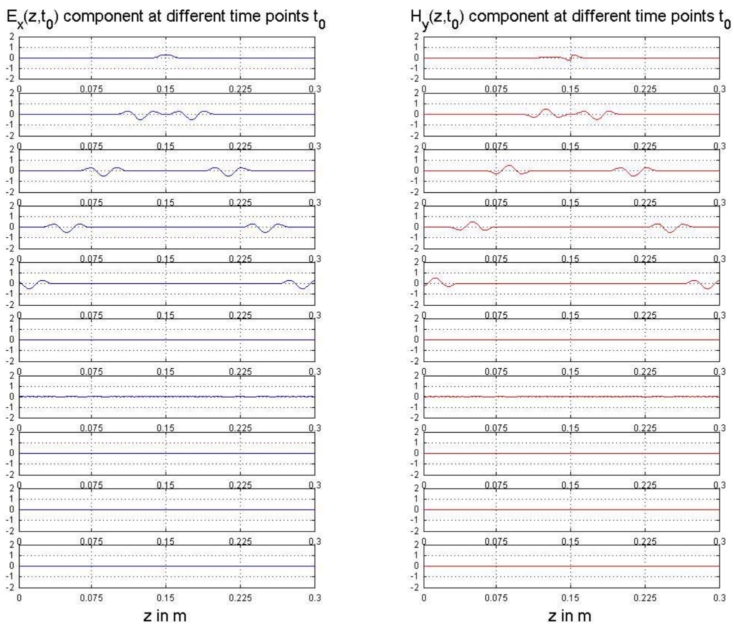

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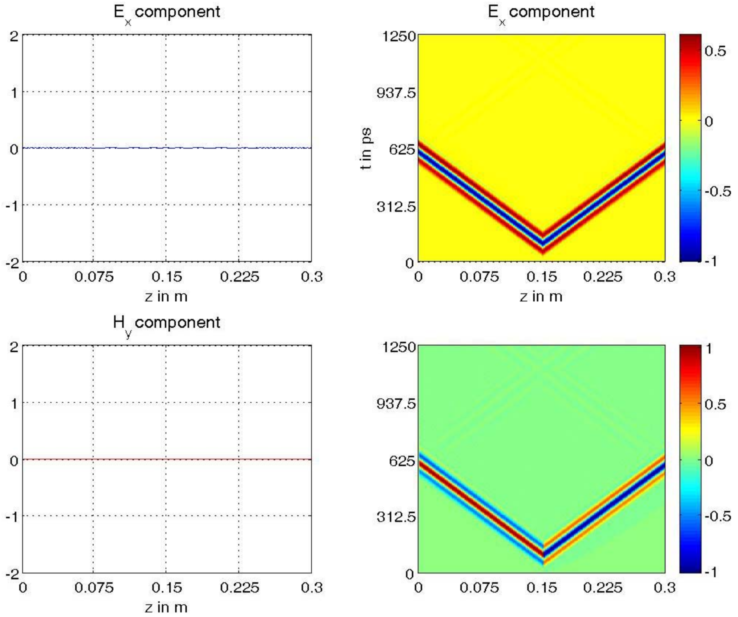
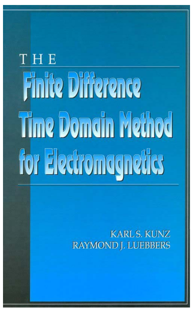
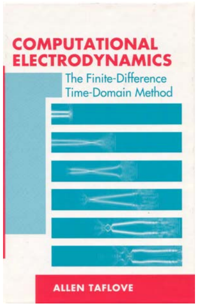




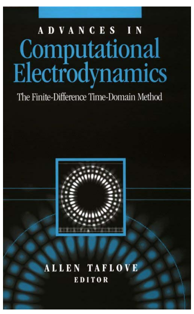

## FDTD Books / FDTD-Bücher



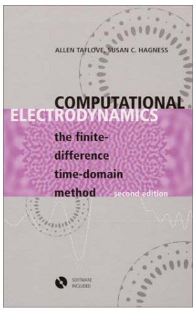

Kunz, K. S., Luebbers, R. J.: *The Finite Difference Time Domain Method for Electromagnetics.* 1993




Taflove, A. (Editor): *Computational Electrodynamics: The Finite-Difference Time-Domain Method.* Artech House, Boston, 1995.

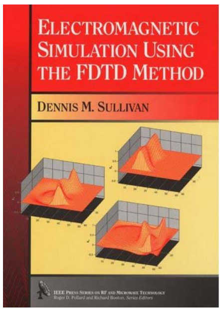



Taflove, A. (Editor): *Advances in Computational Electrodynamics: The Finite-Difference Time-Domain Method.* Artech House, 1998.




Taflove, A. (Editor): *Computational Electrodynamics: The Finite-Difference Time-Domain Method.* 2nd Editon, Artech House, Boston, 2000.

## FDTD Books / FDTD-Bücher




Sullivan, D. M.: *Electromagnetic Simulation Using the FDTD Method*. IEEE Press, New York, 2000.



# End of Lecture 7 / Ende der 7. Vorlesung