Numerical Methods of Electromagnetic Field Theory I (NFT I)
Numerische Methoden der Elektromagnetischen Feldtheorie I (NFT I) /

9th Lecture / 9. Vorlesung

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3-D FIT - Derivation of the Discrete Grid Equations / 3D-FIT - Ableitung der diskreten Gittergleichungen

Local grid equations in local notation /
Lokale Gittergleichungen in lokaler Notation

$$\frac{\mathrm{d}}{\mathrm{d}t}B_x^{(m)}(t)\Delta y\Delta z=-\left[E_y^{(u)}(t)\Delta y+E_z^{(f)}(t)\Delta z-E_y^{(d)}(t)\Delta y-E_z^{(b)}(t)\Delta z\right]-J_{\mathrm{m}x}^{(m)}(t)\Delta y\Delta z$$

$$\frac{\mathrm{d}}{\mathrm{d}t}B_y^{(m)}(t)\Delta x\Delta z=-\left[-E_x^{(u)}(t)\Delta x+E_z^{(l)}(t)\Delta z+E_x^{(d)}(t)\Delta x-E_z^{(r)}(t)\Delta z\right]-J_{\mathrm{m}y}^{(m)}(t)\Delta x\Delta z$$

$$\frac{\mathrm{d}}{\mathrm{d}t}B_z^{(m)}(t)\Delta x\Delta y=-\left[E_x^{(b)}(t)\Delta x+E_y^{(r)}(t)\Delta y-E_x^{(f)}(t)\Delta x-E_y^{(l)}(t)\Delta y\right]-J_{\mathrm{m}z}^{(m)}(t)\Delta x\Delta y$$

Local grid equations in global grid node notation /
Lokale Gittergleichungen in globaler Gitterknotennotation

$$\frac{\mathrm{d}}{\mathrm{d}t}B_x^{(n)}(t)\Delta y\Delta z=-\left\{\left[E_y^{(n-M_z)}(t)-E_y^{(n)}(t)\right]\Delta y+\left[E_z^{(n)}(t)-E_z^{(n-M_y)}(t)\right]\Delta z\right\}-J_{\mathrm{m}x}^{(n)}(t)\Delta y\Delta z$$

$$\frac{\mathrm{d}}{\mathrm{d}t}B_y^{(n)}(t)\Delta x\Delta z=-\left\{\left[E_x^{(n)}(t)-E_x^{(n-M_z)}(t)\right]\Delta x+\left[E_z^{(n-M_x)}(t)-E_z^{(n)}(t)\right]\Delta z\right\}-J_{\mathrm{m}y}^{(n)}(t)\Delta x\Delta z$$

$$\frac{\mathrm{d}}{\mathrm{d}t}B_z^{(n)}(t)\Delta x\Delta y=-\left\{\left[E_x^{(n-M_y)}(t)-E_x^{(n)}(t)\right]\Delta x+\left[E_y^{(n)}(t)-E_y^{(n-M_x)}(t)\right]\Delta y\right\}-J_{\mathrm{m}z}^{(n)}(t)\Delta x\Delta y$$

3-D FIT – Derivation of the Discrete Grid Equations / 3D-FIT – Ableitung der diskreten Gittergleichungen

Local grid equations in global grid node notation /
Lokale Gittergleichungen in globaler Gitterknotennotation

$$\frac{\mathrm{d}}{\mathrm{d}t} B_x^{(n)}(t)\Delta y\Delta z = -\left\{\left[E_y^{(n-M_z)}(t) - E_y^{(n)}(t)\right]\Delta y + \left[E_z^{(n)}(t) - E_z^{(n-M_y)}(t)\right]\Delta z\right\} - J_{\mathrm{m}x}^{(n)}(t)\Delta y\Delta z$$

$$\frac{\mathrm{d}}{\mathrm{d}t} B_y^{(n)}(t)\Delta x\Delta z = -\left\{\left[E_x^{(n)}(t) - E_x^{(n-M_z)}(t)\right]\Delta x + \left[E_z^{(n-M_x)}(t) - E_z^{(n)}(t)\right]\Delta z\right\} - J_{\mathrm{m}y}^{(n)}(t)\Delta x\Delta z$$

$$\frac{\mathrm{d}}{\mathrm{d}t} B_z^{(n)}(t)\Delta x\Delta y = -\left\{\left[E_x^{(n-M_y)}(t) - E_x^{(n)}(t)\right]\Delta x + \left[E_y^{(n)}(t) - E_y^{(n-M_x)}(t)\right]\Delta y\right\} - J_{\mathrm{m}z}^{(n)}(t)\Delta x\Delta y$$

Local spatial shift operators / Lokale räumliche Schiebeoperatoren

$$S_{\pm M_i} f^{(n)} = f^{(n\pm M_i)}$$

$$S_0 f^{(n)} = f^{(n)}$$

$$S_0 = I$$

$$I\, f^{(n)} = f^{(n)}$$

Local grid equations with local spatial shift operators in global grid node notation /
Lokale Gittergleichungen mit lokalen räumlichen Schiebeoperatoren in globaler Gitterknotennotation

$$\frac{\mathrm{d}}{\mathrm{d}t} B_x^{(n)}(t)\Delta y\Delta z = -\left\{\left[S_{-M_z} - I\right]E_y^{(n)}(t)\Delta y + \left[I - S_{M_y}\right]E_z^{(n)}(t)\Delta z\right\} - J_{\mathrm{m}x}^{(n)}(t)\Delta y\Delta z$$

$$\frac{\mathrm{d}}{\mathrm{d}t} B_y^{(n)}(t)\Delta x\Delta z = -\left\{\left[I - S_{-M_z}\right]E_x^{(n)}(t)\Delta x + \left[S_{-M_x} - I\right]E_z^{(n)}(t)\Delta z\right\} - J_{\mathrm{m}y}^{(n)}(t)\Delta x\Delta z$$

$$\frac{\mathrm{d}}{\mathrm{d}t} B_z^{(n)}(t)\Delta x\Delta y = -\left\{\left[S_{-M_y} - I\right]E_x^{(n)}(t)\Delta x + \left[I - S_{-M_x}\right]E_y^{(n)}(t)\Delta y\right\} - J_{\mathrm{m}z}^{(n)}(t)\Delta x\Delta y$$

3-D FIT – Local Spatial Shift Operators / 3D-FIT – Lokale räumliche Schiebeoperatoren

1. Simple spatial shift operation / Einfache räumliche Schiebeoperation

$$S_{\pm M_i} f^{(n)} = f^{(n\pm M_i)}$$

2. Identity operation / Identitätsoperation

$$I\, f^{(n)} = f^{(n)}$$

3. Multiple shift operations / Zusammengesetzte Schiebeoperationen

$$S_{\pm M_i} S_{\pm M_j} f^{(n)} = S_{\pm M_j} S_{\pm M_i} f^{(n)} = f^{(n\pm M_i \pm M_j)}$$

Special case for $M_j = -M_i$ / Speziell folgt für $M_j = -M_i$

$$S_{\pm M_i} S_{\mp M_i} = I$$

4. Local difference operator / Lokaler Differenzoperator

$$P_{\pm M_i} = \mp I \pm S_{\pm M_i}$$

5. Local averaging operator / Lokaler Mittelungsoperator

$$A_{\pm M_i} = \frac{1}{2}\left(I + S_{\pm M_i}\right)$$

3-D FIT – Derivation of the Discrete Grid Equations / 3D-FIT – Ableitung der diskreten Gittergleichungen

Local grid equations with local spatial shift operators in global grid node notation /
Lokale Gittergleichungen mit lokalen räumlichen Schiebeoperatoren in globaler Gitterknotennotation

$$\frac{\mathrm{d}}{\mathrm{d}t} B_x^{(n)}(t)\Delta y\Delta z = -\left\{\left[S_{-M_z} - I\right] E_y^{(n)}(t)\Delta y + \left[I - S_{M_y}\right] E_z^{(n)}(t)\Delta z\right\} - J_{\mathrm{m}x}^{(n)}(t)\Delta y\Delta z$$

$$\frac{\mathrm{d}}{\mathrm{d}t} B_y^{(n)}(t)\Delta x\Delta z = -\left\{\left[I - S_{-M_z}\right] E_x^{(n)}(t)\Delta x + \left[S_{-M_x} - I\right] E_z^{(n)}(t)\Delta z\right\} - J_{\mathrm{m}y}^{(n)}(t)\Delta x\Delta z$$

$$\frac{\mathrm{d}}{\mathrm{d}t} B_z^{(n)}(t)\Delta x\Delta y = -\left\{\left[S_{-M_y} - I\right] E_x^{(n)}(t)\Delta x + \left[I - S_{-M_x}\right] E_y^{(n)}(t)\Delta y\right\} - J_{\mathrm{m}z}^{(n)}(t)\Delta x\Delta y$$

... in local matrix form / ... in lokaler Matrixform

$$\underbrace{\begin{bmatrix} \Delta y\Delta z & & \\ & \Delta x\Delta z & \\ & & \Delta x\Delta y \end{bmatrix}}_{=[S]} \frac{\mathrm{d}}{\mathrm{d}t} \underbrace{\begin{Bmatrix} B_x^{(n)}(t) \\ B_y^{(n)}(t) \\ B_z^{(n)}(t) \end{Bmatrix}^{(n)}}_{=\{B\}^{(n)}(t)} = -\underbrace{\begin{bmatrix} 0 & S_{-M_z} - I & I - S_{M_y} \\ I - S_{-M_z} & 0 & S_{-M_x} - I \\ S_{-M_y} - I & I - S_{-M_x} & 0 \end{bmatrix}}_{=[\mathrm{curl}]} \underbrace{\begin{bmatrix} \Delta x & & \\ & \Delta y & \\ & & \Delta z \end{bmatrix}}_{=[R]} \underbrace{\begin{Bmatrix} E_x^{(n)}(t) \\ E_y^{(n)}(t) \\ E_z^{(n)}(t) \end{Bmatrix}^{(n)}}_{=\{E\}^{(n)}(t)}$$

$$-\underbrace{\begin{bmatrix} \Delta y\Delta z & & \\ & \Delta x\Delta z & \\ & & \Delta x\Delta y \end{bmatrix}}_{=[S]} \underbrace{\begin{Bmatrix} J_{\mathrm{m}x}^{(n)}(t) \\ J_{\mathrm{m}y}^{(n)}(t) \\ J_{\mathrm{m}z}^{(n)}(t) \end{Bmatrix}^{(n)}}_{=\{J_\mathrm{m}\}^{(n)}(t)}$$

3-D FIT – ... Discrete Grid Equations in Local Matrix Form / 3D-FIT – ... diskreten Gittergleichungen in lokaler Matrixform

$$\underbrace{\begin{bmatrix} \Delta y\Delta z & & \\ & \Delta x\Delta z & \\ & & \Delta x\Delta y \end{bmatrix}}_{=[S]} \frac{\mathrm{d}}{\mathrm{d}t} \underbrace{\begin{Bmatrix} B_x^{(n)}(t) \\ B_y^{(n)}(t) \\ B_z^{(n)}(t) \end{Bmatrix}^{(n)}}_{=\{B\}^{(n)}(t)} = -\underbrace{\begin{bmatrix} 0 & S_{-M_z} - I & I - S_{M_y} \\ I - S_{-M_z} & 0 & S_{-M_x} - I \\ S_{-M_y} - I & I - S_{-M_x} & 0 \end{bmatrix}}_{=[\mathrm{curl}]} \underbrace{\begin{bmatrix} \Delta x & & \\ & \Delta y & \\ & & \Delta z \end{bmatrix}}_{=[R]} \underbrace{\begin{Bmatrix} E_x^{(n)}(t) \\ E_y^{(n)}(t) \\ E_z^{(n)}(t) \end{Bmatrix}^{(n)}}_{=\{E\}^{(n)}(t)}$$

$$-\underbrace{\begin{bmatrix} \Delta y\Delta z & & \\ & \Delta x\Delta z & \\ & & \Delta x\Delta y \end{bmatrix}}_{=[S]} \underbrace{\begin{Bmatrix} J_{\mathrm{m}x}^{(n)}(t) \\ J_{\mathrm{m}x}^{(n)}(t) \\ J_{\mathrm{m}x}^{(n)}(t) \end{Bmatrix}^{(n)}}_{=\{J_\mathrm{m}\}^{(n)}(t)}$$

$$\begin{bmatrix} 0 & S_{-M_z} - I & I - S_{-M_y} \\ I - S_{-M_z} & 0 & S_{-M_x} - I \\ S_{-M_y} - I & I - S_{-M_x} & 0 \end{bmatrix} = \begin{bmatrix} 0 & -P_{-M_z} & P_{-M_y} \\ P_{-M_z} & 0 & -P_{-M_x} \\ -P_{-M_y} & P_{-M_x} & 0 \end{bmatrix} = [\mathrm{curl}]$$

3-D FIT – … Discrete Grid Equations in Local Matrix Form / 3D-FIT – … diskreten Gittergleichungen in lokaler Matrixform

Faraday's induction law in local matrix form / Faradaysches Induktionsgesetz in lokaler Matrixform

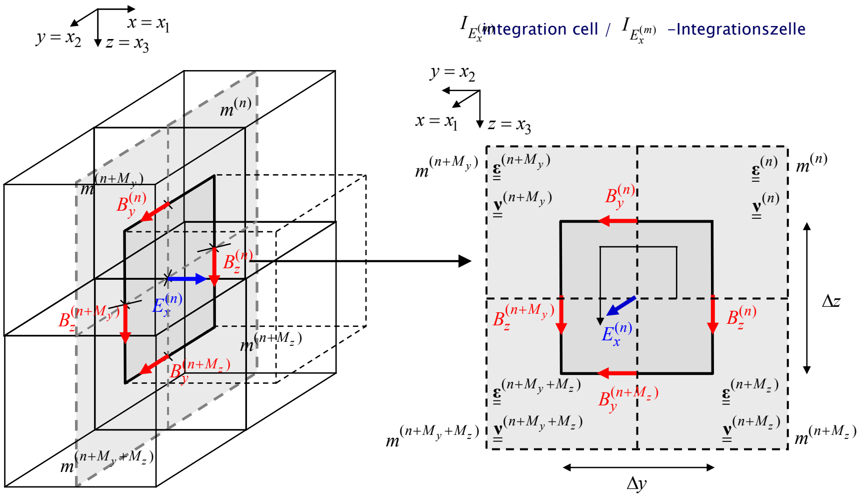
$$\underbrace{\begin{bmatrix}\Delta y\Delta z & & \\ & \Delta x\Delta z & \\ & & \Delta x\Delta y\end{bmatrix}}_{=[S]}\frac{\mathrm{d}}{\mathrm{d}t}\underbrace{\begin{Bmatrix}B_x^{(n)}(t)\\ B_y^{(n)}(t)\\ B_z^{(n)}(t)\end{Bmatrix}^{(n)}}_{=\{B\}^{(n)}(t)} = -\underbrace{\begin{bmatrix}0 & -P_{-M_z} & P_{-M_y}\\ P_{-M_z} & 0 & -P_{-M_x}\\ -P_{-M_y} & P_{-M_x} & 0\end{bmatrix}}_{=[\mathrm{curl}]}\underbrace{\begin{bmatrix}\Delta x & & \\ & \Delta y & \\ & & \Delta z\end{bmatrix}}_{=[R]}\underbrace{\begin{Bmatrix}E_x^{(n)}(t)\\ E_y^{(n)}(t)\\ E_z^{(n)}(t)\end{Bmatrix}^{(n)}}_{=\{E\}^{(n)}(t)} - \underbrace{\begin{bmatrix}\Delta y\Delta z & & \\ & \Delta x\Delta z & \\ & & \Delta x\Delta y\end{bmatrix}}_{=[S]}\underbrace{\begin{Bmatrix}J_{\mathrm{mx}}^{(n)}(t)\\ J_{\mathrm{mx}}^{(n)}(t)\\ J_{\mathrm{mx}}^{(n)}(t)\end{Bmatrix}^{(n)}}_{=\{J_\mathrm{m}\}^{(n)}(t)}$$

$$[S]\frac{\mathrm{d}}{\mathrm{d}t}\{B\}^{(n)}(t) = -[\mathrm{curl}][R]\{E\}^{(n)}(t) - [S]\{J_\mathrm{m}\}^{(n)}(t)$$

$[S]$	$\in \mathbb{R}^{3\times 3}$	Diagonal matrix of elementary surfaces on the grid G / Diagonalmatrix der Elementarflächen auf dem Gitter G
$\{B\}^{(n)}(t)$	$\in \mathbb{R}^{3}$	Algebraic magnetic flux density vector / Algebraischer magnetischer Flussdichtevektor
$[\mathrm{curl}]$	$\in \mathbb{R}^{3\times 3}$	Topological curl operator in matrix form on the grid G / Topologischer Rotationsoperator in Matrixform auf dem Gitter G
$[R]$	$\in \mathbb{R}^{3\times 3}$	Diagonal matrix of elementary lines on the grid G / Diagonalmatrix der Elementarstrecken auf dem Gitter G
$\{E\}^{(n)}(t)$	$\in \mathbb{R}^{3}$	Algebraic electric field strength vector / Algebraischer elektrische Feldstärkevektor
$\{J_\mathrm{m}\}^{(n)}(t)$	$\in \mathbb{R}^{3}$	Algebraic magnetic current density vector / Algebraischer magnetischer Stromdichtevektor

3-D FIT – Derivation of the Discrete Grid Equations / 3D-FIT – Ableitung der diskreten Gittergleichungen

$$\frac{\mathrm{d}}{\mathrm{d}t}\iint_S \left[\underline{\underline{\boldsymbol{\varepsilon}}}(\mathbf{R})\cdot\mathbf{E}(\mathbf{R},t)\right]\cdot\mathbf{dS} = \oint_{C=\partial S}\left[\underline{\underline{\boldsymbol{\nu}}}(\mathbf{R})\cdot\mathbf{B}(\mathbf{R},t)\right]\cdot\mathbf{dR} - \iint_S \mathbf{J}_\mathrm{e}(\mathbf{R},t)\cdot\mathbf{dS}$$

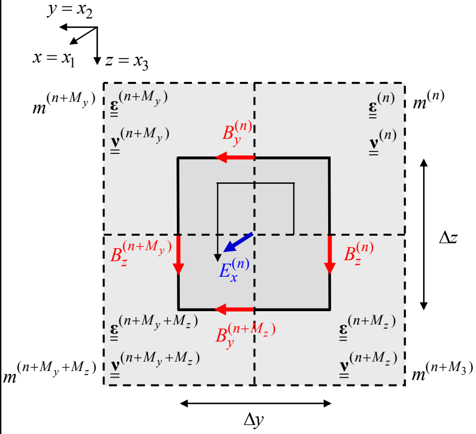


3-D FIT – Derivation of the Discrete Grid Equations / 3D-FIT – Ableitung der diskreten Gittergleichungen

$$\frac{\mathrm{d}}{\mathrm{d}t}\iint_S \left[\underline{\underline{\boldsymbol{\varepsilon}}}(\mathbf{R})\cdot\mathbf{E}(\mathbf{R},t)\right]\cdot\mathbf{dS} = \oint_{C=\partial S}\left[\underline{\underline{\boldsymbol{\nu}}}(\mathbf{R})\cdot\mathbf{B}(\mathbf{R},t)\right]\cdot\mathbf{dR} - \iint_S \mathbf{J}_\mathrm{e}(\mathbf{R},t)\cdot\mathbf{dS}$$

$$\iint_S \left[\underline{\underline{\boldsymbol{\varepsilon}}}(\mathbf{R})\cdot\mathbf{E}(\mathbf{R},t)\right]\cdot\mathbf{dS} = \iint_S \underline{\mathbf{e}}_x\cdot\left[\underline{\underline{\boldsymbol{\varepsilon}}}(\mathbf{R})\cdot\mathbf{E}(\mathbf{R},t)\right]\mathrm{d}S$$
$$= \iint_S \varepsilon_{xx}(\mathbf{R})\,E_x(\mathbf{R},t)\,\mathrm{d}S$$
$$= E_x^{(n)}(t)\iint_S \varepsilon_{xx}(\mathbf{R})\,\mathrm{d}S + \mathrm{O}\left[(\Delta y)^3\Delta z + \Delta y(\Delta z)^3\right]$$

$I_{E_x^{(n)}}$-integration cell / $I_{E_x^{(m)}}$ -Integrationszelle



$$\iint_S \varepsilon_{xx}(\mathbf{R})\,\mathrm{d}S = \underbrace{\frac{1}{4}\left[\varepsilon_{xx}^{(n)} + \varepsilon_{xx}^{(n+M_y)} + \varepsilon_{xx}^{(n+M_z)} + \varepsilon_{xx}^{(n+M_y+M_z)}\right]}_{=\tilde{\varepsilon}_{xx}^{(n)}}\Delta y\Delta z = \tilde{\varepsilon}_{xx}^{(n)}\Delta y\Delta z$$

$$\iint_S \left[\underline{\underline{\boldsymbol{\varepsilon}}}(\mathbf{R})\cdot\mathbf{E}(\mathbf{R},t)\right]\cdot\mathbf{dS} = E_x^{(n)}(t)\tilde{\varepsilon}_{xx}^{(n)}\Delta y\Delta z + \mathrm{O}\left[(\Delta y)^3\Delta z + \Delta y(\Delta z)^3\right]$$

$$\iint_S \mathbf{J}_\mathrm{e}(\mathbf{R},t)\cdot\mathbf{dS} = J_{\mathrm{e}x}^{(n)}(t)\Delta y\Delta z + \mathrm{O}\left[(\Delta y)^3\Delta z + \Delta y(\Delta z)^3\right]$$

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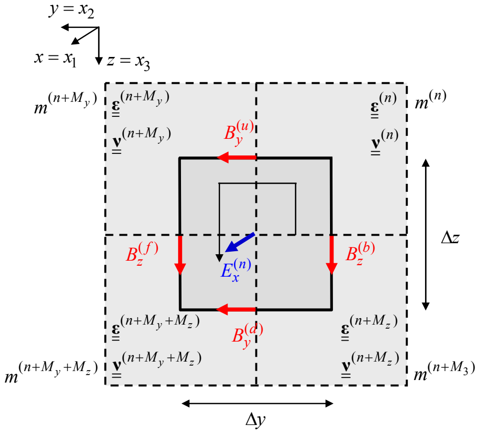
3-D FIT – Derivation of the Discrete Grid Equations / 3D-FIT – Ableitung der diskreten Gittergleichungen

$$\frac{\mathrm{d}}{\mathrm{d}t}\iint_S \left[\underline{\underline{\boldsymbol{\varepsilon}}}(\mathbf{R})\cdot\mathbf{E}(\mathbf{R},t)\right]\cdot\mathbf{dS} = \oint_{C=\partial S}\left[\underline{\underline{\boldsymbol{\nu}}}(\mathbf{R})\cdot\mathbf{B}(\mathbf{R},t)\right]\cdot\mathbf{dR} - \iint_S \mathbf{J}_\mathrm{e}(\mathbf{R},t)\cdot\mathbf{dS}$$

$$\oint_{C=\partial S}\left[\underline{\underline{\boldsymbol{\nu}}}(\mathbf{R})\cdot\mathbf{B}(\mathbf{R},t)\right]\cdot\mathbf{dR} = ?$$

$$\mathbf{dS} = \underline{\mathbf{n}}\,\mathrm{d}S = \underline{\mathbf{e}}_x\,\mathrm{d}y\,\mathrm{d}z$$
$$\mathbf{dR}_y = \underline{\mathbf{s}}\,\mathrm{d}R = \underline{\mathbf{e}}_y\,\mathrm{d}y$$
$$\mathbf{dR}_z = \underline{\mathbf{s}}\,\mathrm{d}R = \underline{\mathbf{e}}_z\,\mathrm{d}z$$

$I_{E_x^{(n)}}$-integration cell / $I_{E_x^{(m)}}$ -Integrationszelle



$$\oint_{C=\partial S}\left[\underline{\underline{\boldsymbol{\nu}}}(\mathbf{R})\cdot\mathbf{B}(\mathbf{R},t)\right]\cdot\mathbf{dR} = \int_{C^{(u)}}\left[\underline{\underline{\boldsymbol{\nu}}}(\mathbf{R})\cdot\mathbf{B}(\mathbf{R},t)\right]\cdot\mathbf{dR} + \int_{C^{(f)}}\left[\underline{\underline{\boldsymbol{\nu}}}(\mathbf{R})\cdot\mathbf{B}(\mathbf{R},t)\right]\cdot\mathbf{dR} + \int_{C^{(d)}}\left[\underline{\underline{\boldsymbol{\nu}}}(\mathbf{R})\cdot\mathbf{B}(\mathbf{R},t)\right]\cdot\mathbf{dR} + \int_{C^{(b)}}\left[\underline{\underline{\boldsymbol{\nu}}}(\mathbf{R})\cdot\mathbf{B}(\mathbf{R},t)\right]\cdot\mathbf{dR}$$

$$\oint_{C=\partial S}\left[\underline{\underline{\boldsymbol{\nu}}}(\mathbf{R})\cdot\mathbf{B}(\mathbf{R},t)\right]\cdot\mathbf{dR} = \int_{C^{(u)}}\nu_{yy}(\mathbf{R})B_y(\mathbf{R},t)\,\mathrm{d}y + \int_{C^{(f)}}\nu_{zz}(\mathbf{R})B_z(\mathbf{R},t)\,\mathrm{d}z - \int_{C^{(d)}}\nu_{yy}(\mathbf{R})B_y(\mathbf{R},t)\,\mathrm{d}y - \int_{C^{(b)}}\nu_{zz}(\mathbf{R})B_z(\mathbf{R},t)\,\mathrm{d}z$$

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3-D FIT - Derivation of the Discrete Grid Equations / 3D-FIT - Ableitung der diskreten Gittergleichungen

$$\oint_{C=\partial S}\left[\underline{\underline{\mathbf{v}}}(\mathbf{R})\bullet\mathbf{B}(\mathbf{R},t)\right]\bullet\mathbf{dR}$$
$$=\int_{C^{(u)}}\nu_{yy}(\mathbf{R})B_y(\mathbf{R},t)\,\mathrm{d}y+\int_{C^{(f)}}\nu_{zz}(\mathbf{R})B_z(\mathbf{R},t)\,\mathrm{d}z-\int_{C^{(d)}}\nu_{yy}(\mathbf{R})B_y(\mathbf{R},t)\,\mathrm{d}y-\int_{C^{(b)}}\nu_{zz}(\mathbf{R})B_z(\mathbf{R},t)\,\mathrm{d}z$$

$$\int_{C^{(u)}}\nu_{yy}(\mathbf{R})B_y(\mathbf{R},t)\,\mathrm{d}y=B_y^{(u)}(t)\int_{C^{(u)}}\nu_{yy}(\mathbf{R})\,\mathrm{d}y+O\left[(\Delta y)^3\right]\qquad \int_{C^{(u)}}\nu_{yy}(\mathbf{R})\,\mathrm{d}y=\frac{1}{2}\left[\nu_{yy}^{(n)}+\nu_{yy}^{(n+M_y)}\right]\Delta y$$

$$\int_{C^{(f)}}\nu_{zz}(\mathbf{R})B_z(\mathbf{R},t)\,\mathrm{d}z=B_z^{(f)}(t)\int_{C^{(f)}}\nu_{zz}(\mathbf{R})\,\mathrm{d}z+O\left[(\Delta z)^3\right]\qquad \int_{C^{(f)}}\nu_{zz}(\mathbf{R})\,\mathrm{d}z=\frac{1}{2}\left[\nu_{yy}^{(n+M_z)}+\nu_{yy}^{(n+M_y+M_z)}\right]\Delta z$$

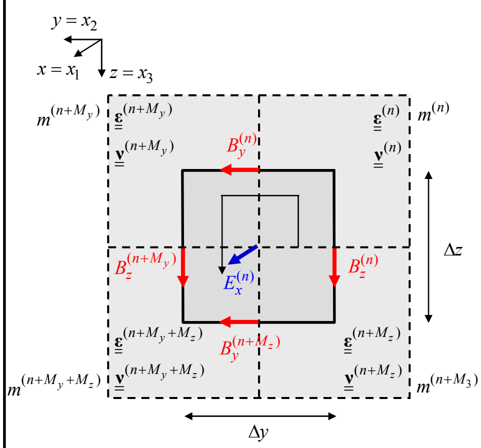
$$\int_{C^{(d)}}\nu_{yy}(\mathbf{R})B_y(\mathbf{R},t)\,\mathrm{d}y=B_y^{(d)}(t)\int_{C^{(d)}}\nu_{yy}(\mathbf{R})\,\mathrm{d}y+O\left[(\Delta y)^3\right]\qquad \int_{C^{(d)}}\nu_{yy}(\mathbf{R})\,\mathrm{d}y=\frac{1}{2}\left[\nu_{zz}^{(n+M_y)}+\nu_{zz}^{(n+M_y+M_z)}\right]\Delta y$$

$$\int_{C^{(b)}}\nu_{zz}(\mathbf{R})B_z(\mathbf{R},t)\,\mathrm{d}z=B_z^{(b)}(t)\int_{C^{(b)}}\nu_{zz}(\mathbf{R})\,\mathrm{d}z+O\left[(\Delta z)^3\right]\qquad \int_{C^{(b)}}\nu_{zz}(\mathbf{R})\,\mathrm{d}z=\frac{1}{2}\left[\nu_{zz}^{(n)}+\nu_{zz}^{(n+M_z)}\right]\Delta z$$

3-D FIT - Derivation of the Discrete Grid Equations / 3D-FIT - Ableitung der diskreten Gittergleichungen

$$\frac{\mathrm{d}}{\mathrm{d}t}\iint_S\left[\underline{\underline{\boldsymbol{\varepsilon}}}(\mathbf{R})\bullet\mathbf{E}(\mathbf{R},t)\right]\bullet\mathbf{dS}=\oint_{C=\partial S}\left[\underline{\underline{\mathbf{v}}}(\mathbf{R})\bullet\mathbf{B}(\mathbf{R},t)\right]\bullet\mathbf{dR}-\iint_S\mathbf{J}_{\mathrm{e}}(\mathbf{R},t)\bullet\mathbf{dS}$$

$I_{E_x^{(m)}}$ -integration cell / $I_{E_x^{(m)}}$ -Integrationszelle



$$\oint_{C=\partial S}\left[\underline{\underline{\mathbf{v}}}(\mathbf{R})\bullet\mathbf{B}(\mathbf{R},t)\right]\bullet\mathbf{dR}$$
$$=\underbrace{\frac{1}{2}\left[\nu_{yy}^{(n)}+\nu_{yy}^{(n+M_y)}\right]}_{=\tilde{\nu}_{yy}^{(n)}}B_y^{(n)}(t)\Delta y$$
$$-\underbrace{\frac{1}{2}\left[\nu_{yy}^{(n+M_z)}+\nu_{yy}^{(n+M_y+M_z)}\right]}_{=\tilde{\nu}_{yy}^{(n+M_y)}}B_y^{(n+M_z)}(t)\Delta y$$
$$+\underbrace{\frac{1}{2}\left[\nu_{zz}^{(n+M_y)}+\nu_{zz}^{(n+M_y+M_z)}\right]}_{=\tilde{\nu}_{zz}^{(n+M_z)}}B_z^{(n+M_y)}(t)\Delta z$$
$$-\underbrace{\frac{1}{2}\left[\nu_{zz}^{(n)}+\nu_{zz}^{(n+M_z)}\right]}_{=\tilde{\nu}_{zz}^{(n)}}B_z^{(n)}(t)\Delta z$$
$$=\tilde{\nu}_{yy}^{(n)}B_y^{(n)}(t)\Delta y-\tilde{\nu}_{yy}^{(n+M_y)}B_y^{(n+M_y)}(t)\Delta y$$
$$+\tilde{\nu}_{zz}^{(n+M_z)}B_z^{(n+M_y)}(t)\Delta z-\tilde{\nu}_{zz}^{(n)}B_z^{(n)}(t)\Delta z$$

3-D FIT – Derivation of the Discrete Grid Equations / 3D-FIT – Ableitung der diskreten Gittergleichungen

$$\frac{\mathrm{d}}{\mathrm{d}t}\iint_S \left[\underline{\underline{\boldsymbol{\varepsilon}}}(\mathbf{R})\bullet\mathbf{E}(\mathbf{R},t)\right]\bullet\mathbf{dS} = \oint_{C=\partial S}\left[\underline{\underline{\boldsymbol{\nu}}}(\mathbf{R})\bullet\mathbf{B}(\mathbf{R},t)\right]\bullet\mathbf{dR} - \iint_S \mathbf{J}_{\mathrm{e}}(\mathbf{R},t)\bullet\mathbf{dS}$$

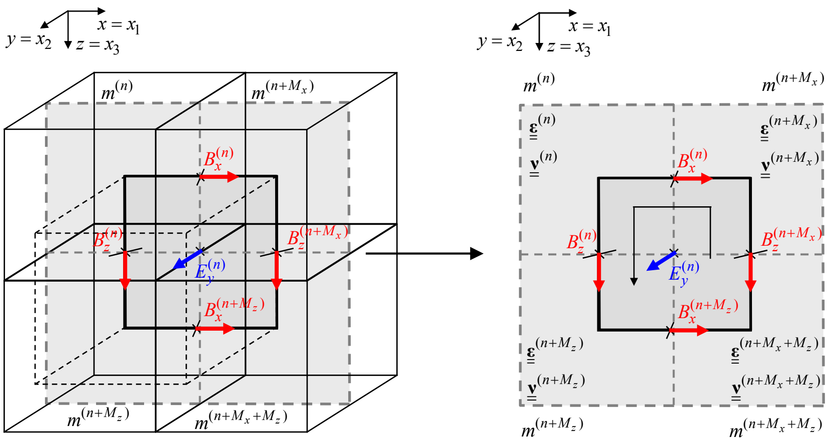
$I_{E_x^{(m)}}$ integration cell / $I_{E_x^{(m)}}$ –Integrationszelle

$$\tilde{\varepsilon}_{xx}^{(n)}\frac{\mathrm{d}}{\mathrm{d}t}E_x^{(n)}(t)\Delta y\Delta z = \tilde{\nu}_{yy}^{(n)}B_y^{(n)}(t)\Delta y - \tilde{\nu}_{yy}^{(n+M_z)}B_y^{(n+M_z)}(t)\Delta y$$
$$+\tilde{\nu}_{zz}^{(n+M_y)}B_z^{(n+M_y)}(t)\Delta z - \tilde{\nu}_{zz}^{(n)}B_z^{(n)}(t)\Delta z - J_{\mathrm{ex}}^{(n)}(t)\Delta y\Delta z$$
$$=\left(I - S_{M_z}\right)\tilde{\nu}_{yy}^{(n)}B_y^{(n)}(t)\Delta y + \left(S_{M_y} - I\right)\tilde{\nu}_{zz}^{(n)}B_z^{(n)}(t)\Delta z - J_{\mathrm{ex}}^{(n)}(t)\Delta y\Delta z$$

3-D FIT – Derivation of the Discrete Grid Equations / 3D-FIT – Ableitung der diskreten Gittergleichungen

$$\frac{\mathrm{d}}{\mathrm{d}t}\iint_S \left[\underline{\underline{\boldsymbol{\varepsilon}}}(\mathbf{R})\bullet\mathbf{E}(\mathbf{R},t)\right]\bullet\mathbf{dS} = \oint_{C=\partial S}\left[\underline{\underline{\boldsymbol{\nu}}}(\mathbf{R})\bullet\mathbf{B}(\mathbf{R},t)\right]\bullet\mathbf{dR} - \iint_S \mathbf{J}_{\mathrm{e}}(\mathbf{R},t)\bullet\mathbf{dS}$$

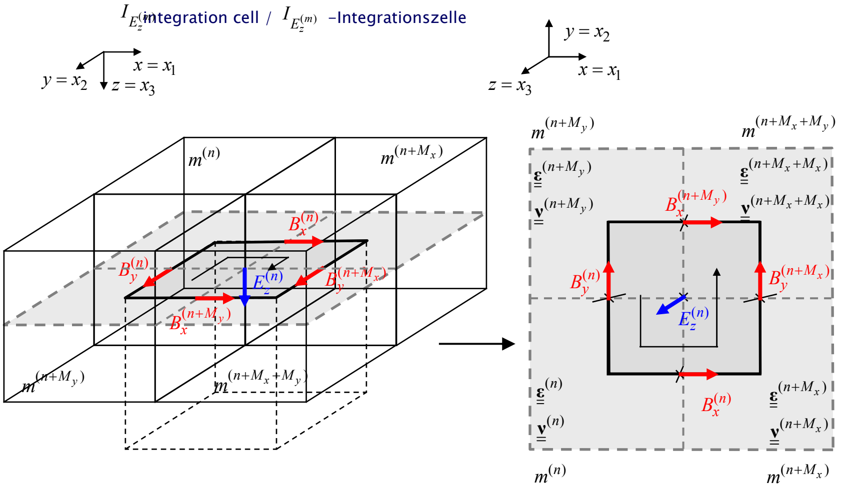
$I_{E_y^{(m)}}$ integration cell / $I_{E_y^{(m)}}$ –Integrationszelle



3-D FIT – Derivation of the Discrete Grid Equations / 3D-FIT – Ableitung der diskreten Gittergleichungen

$$\frac{\mathrm{d}}{\mathrm{d}t}\iint_S \left[\underline{\underline{\varepsilon}}(\mathbf{R})\cdot\mathbf{E}(\mathbf{R},t)\right]\cdot\mathbf{dS} = \oint_{C=\partial S}\left[\underline{\underline{\nu}}(\mathbf{R})\cdot\mathbf{B}(\mathbf{R},t)\right]\cdot\mathbf{dR} - \iint_S \mathbf{J}_\mathrm{e}(\mathbf{R},t)\cdot\mathbf{dS}$$

$I_{E_z^{(n)}}$ -integration cell / $I_{E_z^{(m)}}$ -Integrationszelle



3-D FIT – Derivation of the Discrete Grid Equations / 3D-FIT – Ableitung der diskreten Gittergleichungen

$$\frac{\mathrm{d}}{\mathrm{d}t}\iint_S \left[\underline{\underline{\varepsilon}}(\mathbf{R})\cdot\mathbf{E}(\mathbf{R},t)\right]\cdot\mathbf{dS} = \oint_{C=\partial S}\left[\underline{\underline{\nu}}(\mathbf{R})\cdot\mathbf{B}(\mathbf{R},t)\right]\cdot\mathbf{dR} - \iint_S \mathbf{J}_\mathrm{e}(\mathbf{R},t)\cdot\mathbf{dS}$$

$$\Downarrow$$

$$\begin{aligned}
\tilde{\varepsilon}_{xx}^{(n)}\frac{\mathrm{d}}{\mathrm{d}t}E_x^{(n)}(t)\Delta y\Delta z &= \tilde{\nu}_{yy}^{(n)}B_y^{(n)}(t)\Delta y - \tilde{\nu}_{yy}^{(n+M_z)}B_y^{(n+M_z)}(t)\Delta y \\
&\quad + \tilde{\nu}_{zz}^{(n+M_y)}B_z^{(n+M_y)}(t)\Delta z - \tilde{\nu}_{zz}^{(n)}B_z^{(n)}(t)\Delta z - J_{\mathrm{ex}}^{(n)}(t)\Delta y\Delta z \\
&= \left(I - S_{M_z}\right)\tilde{\nu}_{yy}^{(n)}B_y^{(n)}(t)\Delta y + \left(S_{M_y} - I\right)\tilde{\nu}_{zz}^{(n)}B_z^{(n)}(t)\Delta z - J_{\mathrm{ex}}^{(n)}(t)\Delta y\Delta z \\
\tilde{\varepsilon}_{yy}^{(n)}\frac{\mathrm{d}}{\mathrm{d}t}E_y^{(n)}(t)\Delta x\Delta z &= \tilde{\nu}_{xx}^{(n+M_3)}B_x^{(n+M_3)}(t)\Delta x - \tilde{\nu}_{xx}^{(n_z)}B_x^{(n)}(t)\Delta x \\
&\quad + \tilde{\nu}_{zz}^{(n)}B_z^{(n)}(t)\Delta z - \tilde{\nu}_{zz}^{(n+M_x)}B_z^{(n+M_x)}(t)\Delta z - J_{\mathrm{ey}}^{(n)}(t)\Delta x\Delta z \\
&= \left(S_{M_z} - I\right)\tilde{\nu}_{xx}^{(n)}B_x^{(n)}(t)\Delta x + \left(I - S_{M_x}\right)\tilde{\nu}_{zz}^{(n)}B_z^{(n)}(t)\Delta z - J_{\mathrm{ey}}^{(n)}(t)\Delta y\Delta z \\
\tilde{\varepsilon}_{zz}^{(n)}\frac{\mathrm{d}}{\mathrm{d}t}E_z^{(n)}(t)\Delta x\Delta y &= \tilde{\nu}_{xx}^{(n)}B_x^{(n)}(t)\Delta x - \tilde{\nu}_{xx}^{(n+M_y)}B_x^{(n+M_y)}(t)\Delta x \\
&\quad + \tilde{\nu}_{yy}^{(n+M_x)}B_y^{(n+M_x)}(t)\Delta y - \tilde{\nu}_{yy}^{(n)}B_y^{(n)}(t)\Delta y - J_{\mathrm{ez}}^{(n)}(t)\Delta x\Delta y \\
&= \left(I - S_{M_y}\right)\tilde{\nu}_{xx}^{(n)}B_x^{(n)}(t)\Delta x + \left(S_{M_x} - I\right)\tilde{\nu}_{yy}^{(n)}B_y^{(n)}(t)\Delta y - J_{\mathrm{ex}}^{(n)}(t)\Delta x\Delta y
\end{aligned}$$

3-D FIT – … Discrete Grid Equations in Local Matrix Form / 3D-FIT – ... diskreten Gittergleichungen in lokaler Matrixform

$$\underbrace{\begin{bmatrix}\tilde{\varepsilon}_{xx}^{(n)} & & \\ & \tilde{\varepsilon}_{yy}^{(n)} & \\ & & \tilde{\varepsilon}_{zz}^{(n)}\end{bmatrix}}_{=[\tilde{\varepsilon}]^{(n)}}\underbrace{\begin{bmatrix}\Delta y\Delta z & & \\ & \Delta x\Delta z & \\ & & \Delta x\Delta y\end{bmatrix}}_{=[\tilde{S}]}\frac{\mathrm{d}}{\mathrm{d}t}\underbrace{\begin{Bmatrix}E_x^{(n)}(t)\\ E_y^{(n)}(t)\\ E_z^{(n)}(t)\end{Bmatrix}}_{=\{E\}^{(n)}(t)}$$

$$=\underbrace{\begin{bmatrix}0 & I-S_{M_z} & S_{M_y}-I\\ S_{M_z}-I & 0 & i-S_{M_x}\\ I-S_{M_y} & S_{M_x}-I & 0\end{bmatrix}}_{=[\mathrm{curl}]}\underbrace{\begin{bmatrix}\tilde{\nu}_{xx}^{(n)} & & \\ & \tilde{\nu}_{yy}^{(n)} & \\ & & \tilde{\nu}_{zz}^{(n)}\end{bmatrix}}_{=[\tilde{\nu}]^{(n)}}\underbrace{\begin{bmatrix}\Delta x & & \\ & \Delta y & \\ & & \Delta z\end{bmatrix}}_{=[\tilde{R}]}\underbrace{\begin{Bmatrix}B_x^{(n)}(t)\\ B_y^{(n)}(t)\\ B_z^{(n)}(t)\end{Bmatrix}}_{=\{B\}^{(n)}(t)}-\underbrace{\begin{bmatrix}\Delta y\Delta z & & \\ & \Delta x\Delta z & \\ & & \Delta x\Delta y\end{bmatrix}}_{=[\tilde{S}]}\underbrace{\begin{Bmatrix}J_{\mathrm{ex}}^{(n)}(t)\\ J_{\mathrm{ex}}^{(n)}(t)\\ J_{\mathrm{ex}}^{(n)}(t)\end{Bmatrix}}_{=\{J_{\mathrm{e}}\}^{(n)}(t)}$$

$$\begin{bmatrix}0 & I-S_{M_z} & S_{M_y}-I\\ S_{M_z}-I & 0 & i-S_{M_x}\\ I-S_{M_y} & S_{M_x}-I & 0\end{bmatrix}=\begin{bmatrix}0 & -P_{M_z} & P_{M_y}\\ P_{M_z} & 0 & -P_{M_x}\\ -P_{M_y} & P_{M_x} & 0\end{bmatrix}=[\widetilde{\mathrm{curl}}]$$

3-D FIT – … Discrete Grid Equations in Local Matrix Form / 3D-FIT – ... diskreten Gittergleichungen in lokaler Matrixform

$$\underbrace{\begin{bmatrix}\tilde{\varepsilon}_{xx}^{(n)} & & \\ & \tilde{\varepsilon}_{yy}^{(n)} & \\ & & \tilde{\varepsilon}_{zz}^{(n)}\end{bmatrix}}_{=[\tilde{\varepsilon}]^{(n)}}\underbrace{\begin{bmatrix}\Delta y\Delta z & & \\ & \Delta x\Delta z & \\ & & \Delta x\Delta y\end{bmatrix}}_{=[\tilde{S}]}\frac{d}{dt}\underbrace{\begin{Bmatrix}E_x^{(n)}(t)\\ E_y^{(n)}(t)\\ E_z^{(n)}(t)\end{Bmatrix}}_{=\{E\}^{(n)}(t)}=\underbrace{\begin{bmatrix}0 & -P_{M_z} & P_{M_y}\\ P_{M_z} & 0 & -P_{M_x}\\ -P_{M_y} & P_{M_x} & 0\end{bmatrix}}_{=[\widetilde{\mathrm{curl}}]}\underbrace{\begin{bmatrix}\tilde{\nu}_{xx}^{(n)} & & \\ & \tilde{\nu}_{yy}^{(n)} & \\ & & \tilde{\nu}_{zz}^{(n)}\end{bmatrix}}_{=[\tilde{\nu}]^{(n)}}\underbrace{\begin{bmatrix}\Delta x & & \\ & \Delta y & \\ & & \Delta z\end{bmatrix}}_{=[\tilde{R}]}\underbrace{\begin{Bmatrix}B_x^{(n)}(t)\\ B_y^{(n)}(t)\\ B_z^{(n)}(t)\end{Bmatrix}}_{=\{B\}^{(n)}(t)}-\underbrace{\begin{bmatrix}\Delta y\Delta z & & \\ & \Delta x\Delta z & \\ & & \Delta x\Delta y\end{bmatrix}}_{=[\tilde{S}]}\underbrace{\begin{Bmatrix}J_{\mathrm{ex}}^{(n)}(t)\\ J_{\mathrm{ex}}^{(n)}(t)\\ J_{\mathrm{ex}}^{(n)}(t)\end{Bmatrix}}_{=\{J_{\mathrm{e}}\}^{(n)}(t)}$$

$$[\tilde{\varepsilon}]^{(n)}[\tilde{S}]\frac{\mathrm{d}}{\mathrm{d}t}\{E\}^{(n)}(t)=[\widetilde{\mathrm{curl}}][\tilde{\nu}]^{(n)}[\tilde{R}]\{B\}^{(n)}(t)-[\tilde{S}]\{J_{\mathrm{e}}\}^{(n)}(t)$$

$[\tilde{\varepsilon}]^{(n)}$	$\in\mathbb{R}^{3\times 3}$	Diagonal matrix of permittivities on the grid $\tilde{G}$ / Diagonalmatrix der Permittivitäten auf dem Gitter $\tilde{G}$
$[\tilde{S}]$	$\in\mathbb{R}^{3\times 3}$	Diagonal matrix of elementary surfaces on the grid $\tilde{G}$ / Diagonalmatrix der Elementarflächen auf dem Gitter $\tilde{G}$
$\{E\}^{(n)}(t)$	$\in\mathbb{R}^{3}$	Algebraic electric field strength vector / Algebraischer elektrischer Feldstärkevektor
$[\widetilde{\mathrm{curl}}]$	$\in\mathbb{R}^{3\times 3}$	Topological curl operator in matrix form on the grid $\tilde{G}$ / Topologischer Rotationsoperator in Matrixform auf dem Gitter $\tilde{G}$
$[\tilde{\nu}]^{(n)}$	$\in\mathbb{R}^{3\times 3}$	Diagonal matrix of impermeabilities on the grid $\tilde{G}$ / Diagonalmatrix der Impermeabilitäten auf dem Gitter $\tilde{G}$
$[\tilde{R}]$	$\in\mathbb{R}^{3\times 3}$	Diagonal matrix of elementary lines on the grid $\tilde{G}$ / Diagonalmatrix der Elementarstrecken auf dem Gitter $\tilde{G}$
$\{B\}^{(n)}(t)$	$\in\mathbb{R}^{3}$	Algebraic magnetic flux density vector / Algebraischer magnetischer Flussdichtevektor
$\{J_{\mathrm{e}}\}^{(n)}(t)$	$\in\mathbb{R}^{3}$	Algebraic electric current density vector / Algebraischer elektrischer Stromdichtevektor

3-D FIT – ... Discrete Grid Equations in Local and Global Matrix Form / 3D-FIT – ... diskreten Gittergleichungen in lokaler und globaler Matrixform

Maxwell's equations in integral form / Maxwellsche Gleichungen in Integralform

$$\frac{\mathrm{d}}{\mathrm{d}t}\iint_S \underline{\mathbf{B}}(\underline{\mathbf{R}},t)\bullet\underline{\mathbf{dS}} = -\oint_{C=\partial S} \underline{\mathbf{E}}(\underline{\mathbf{R}},t)\bullet\underline{\mathbf{dR}} - \iint_S \underline{\mathbf{J}}_\mathrm{m}(\underline{\mathbf{R}},t)\bullet\underline{\mathbf{dS}}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\iint_S \underline{\underline{\boldsymbol{\varepsilon}}}(\underline{\mathbf{R}})\bullet\underline{\mathbf{E}}(\underline{\mathbf{R}},t)\bullet\underline{\mathbf{dS}} = \oint_{C=\partial S} \underline{\underline{\boldsymbol{\nu}}}(\underline{\mathbf{R}})\bullet\underline{\mathbf{B}}(\underline{\mathbf{R}},t)\bullet\underline{\mathbf{dR}} - \iint_S \underline{\mathbf{J}}_\mathrm{e}(\underline{\mathbf{R}},t)\bullet\underline{\mathbf{dS}}$$

Discrete grid equations in local matrix form / Diskrete Gittergleichungen in lokaler Matrixform

$$[S]\frac{\mathrm{d}}{\mathrm{d}t}\{B\}^{(n)}(t) = -[\mathrm{curl}][R]\{E\}^{(n)}(t) - [S]\{J_\mathrm{m}\}^{(n)}(t)$$

$$[\widetilde{\varepsilon}]^{(n)}[\widetilde{S}]\frac{\mathrm{d}}{\mathrm{d}t}\{E\}^{(n)}(t) = [\widetilde{\mathrm{curl}}][\widetilde{\nu}]^{(n)}[\widetilde{R}]\{B\}^{(n)}(t) - [\widetilde{S}]\{J_\mathrm{e}\}^{(n)}(t)$$

Discrete grid equations in global matrix form / Diskrete Gittergleichungen in globaler Matrixform

$$[\mathbf{S}]\frac{\mathrm{d}}{\mathrm{d}t}\{\mathbf{B}\}(t) = -[\mathbf{curl}][\mathbf{R}]\{\mathbf{E}\}(t) - [\mathbf{S}]\{\mathbf{J}_\mathrm{m}\}(t)$$

$$[\widetilde{\boldsymbol{\varepsilon}}][\widetilde{\mathbf{S}}]\frac{\mathrm{d}}{\mathrm{d}t}\{\mathbf{E}\}(t) = [\widetilde{\mathbf{curl}}][\widetilde{\boldsymbol{\nu}}][\widetilde{\mathbf{R}}]\{\mathbf{B}\}(t) - [\widetilde{\mathbf{S}}]\{\mathbf{J}_\mathrm{e}\}(t)$$

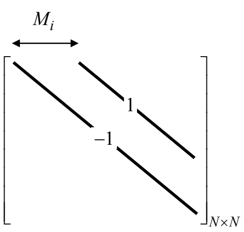
Elementary Difference Matrix [P$_i$] (P Matrix) / Elementare Differenzmatrix [P$_i$] (P-Matrix)

Elementary difference operator in global matrix form (P matrix) / Elementarer Differenzoperator in globaler Matrixform (P-Matrix)

$$[\mathbf{P}_{\pm i}] := \left([\mathbf{P}_{\pm i}]\right)_{jk}, \quad j,k \in \{1,2,\ldots,N\}$$

$$\left([\mathbf{P}_{\pm i}]\right)_{jk} = \begin{cases} \mp 1 & j = k \\ \pm 1 & j = k \mp M_i \text{ or / bzw. } k = j \pm M_i; \quad i = x,y,z; \; j,k \in \{1,2,\ldots,N\} \\ 0 & \text{else / sonst} \end{cases}$$

$[\mathbf{P}_i] =$

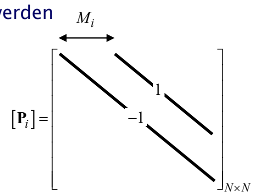


The P matrix has only two bands / Die P-Matrix hat nur zwei Bänder

Elementary Difference Matrix $[\mathbf{P}_i]$ (P Matrix) (...) / Elementare Differenzmatrix $[\mathbf{P}_i]$ (P-Matrix) (...)

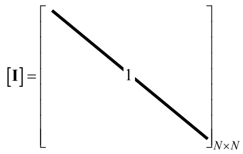
The P matrix can be represented by a sum of an identity matrix [I] and a band matrix [B] /
Die P-Matrix kann als Summe aus einer Einheitsmatrix (Identitätsmatrix) [I] und Bandmatrix [B] dargestellt werden

$$[\mathbf{P}_{\pm i}] := \mp[\mathbf{I}] \pm [\mathbf{B}_{\pm i}], \quad i = \{x, y, z\}$$



Identity matrix / Einheitsmatrix (Identitätsmatrix)

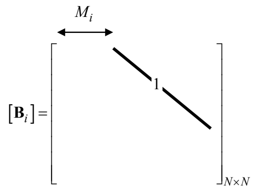
$$([\mathbf{I}])_{ij} = \delta_{ij} \qquad i, j \in \{1, 2, \ldots, N\}$$



Band matrix / Bandmatrix

$$([\mathbf{B}]_{\pm i})_{jk} = \begin{cases} 1 & j = k \mp M_i \text{ or / bzw. } k = j \pm M_i \\ 0 & \text{else / sonst} \end{cases}$$

$$i = x, y, z; \; j, k \in \{1, 2, \ldots, N\}$$

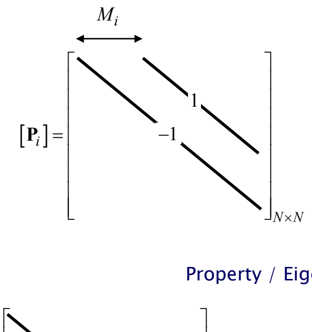


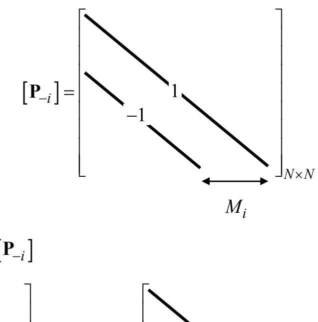
Properties of the Difference Matrix $[\mathbf{P}_i]$ (P Matrix) / Eigenschaften der Differenzmatrix $[\mathbf{P}_i]$ (P-Matrix)

$$[\mathbf{P}_{\pm i}] := \mp[\mathbf{I}] \pm [\mathbf{B}_{\pm i}], \quad i = \{x, y, z\}$$

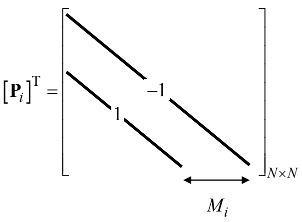
$$[\mathbf{P}_i] := -[\mathbf{I}] + [\mathbf{B}_i], \quad i = \{x, y, z\}$$

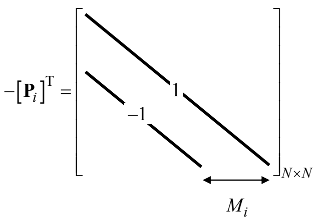
$$[\mathbf{P}_{-i}] := [\mathbf{I}] - [\mathbf{B}_{-i}], \quad i = \{x, y, z\}$$

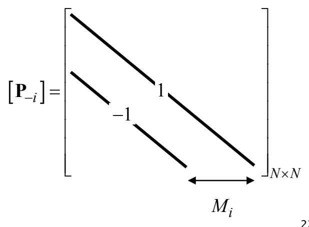




Property / Eigenschaft $\quad -[\mathbf{P}_i]^{\mathrm{T}} = [\mathbf{P}_{-i}]$







Discrete Global Gradient, Divergence, and Curl Operator / Diskreter globaler Gradienten-, Divergenz- und Rotationsoperator

Discrete gradient operator /
Diskreter Gradientenoperator

$$[\mathbf{grad}] = \begin{bmatrix} -[\mathbf{P}_x]^{\mathrm{T}} \\ -[\mathbf{P}_y]^{\mathrm{T}} \\ -[\mathbf{P}_z]^{\mathrm{T}} \end{bmatrix}_{3N\times N}$$

$$[\widetilde{\mathbf{grad}}] = \begin{bmatrix} [\mathbf{P}_x] \\ [\mathbf{P}_y] \\ [\mathbf{P}_z] \end{bmatrix}_{3N\times N}$$

Discrete curl operator /
Diskreter Rotationsoperator

$$[\mathbf{curl}] = \begin{bmatrix} [\mathbf{0}] & [\mathbf{P}_z]^{\mathrm{T}} & -[\mathbf{P}_y]^{\mathrm{T}} \\ -[\mathbf{P}_z]^{\mathrm{T}} & [\mathbf{0}] & [\mathbf{P}_x]^{\mathrm{T}} \\ [\mathbf{P}_y]^{\mathrm{T}} & -[\mathbf{P}_x]^{\mathrm{T}} & [\mathbf{0}] \end{bmatrix}_{3N\times 3N}$$

$$[\widetilde{\mathbf{curl}}] = \begin{bmatrix} [\mathbf{0}] & -[\mathbf{P}_z] & [\mathbf{P}_y] \\ [\mathbf{P}_z] & [\mathbf{0}] & -[\mathbf{P}_x] \\ -[\mathbf{P}_y] & [\mathbf{P}_x] & [\mathbf{0}] \end{bmatrix}_{3N\times 3N}$$

Discrete divergence operator /
Diskreter Divergenzoperator

$$[\mathbf{div}] := \left[-[\mathbf{P}_x]^{\mathrm{T}}, -[\mathbf{P}_y]^{\mathrm{T}}, -[\mathbf{P}_z]^{\mathrm{T}}\right]_{N\times 3N}$$

$$[\widetilde{\mathbf{div}}] := \left[[\mathbf{P}_x], [\mathbf{P}_y], [\mathbf{P}_z]\right]_{N\times 3N}$$

The matrix operators /
Die Matrixoperatoren

$$[\mathbf{grad}] \quad [\widetilde{\mathbf{grad}}]$$
$$[\mathbf{div}] \quad [\widetilde{\mathbf{div}}]$$
$$[\mathbf{curl}] \quad [\widetilde{\mathbf{curl}}]$$

are **global** matrix operators /
sind **globale** Matrixoperatoren

Properties of the Global Matrix Operators / Eigenschaften der globalen Matrixoperatoren

Some properties of the global matrix operators of the dual grid system /
Einige Eigenschaften der globalen Matrixoperatoren des dualen Gittersystems

$$-[\widetilde{\mathbf{div}}] = [\mathbf{grad}]^{\mathrm{T}}$$
$$[\widetilde{\mathbf{grad}}]^{\mathrm{T}} = [\mathbf{div}]$$
$$[\mathbf{curl}] = [\widetilde{\mathbf{curl}}]^{\mathrm{T}}$$

Conservation of important vector identities /
Erhaltung von wichtigen Vektoridentitäten

Vector identities /
Vektoridentitäten

$$\text{curl grad} = \nabla \times \nabla = \underline{\mathbf{0}}$$
$$\text{div curl} = \nabla \bullet \nabla = 0$$

⇕

are conserved in the dual grid system /
bleiben im dualen Gittersystem erhalten

$$[\mathbf{curl}][\mathbf{grad}] = [\mathbf{0}]$$
$$[\widetilde{\mathbf{curl}}][\widetilde{\mathbf{grad}}] = [\mathbf{0}]$$
$$[\mathbf{div}][\mathbf{curl}] = [\mathbf{0}]$$
$$[\widetilde{\mathbf{div}}][\widetilde{\mathbf{curl}}] = [\mathbf{0}]$$

Properties of the Global Matrix Operators / Eigenschaften der globalen Matrixoperatoren

Consistency test / Konsistenztest

$$\widetilde{[\mathbf{curl}]}\,\widetilde{[\mathbf{grad}]} = \begin{bmatrix} [\mathbf{0}] & -[\mathbf{P}_z] & [\mathbf{P}_y] \\ [\mathbf{P}_z] & [\mathbf{0}] & -[\mathbf{P}_x] \\ -[\mathbf{P}_y] & [\mathbf{P}_x] & [\mathbf{0}] \end{bmatrix} \begin{bmatrix} [\mathbf{P}_x] \\ [\mathbf{P}_y] \\ [\mathbf{P}_z] \end{bmatrix}$$

$$= \begin{bmatrix} [\mathbf{P}_y][\mathbf{P}_z] - [\mathbf{P}_z][\mathbf{P}_y] \\ [\mathbf{P}_z][\mathbf{P}_x] - [\mathbf{P}_x][\mathbf{P}_z] \\ [\mathbf{P}_x][\mathbf{P}_y] - [\mathbf{P}_y][\mathbf{P}_x] \end{bmatrix}$$

$$\begin{aligned}
[\mathbf{P}_i][\mathbf{P}_j] - [\mathbf{P}_j][\mathbf{P}_i] &= \left(-[\mathbf{I}] + [\mathbf{B}_i]\right)\left(-[\mathbf{I}] + [\mathbf{B}_j]\right) - \left(-[\mathbf{I}] + [\mathbf{B}_j]\right)\left(-[\mathbf{I}] + [\mathbf{B}_i]\right) \\
&= \left(-[\mathbf{I}][\mathbf{I}] - [\mathbf{I}][\mathbf{B}_j] - [\mathbf{B}_i][\mathbf{I}] + [\mathbf{B}_i][\mathbf{B}_j]\right) \\
&\quad - \left(-[\mathbf{I}][\mathbf{I}] - [\mathbf{I}][\mathbf{B}_i] - [\mathbf{B}_j][\mathbf{I}] + [\mathbf{B}_j][\mathbf{B}_i]\right) \\
&= \left(-[\mathbf{I}] - [\mathbf{B}_j] - [\mathbf{B}_i] + [\mathbf{B}_i][\mathbf{B}_j]\right) \\
&\quad - \left(-[\mathbf{I}] - [\mathbf{B}_i] - [\mathbf{B}_j] + [\mathbf{B}_j][\mathbf{B}_i]\right) \\
&= -[\mathbf{I}] - [\mathbf{B}_j] - [\mathbf{B}_i] + [\mathbf{B}_i][\mathbf{B}_j] + [\mathbf{I}] + [\mathbf{B}_i] + [\mathbf{B}_j] - [\mathbf{B}_j][\mathbf{B}_i] \\
&= [\mathbf{B}_i][\mathbf{B}_j] - [\mathbf{B}_j][\mathbf{B}_i]
\end{aligned}$$

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Properties of the Global Matrix Operators / Eigenschaften der globalen Matrixoperatoren

With the property / Mit der Eigenschaft

$$\left([\mathbf{B}_{\pm i}][\mathbf{B}_{\pm j}]\right)_{kl} = \begin{cases} 1 & k = l \mp M_i \mp M_j \\ 0 & \text{else / sonst} \end{cases}$$

i and *j* can be arbitrarily interchanged / *i* und *j* können beliebig vertauscht werden

This means that the matrices $[\mathbf{B}_{\pm i}]$ and $[\mathbf{B}_{\pm j}]$ as well as $[\mathbf{P}_{\pm i}]$ and $[\mathbf{P}_{\pm j}]$ are commutative! / Das bedeutet, dass die Matrizen $[\mathbf{B}_{\pm i}]$ und $[\mathbf{B}_{\pm j}]$ als auch $[\mathbf{P}_{\pm i}]$ und $[\mathbf{P}_{\pm j}]$ kommutativ sind!

$$[\mathbf{B}_{\pm i}][\mathbf{B}_{\pm j}] = [\mathbf{B}_{\pm j}][\mathbf{B}_{\pm i}]$$

$$[\mathbf{P}_{\pm i}][\mathbf{P}_{\pm j}] = [\mathbf{P}_{\pm j}][\mathbf{P}_{\pm i}]$$

$$\begin{aligned}
\widetilde{[\mathbf{curl}]}\,\widetilde{[\mathbf{grad}]} &= [\mathbf{P}_i][\mathbf{P}_j] - [\mathbf{P}_j][\mathbf{P}_i] \\
&= [\mathbf{B}_i][\mathbf{B}_j] - \underbrace{[\mathbf{B}_j][\mathbf{B}_i]}_{=[\mathbf{B}_i][\mathbf{B}_j]} \\
&= [\mathbf{B}_i][\mathbf{B}_j] - [\mathbf{B}_i][\mathbf{B}_j] \\
&= [\mathbf{0}]
\end{aligned}$$

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3-D FIT – ... Discrete Grid Equations in Local and Global Matrix Form / 3D-FIT – ... diskrete Gittergleichungen in lokaler und globaler Matrixform

Maxwell's equations in integral form /
Maxwellsche Gleichungen in Integralform

$$\frac{\mathrm{d}}{\mathrm{d}t}\iint_S \underline{\mathbf{B}}(\underline{\mathbf{R}},t)\bullet\underline{\mathbf{dS}} = -\oint_{C=\partial S} \underline{\mathbf{E}}(\underline{\mathbf{R}},t)\bullet\underline{\mathbf{dR}} - \iint_S \underline{\mathbf{J}}_{\mathrm{m}}(\underline{\mathbf{R}},t)\bullet\underline{\mathbf{dS}}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\iint_S \underline{\underline{\boldsymbol{\varepsilon}}}(\underline{\mathbf{R}})\bullet\underline{\mathbf{E}}(\underline{\mathbf{R}},t)\bullet\underline{\mathbf{dS}} = \oint_{C=\partial S} \underline{\underline{\boldsymbol{\nu}}}(\underline{\mathbf{R}})\bullet\underline{\mathbf{B}}(\underline{\mathbf{R}},t)\bullet\underline{\mathbf{dR}} - \iint_S \underline{\mathbf{J}}_{\mathrm{e}}(\underline{\mathbf{R}},t)\bullet\underline{\mathbf{dS}}$$

Discrete grid equations in local matrix form /
Diskrete Gittergleichungen in lokaler Matrixform

$$[S]\frac{\mathrm{d}}{\mathrm{d}t}\{B\}^{(n)}(t) = -[\mathrm{curl}][R]\{E\}^{(n)}(t) - [S]\{J_{\mathrm{m}}\}^{(n)}(t)$$

$$\widetilde{[\varepsilon]}^{(n)}\widetilde{[S]}\frac{\mathrm{d}}{\mathrm{d}t}\{E\}^{(n)}(t) = \widetilde{[\mathrm{curl}]}\widetilde{[\nu]}^{(n)}\widetilde{[R]}\{B\}^{(n)}(t) - \widetilde{[S]}\{J_{\mathrm{e}}\}^{(n)}(t)$$

$$n = 1, 2, \ldots, N$$

Discrete grid equations in global matrix form /
Diskrete Gittergleichungen in globaler Matrixform

$$[\mathbf{S}]\frac{\mathrm{d}}{\mathrm{d}t}\{\mathbf{B}\}(t) = -[\mathbf{curl}][\mathbf{R}]\{\mathbf{E}\}(t) - [\mathbf{S}]\{\mathbf{J}_{\mathrm{m}}\}(t)$$

$$\widetilde{[\boldsymbol{\varepsilon}]}\widetilde{[\mathbf{S}]}\frac{\mathrm{d}}{\mathrm{d}t}\{\mathbf{E}\}(t) = \widetilde{[\mathbf{curl}]}\widetilde{[\mathbf{v}]}\widetilde{[\mathbf{R}]}\{\mathbf{B}\}(t) - \widetilde{[\mathbf{S}]}\{\mathbf{J}_{\mathrm{e}}\}(t)$$

3-D FIT – ... Discrete Grid Equations in Global Matrix Form / 3D-FIT – ... diskrete Gittergleichungen in globaler Matrixform

Faraday's induction law in global matrix form /
Faradaysches Induktionsgesetz in globaler Matrixform

$$[\mathbf{S}]\frac{\mathrm{d}}{\mathrm{d}t}\{\mathbf{B}\}(t) = -[\mathbf{curl}][\mathbf{R}]\{\mathbf{E}\}(t) - [\mathbf{S}]\{\mathbf{J}_{\mathrm{m}}\}(t)$$

$[\mathbf{S}]$	$\in \mathbb{R}^{3N\times 3N}$	Diagonal matrix of elementary surfaces on the grid G / Diagonalmatrix der Elementarflächen auf dem Gitter G
$\{\mathbf{B}\}(t)$	$\in \mathbb{R}^{3N}$	Algebraic magnetic flux density vector / Algebraischer magnetischer Flussdichtevektor
$[\mathbf{curl}]$	$\in \mathbb{R}^{3N\times 3N}$	Topological curl operator in matrix form on the grid G / Topologischer Rotationsoperator in Matrixform auf dem Gitter G
$[\mathbf{R}]$	$\in \mathbb{R}^{3N\times 3N}$	Diagonal matrix of elementary lines on the grid G / Diagonalmatrix der Elementarstrecken auf dem Gitter G
$\{\mathbf{E}\}(t)$	$\in \mathbb{R}^{3N}$	Algebraic electric field strength vector / Algebraischer elektrische Feldstärkevektor
$\{\mathbf{J}_{\mathrm{m}}\}(t)$	$\in \mathbb{R}^{3N}$	Algebraic magnetic current density vector / Algebraischer magnetischer Stromdichtevektor

3-D FIT – ... Discrete Grid Equations in Global Matrix Form / 3D-FIT – ... diskrete Gittergleichungen in globaler Matrixform

Faraday's induction law in global matrix form /
Faradaysches Induktionsgesetz in globaler Matrixform

$$[\mathbf{S}]\frac{\mathrm{d}}{\mathrm{d}t}\{\mathbf{B}\}(t)=-[\mathbf{curl}][\mathbf{R}]\{\mathbf{E}\}(t)\quad-[\mathbf{S}]\{\mathbf{J}_{\mathrm{m}}\}(t)$$

$$[\mathbf{S}]=\begin{bmatrix}\Delta y\Delta z &&&&&&&\\ &\ddots&&&&&&\\ &&\Delta y\Delta z&&&&&\\ &&&\Delta x\Delta y&&&&\\ &&&&\ddots&&&\\ &&&&&\Delta x\Delta y&&\\ &&&&&&\Delta x\Delta y&\\ &&&&&&&\ddots\\ &&&&&&&&\Delta x\Delta y\end{bmatrix}_{3N\times 3N}$$

$$=\begin{bmatrix}[\mathrm{diag}\{\Delta y\Delta z\}]_{N\times N} & [0] & [0]\\ [0] & [\mathrm{diag}\{\Delta x\Delta z\}]_{N\times N} & [0]\\ [0] & [0] & [\mathrm{diag}\{\Delta x\Delta y\}]_{N\times N}\end{bmatrix}_{3N\times 3N}$$

$$[\mathbf{curl}]=\begin{bmatrix}[\mathbf{0}] & [\mathbf{P}_z]^{\mathrm{T}} & -[\mathbf{P}_y]^{\mathrm{T}}\\ -[\mathbf{P}_z]^{\mathrm{T}} & [\mathbf{0}] & [\mathbf{P}_x]^{\mathrm{T}}\\ [\mathbf{P}_y]^{\mathrm{T}} & -[\mathbf{P}_x]^{\mathrm{T}} & [\mathbf{0}]\end{bmatrix}_{3N\times 3N}$$

$$[\mathbf{R}]=\begin{bmatrix}\Delta x &&&&&&&&\\ &\ddots&&&&&&&\\ &&\Delta x&&&&&&\\ &&&\Delta y&&&&&\\ &&&&\ddots&&&&\\ &&&&&\Delta y&&&\\ &&&&&&\Delta z&&\\ &&&&&&&\ddots&\\ &&&&&&&&\Delta z\end{bmatrix}_{3N\times 3N}$$

$$=\begin{bmatrix}[\mathrm{diag}\{\Delta x\}]_{N\times N} & [0] & [0]\\ [0] & [\mathrm{diag}\{\Delta y\}]_{N\times N} & [0]\\ [0] & [0] & [\mathrm{diag}\{\Delta z\}]_{N\times N}\end{bmatrix}_{3N\times 3N}$$

$$\{\mathbf{B}\}(t)=\begin{Bmatrix}\{B_x\}(t)\\ \{B_y\}(t)\\ \{B_z\}(t)\end{Bmatrix}_{3N}\qquad \{B_i\}(t)=\begin{Bmatrix}B_i^{(1)}(t)\\ B_i^{(2)}(t)\\ \vdots\\ B_i^{(N)}(t)\end{Bmatrix}_{N}\qquad i=x,y,z$$

3-D FIT – ... Discrete Grid Equations in Global Matrix Form / 3D-FIT – ... diskrete Gittergleichungen in globaler Matrixform

Faraday's induction law in global matrix form /
Faradaysches Induktionsgesetz in globaler Matrixform

$$[\mathbf{S}]\frac{\mathrm{d}}{\mathrm{d}t}\{\mathbf{B}\}(t)=-[\mathbf{curl}][\mathbf{R}]\{\mathbf{E}\}(t)\quad-[\mathbf{S}]\{\mathbf{J}_{\mathrm{m}}\}(t)$$

$$\{\mathbf{E}\}(t)=\begin{Bmatrix}\{E_x\}(t)\\ \{E_y\}(t)\\ \{E_z\}(t)\end{Bmatrix}_{3N}\qquad \{E_i\}(t)=\begin{Bmatrix}E_i^{(1)}(t)\\ E_i^{(2)}(t)\\ \vdots\\ E_i^{(N)}(t)\end{Bmatrix}_{N}\qquad i=x,y,z$$

$$\{\mathbf{J}_{\mathrm{m}}\}(t)=\begin{Bmatrix}\{J_{\mathrm{m}x}\}(t)\\ \{J_{\mathrm{m}y}\}(t)\\ \{J_{\mathrm{m}z}\}(t)\end{Bmatrix}_{3N}\qquad \{J_{\mathrm{m}i}\}(t)=\begin{Bmatrix}J_{\mathrm{m}i}^{(1)}(t)\\ J_{\mathrm{m}i}^{(2)}(t)\\ \vdots\\ J_{\mathrm{m}i}^{(N)}(t)\end{Bmatrix}_{N}\qquad i=x,y,z$$

3-D FIT – ... Discrete Grid Equations in Global Matrix Form / 3D-FIT – ... diskrete Gittergleichungen in globaler Matrixform

Ampère-Maxwell's circuital law in global matrix form /
Ampère-Maxwellsches Durchflutungsgesetz in globaler Matrixform

$$\left[\widetilde{\boldsymbol{\varepsilon}}\right]\left[\widetilde{\mathbf{S}}\right]\frac{\mathrm{d}}{\mathrm{d}t}\left\{\mathbf{E}\right\}(t) = \left[\widetilde{\mathbf{curl}}\right]\left[\widetilde{\mathbf{v}}\right]\left[\widetilde{\mathbf{R}}\right]\left\{\mathbf{B}\right\}(t) - \left[\widetilde{\mathbf{S}}\right]\left\{\mathbf{J}_\mathrm{e}\right\}(t)$$

$\left[\widetilde{\boldsymbol{\varepsilon}}\right]$	$\in \mathbb{R}^{3N\times 3N}$	Diagonal matrix of permittivities on the grid $\widetilde{G}$ / Diagonalmatrix der Permittivitäten auf dem Gitter $\widetilde{G}$
$\left[\widetilde{\mathbf{S}}\right]$	$\in \mathbb{R}^{3N\times 3N}$	Diagonal matrix of elementary surfaces on the grid $\widetilde{G}$ / Diagonalmatrix der Elementarflächen auf dem Gitter $\widetilde{G}$
$\left\{\mathbf{E}\right\}(t)$	$\in \mathbb{R}^{3N}$	Algebraic electric field strength vector / Algebraischer elektrischer Feldstärkevektor
$\left[\widetilde{\mathrm{curl}}\right]$	$\in \mathbb{R}^{3N\times 3N}$	Topological curl operator in matrix form on the grid $\widetilde{G}$ / Topologischer Rotationsoperator in Matrixform auf dem Gitter $\widetilde{G}$
$\left[\widetilde{\mathbf{v}}\right]$	$\in \mathbb{R}^{3N\times 3N}$	Diagonal matrix of impermeabilities on the grid $\widetilde{G}$ / Diagonalmatrix der Impermeabilitäten auf dem Gitter $\widetilde{G}$
$\left[\widetilde{\mathbf{R}}\right]$	$\in \mathbb{R}^{3N\times 3N}$	Diagonal matrix of elementary lines on the grid $\widetilde{G}$ / Diagonalmatrix der Elementarstrecken auf dem Gitter $\widetilde{G}$
$\left\{\mathbf{B}\right\}(t)$	$\in \mathbb{R}^{3N}$	Algebraic magnetic flux density vector / Algebraischer magnetischer Flussdichtevektor
$\left\{\mathbf{J}_\mathrm{e}\right\}(t)$	$\in \mathbb{R}^{3N}$	Algebraic electric current density vector / Algebraischer elektrischer Stromdichtevektor

3-D FIT – ... Discrete Grid Equations in Global Matrix Form / 3D-FIT – ... diskrete Gittergleichungen in globaler Matrixform

Ampère-Maxwell's circuital law in global matrix form /
Ampère-Maxwellsches Durchflutungsgesetz in globaler Matrixform

$$\left[\widetilde{\boldsymbol{\varepsilon}}\right]\left[\widetilde{\mathbf{S}}\right]\frac{\mathrm{d}}{\mathrm{d}t}\left\{\mathbf{E}\right\}(t) = \left[\widetilde{\mathbf{curl}}\right]\left[\widetilde{\mathbf{v}}\right]\left[\widetilde{\mathbf{R}}\right]\left\{\mathbf{B}\right\}(t) - \left[\widetilde{\mathbf{S}}\right]\left\{\mathbf{J}_\mathrm{e}\right\}(t)$$

$$\left[\widetilde{\varepsilon}\right] = \begin{bmatrix} \widetilde{\varepsilon}_{xx}^{(1)} & & & & & & & & \\ & \ddots & & & & & & & \\ & & \widetilde{\varepsilon}_{xx}^{(N)} & & & & & & \\ & & & \widetilde{\varepsilon}_{yy}^{(1)} & & & & & \\ & & & & \ddots & & & & \\ & & & & & \widetilde{\varepsilon}_{yy}^{(N)} & & & \\ & & & & & & \widetilde{\varepsilon}_{zz}^{(1)} & & \\ & & & & & & & \ddots & \\ & & & & & & & & \widetilde{\varepsilon}_{zz}^{(N)} \end{bmatrix}_{3N\times 3N}$$

$$= \begin{bmatrix} \left[\mathrm{diag}\{\widetilde{\varepsilon}_{xx}^{(1)}, \widetilde{\varepsilon}_{xx}^{(2)}, \ldots, \widetilde{\varepsilon}_{xx}^{(N)}\}\right]_{N\times N} & [0] & [0] \\ [0] & \left[\mathrm{diag}\{\widetilde{\varepsilon}_{yy}^{(1)}, \widetilde{\varepsilon}_{yy}^{(2)}, \ldots, \widetilde{\varepsilon}_{yy}^{(N)}\}_N\right]_{N\times N} & [0] \\ [0] & [0] & \left[\mathrm{diag}\{\widetilde{\varepsilon}_{zz}^{(1)}, \widetilde{\varepsilon}_{zz}^{(2)}, \ldots, \widetilde{\varepsilon}_{zz}^{(N)}\}_N\right]_{N\times N} \end{bmatrix}_{3N\times 3N}$$

3-D FIT – ... Discrete Grid Equations in Global Matrix Form / 3D-FIT – ... diskrete Gittergleichungen in globaler Matrixform

Ampère-Maxwell's circuital law in global matrix form /
Ampère-Maxwellsches Durchflutungsgesetz in globaler Matrixform

$$[\tilde{\boldsymbol{\varepsilon}}][\tilde{\mathbf{S}}]\frac{\mathrm{d}}{\mathrm{d}t}\{\mathbf{E}\}(t) = [\widetilde{\mathbf{curl}}][\tilde{\boldsymbol{\nu}}][\tilde{\mathbf{R}}]\{\mathbf{B}\}(t) - [\tilde{\mathbf{S}}]\{\mathbf{J}_\mathrm{e}\}(t)$$

$$[\tilde{\mathbf{S}}] = \begin{bmatrix} \Delta y\Delta z & & & & & & & & \\ & \ddots & & & & & & & \\ & & \Delta y\Delta z & & & & & & \\ & & & \Delta x\Delta z & & & & & \\ & & & & \ddots & & & & \\ & & & & & \Delta x\Delta z & & & \\ & & & & & & \Delta x\Delta y & & \\ & & & & & & & \ddots & \\ & & & & & & & & \Delta y\Delta z \end{bmatrix}_{3N\times 3N} = \begin{bmatrix} [\mathrm{diag}\{\Delta y\Delta z\}]_{N\times N} & [0] & [0] \\ [0] & [\mathrm{diag}\{\Delta x\Delta z\}]_{N\times N} & [0] \\ [0] & [0] & [\mathrm{diag}\{\Delta x\Delta y\}]_{N\times N} \end{bmatrix}_{3N\times 3N}$$

$$[\tilde{\mathbf{R}}] = \begin{bmatrix} \Delta x & & & & & & & & \\ & \ddots & & & & & & & \\ & & \Delta x & & & & & & \\ & & & \Delta y & & & & & \\ & & & & \ddots & & & & \\ & & & & & \Delta y & & & \\ & & & & & & \Delta z & & \\ & & & & & & & \ddots & \\ & & & & & & & & \Delta z \end{bmatrix}_{3N\times 3N} = \begin{bmatrix} [\mathrm{diag}\{\Delta x\}]_{N\times N} & [0] & [0] \\ [0] & [\mathrm{diag}\{\Delta y\}]_{N\times N} & [0] \\ [0] & [0] & [\mathrm{diag}\{\Delta z\}]_{N\times N} \end{bmatrix}_{3N\times 3N}$$

$$[\widetilde{\mathbf{curl}}] = \begin{bmatrix} [\mathbf{0}] & -[\mathbf{P}_z] & [\mathbf{P}_y] \\ [\mathbf{P}_z] & [\mathbf{0}] & -[\mathbf{P}_x] \\ -[\mathbf{P}_y] & [\mathbf{P}_x] & [\mathbf{0}] \end{bmatrix}_{3N\times 3N}$$

$$\{\mathbf{J}_\mathrm{e}\}(t) = \begin{Bmatrix} \{J_{\mathrm{e}x}\}(t) \\ \{J_{\mathrm{e}y}\}(t) \\ \{J_{\mathrm{e}z}\}(t) \end{Bmatrix}_{3N} \qquad \{J_{\mathrm{e}i}\}(t) = \begin{Bmatrix} J_{\mathrm{e}i}^{(1)}(t) \\ J_{\mathrm{e}i}^{(2)}(t) \\ \vdots \\ J_{\mathrm{e}i}^{(N)}(t) \end{Bmatrix}_N \qquad i = x, y, z$$

3-D FIT – ... Discrete Grid Equations in Global Matrix Form / 3D-FIT – ... diskrete Gittergleichungen in globaler Matrixform

Ampère-Maxwell's circuital law in global matrix form /
Ampère-Maxwellsches Durchflutungsgesetz in globaler Matrixform

$$[\tilde{\boldsymbol{\varepsilon}}][\tilde{\mathbf{S}}]\frac{\mathrm{d}}{\mathrm{d}t}\{\mathbf{E}\}(t) = [\widetilde{\mathbf{curl}}][\tilde{\boldsymbol{\nu}}][\tilde{\mathbf{R}}]\{\mathbf{B}\}(t) - [\tilde{\mathbf{S}}]\{\mathbf{J}_\mathrm{e}\}(t)$$

$$[\tilde{\nu}] = \begin{bmatrix} \tilde{\nu}_{xx}^{(1)} & & & & & & & & \\ & \ddots & & & & & & & \\ & & \tilde{\nu}_{xx}^{(N)} & & & & & & \\ & & & \tilde{\nu}_{yy}^{(1)} & & & & & \\ & & & & \ddots & & & & \\ & & & & & \tilde{\nu}_{yy}^{(N)} & & & \\ & & & & & & \tilde{\nu}_{zz}^{(1)} & & \\ & & & & & & & \ddots & \\ & & & & & & & & \tilde{\nu}_{zz}^{(N)} \end{bmatrix}_{3N\times 3N}$$

$$= \begin{bmatrix} [\mathrm{diag}\{\tilde{\nu}_{xx}^{(1)}, \tilde{\nu}_{xx}^{(2)}, \ldots, \tilde{\nu}_{xx}^{(N)}\}]_{N\times N} & [0] & [0] \\ [0] & [\mathrm{diag}\{\tilde{\nu}_{yy}^{(1)}, \tilde{\nu}_{yy}^{(2)}, \ldots, \tilde{\nu}_{yy}^{(N)}\}_N]_{N\times N} & [0] \\ [0] & [0] & [\mathrm{diag}\{\tilde{\nu}_{zz}^{(1)}, \tilde{\nu}_{zz}^{(2)}, \ldots, \tilde{\nu}_{zz}^{(N)}\}_N]_{N\times N} \end{bmatrix}_{3N\times 3N}$$

3-D FIT - ... Discrete Grid Equations in Local and Global Matrix Form / 3D-FIT - ... diskrete Gittergleichungen in lokaler und globaler Matrixform

The two discrete grid equations in global matrix form read /
Die beiden diskreten Gittergleichungen in globaler Matrixform lauten

$$[\mathbf{S}]\frac{\mathrm{d}}{\mathrm{d}t}\{\mathbf{B}\}(t) = -[\mathbf{curl}][\mathbf{R}]\{\mathbf{E}\}(t) \quad -[\mathbf{S}]\{\mathbf{J}_\mathrm{m}\}(t)$$

$$[\widetilde{\boldsymbol{\varepsilon}}][\widetilde{\mathbf{S}}]\frac{\mathrm{d}}{\mathrm{d}t}\{\mathbf{E}\}(t) = [\widetilde{\mathbf{curl}}][\widetilde{\boldsymbol{\nu}}][\widetilde{\mathbf{R}}]\{\mathbf{B}\}(t) - [\widetilde{\mathbf{S}}]\{\mathbf{J}_\mathrm{e}\}(t)$$

We arrange the last equations in the form /
Wir bringen die letzten beiden Gleichungen in die Form

$$\frac{\mathrm{d}}{\mathrm{d}t}\{\mathbf{B}\}(t) = -[\mathbf{S}]^{-1}[\mathbf{curl}][\mathbf{R}]\{\mathbf{E}\}(t) - [\mathbf{S}]^{-1}[\mathbf{S}]\{\mathbf{J}_\mathrm{m}\}(t)$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\{\mathbf{E}\}(t) = [\widetilde{\mathbf{S}}]^{-1}[\widetilde{\boldsymbol{\varepsilon}}]^{-1}[\widetilde{\mathbf{curl}}][\widetilde{\boldsymbol{\nu}}][\widetilde{\mathbf{R}}]\{\mathbf{B}\}(t) - [\widetilde{\mathbf{S}}]^{-1}[\widetilde{\boldsymbol{\varepsilon}}]^{-1}[\widetilde{\mathbf{S}}]\{\mathbf{J}_\mathrm{e}\}(t)$$

$$[\mathbf{S}]^{-1}[\mathbf{S}] = [\mathbf{I}]$$

$$[\widetilde{\mathbf{S}}]^{-1}[\widetilde{\boldsymbol{\varepsilon}}]^{-1}[\widetilde{\mathbf{S}}] = \underbrace{[\widetilde{\mathbf{S}}]^{-1}[\widetilde{\mathbf{S}}]}_{=[\mathbf{I}]}[\widetilde{\boldsymbol{\varepsilon}}]^{-1} = [\widetilde{\boldsymbol{\varepsilon}}]^{-1}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\{\mathbf{B}\}(t) = -[\mathbf{S}]^{-1}[\mathbf{curl}][\mathbf{R}]\{\mathbf{E}\}(t) - \{\mathbf{J}_\mathrm{m}\}(t)$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\{\mathbf{E}\}(t) = [\widetilde{\mathbf{S}}]^{-1}[\widetilde{\boldsymbol{\varepsilon}}]^{-1}[\widetilde{\mathbf{curl}}][\widetilde{\boldsymbol{\nu}}][\widetilde{\mathbf{R}}]\{\mathbf{B}\}(t) - [\widetilde{\boldsymbol{\varepsilon}}]^{-1}\{\mathbf{J}_\mathrm{e}\}(t)$$

3-D FIT - ... Discrete Grid Equations in Local and Global Matrix Form / 3D-FIT - ... diskrete Gittergleichungen in lokaler und globaler Matrixform

The two discrete grid equations in global matrix form read /
Die beiden diskreten Gittergleichungen in globaler Matrixform lauten

$$[\mathbf{S}]\frac{\mathrm{d}}{\mathrm{d}t}\{\mathbf{B}\}(t) = -[\mathbf{curl}][\mathbf{R}]\{\mathbf{E}\}(t) \quad -[\mathbf{S}]\{\mathbf{J}_\mathrm{m}\}(t)$$

$$[\widetilde{\boldsymbol{\varepsilon}}][\widetilde{\mathbf{S}}]\frac{\mathrm{d}}{\mathrm{d}t}\{\mathbf{E}\}(t) = [\widetilde{\mathbf{curl}}][\widetilde{\boldsymbol{\nu}}][\widetilde{\mathbf{R}}]\{\mathbf{B}\}(t) - [\widetilde{\mathbf{S}}]\{\mathbf{J}_\mathrm{e}\}(t)$$

We arrange the last equations in the form /
Wir bringen die letzten beiden Gleichungen in die Form

$$\frac{\mathrm{d}}{\mathrm{d}t}\{\mathbf{B}\}(t) = -[\mathbf{S}]^{-1}[\mathbf{curl}][\mathbf{R}]\{\mathbf{E}\}(t) - [\mathbf{S}]^{-1}[\mathbf{S}]\{\mathbf{J}_\mathrm{m}\}(t)$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\{\mathbf{E}\}(t) = [\widetilde{\mathbf{S}}]^{-1}[\widetilde{\boldsymbol{\varepsilon}}]^{-1}[\widetilde{\mathbf{curl}}][\widetilde{\boldsymbol{\nu}}][\widetilde{\mathbf{R}}]\{\mathbf{B}\}(t) - [\widetilde{\mathbf{S}}]^{-1}[\widetilde{\boldsymbol{\varepsilon}}]^{-1}[\widetilde{\mathbf{S}}]\{\mathbf{J}_\mathrm{e}\}(t)$$

$$[\mathbf{S}]^{-1}[\mathbf{S}] = [\mathbf{I}]$$

$$[\widetilde{\mathbf{S}}]^{-1}[\widetilde{\boldsymbol{\varepsilon}}]^{-1}[\widetilde{\mathbf{S}}] = \underbrace{[\widetilde{\mathbf{S}}]^{-1}[\widetilde{\mathbf{S}}]}_{=[\mathbf{I}]}[\widetilde{\boldsymbol{\varepsilon}}]^{-1} = [\widetilde{\boldsymbol{\varepsilon}}]^{-1}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\{\mathbf{B}\}(t) = -[\mathbf{S}]^{-1}[\mathbf{curl}][\mathbf{R}]\{\mathbf{E}\}(t) - \{\mathbf{J}_\mathrm{m}\}(t)$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\{\mathbf{E}\}(t) = [\widetilde{\mathbf{S}}]^{-1}[\widetilde{\boldsymbol{\varepsilon}}]^{-1}[\widetilde{\mathbf{curl}}][\widetilde{\boldsymbol{\nu}}][\widetilde{\mathbf{R}}]\{\mathbf{B}\}(t) - [\widetilde{\boldsymbol{\varepsilon}}]^{-1}\{\mathbf{J}_\mathrm{e}\}(t)$$

3-D FIT – ... Discrete Grid Equations in Local and Global Matrix Form / 3D-FIT – ... diskrete Gittergleichungen in lokaler und globaler Matrixform

The two discrete grid equations in global matrix form read /
Die beiden diskreten Gittergleichungen in globaler Matrixform lauten

$$\frac{\mathrm{d}}{\mathrm{d}t}\{\mathbf{B}\}(t) = -[\mathbf{S}]^{-1} \quad [\mathbf{curl}] \quad [\mathbf{R}]\{\mathbf{E}\}(t) - \quad \{\mathbf{J}_\mathrm{m}\}(t)$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\{\mathbf{E}\}(t) = [\widetilde{\mathbf{S}}]^{-1}[\widetilde{\boldsymbol{\varepsilon}}]^{-1}[\widetilde{\mathbf{curl}}][\widetilde{\boldsymbol{\nu}}][\widetilde{\mathbf{R}}]\{\mathbf{B}\}(t) - [\widetilde{\boldsymbol{\varepsilon}}]^{-1}\{\mathbf{J}_\mathrm{e}\}(t)$$

Now we write these two matrix equations in matrix form and find a first-order system of differential equations / Nun schreiben wir die beiden Matrixgleichungen in Matrixform und finden das folgende System von Differentialgleichungen erster Ordnung

$$\frac{\mathrm{d}}{\mathrm{d}t}\{\mathbf{y}\}(t) = [\mathbf{A}]\{\mathbf{y}\}(t) + \{\mathbf{q}\}(t)$$

with / mit

Solution vector / Lösungsvektor
$$\{\mathbf{y}\}(t) = \begin{Bmatrix} \{\mathbf{B}\}(t) \\ \{\mathbf{E}\}(t) \end{Bmatrix}$$

System matrix / Systemmatrix
$$[\mathbf{A}] = \begin{bmatrix} [0] & [\mathbf{S}]^{-1}[\mathbf{curl}][\mathbf{R}] \\ [\widetilde{\mathbf{S}}]^{-1}[\widetilde{\boldsymbol{\varepsilon}}]^{-1}[\widetilde{\mathbf{curl}}][\widetilde{\boldsymbol{\nu}}][\widetilde{\mathbf{R}}] & [0] \end{bmatrix}$$

Source vector / Quellvektor
$$\{\mathbf{q}\}(t) = \begin{Bmatrix} - \quad \{\mathbf{J}_\mathrm{m}\}(t) \\ -[\widetilde{\boldsymbol{\varepsilon}}]^{-1}\{\mathbf{J}_\mathrm{e}\}(t) \end{Bmatrix}$$

3-D FIT – ... Solution of the Initial Value Problem (IVP) / 3D-FIT – Lösung des Anfangswertproblems (AWP)

A general solution of the initial value problem (IVP) with the initial value $\{\mathbf{y}\}(t_0)$ is /
Eine allgemeine Lösung des Anfangswertproblems (AWP) mit dem Anfangswert $\{\mathbf{y}\}(t_0)$ ist

$$\{\mathbf{y}\}(t) = \{\mathbf{y}\}(t_0) + \underbrace{\int_{t'=t_0}^{t} \underbrace{\{[\mathbf{A}]\{\mathbf{y}\}(t) + \{\mathbf{q}\}(t)\}}_{=\{\dot{\mathbf{y}}\}(t)} \mathrm{d}t}_{\text{time integration / zeitliche Integration}}$$

- implicit time integration / implizierte Zeitintegration
- explicit time integration / explizite Zeitintegration

Explicit time integration / Explizite Zeitintegration

$$\{\mathbf{B}\}(t) = \{\mathbf{B}\}(t_0) + \int_{t'=t_0}^{t} \{\dot{\mathbf{B}}\}(t')\,\mathrm{d}t'$$

$$\{\mathbf{E}\}(t) = \{\mathbf{E}\}(t_0) + \int_{t'=t_0}^{t} \{\dot{\mathbf{E}}\}(t')\,\mathrm{d}t'$$

$t = [0, T]$; T: time interval to be simulated / zu simulierendes Zeitinterval

Initial value / Anfangswert

3-D FIT – ... Solution of the Initial Value Problem (IVP) / 3D-FIT – Lösung des Anfangswertproblems (AWP)

Discretization in time on a staggered grid in time /
Diskretisierung in der Zeit auf einem versetzten Gitter in der Zeit

$$\{\mathbf{B}\}(t) \rightarrow \{\mathbf{B}\}(n_t\Delta t) \rightarrow \{\mathbf{B}\}^{(n_t)}$$

$$\{\mathbf{E}\}(t) \rightarrow \{\mathbf{E}\}\left[\left(n_t + \frac{1}{2}\right)\Delta t\right] \rightarrow \{\mathbf{E}\}^{(n_t+1/2)}$$

$$\{\mathbf{B}\}(t) = \{\mathbf{B}\}(t_0) + \int_{t'=t_0}^{t} \{\dot{\mathbf{B}}\}(t')\,\mathrm{d}t'$$

$$\{\mathbf{E}\}(t) = \{\mathbf{E}\}(t_0) + \int_{t'=t_0}^{t} \{\dot{\mathbf{E}}\}(t')\,\mathrm{d}t'$$

$$\{\mathbf{B}\}^{(n_t)} = \{\mathbf{B}\}^{(n_t-1)} + \int_{t'=(n_t-1)\Delta t}^{n_t\Delta t} \{\dot{\mathbf{B}}\}(t')\,\mathrm{d}t'$$

$$\{\mathbf{E}\}^{(n_t+1/2)} = \{\mathbf{E}\}^{(n_t-1/2)} + \int_{t'=(n_t-1/2)\Delta t}^{(n_t+1/2)\Delta t} \{\dot{\mathbf{E}}\}(t')\,\mathrm{d}t'$$

Mid point rule /
Mittelpunktsregel

$$\int_{t'=(n_t-1)\Delta t}^{n_t\Delta t} \{\dot{\mathbf{B}}\}(t')\,\mathrm{d}t' = \{\dot{\mathbf{B}}\}[(n_t-1/2)\Delta t]\Delta t = \{\dot{\mathbf{B}}\}^{(n_t-1/2)}\Delta t$$

$$\int_{t'=(n_t-1/2)\Delta t}^{(n_t+1/2)\Delta t} \{\dot{\mathbf{E}}\}(t')\,\mathrm{d}t' = \{\dot{\mathbf{E}}\}(n_t\Delta t)\Delta t = \{\dot{\mathbf{E}}\}^{(n_t)}\Delta t$$

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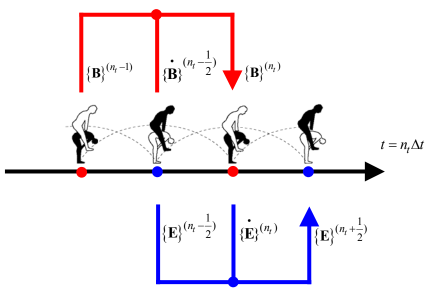
3-D FIT – ... Solution of the Initial Value Problem (IVP) / 3D-FIT – Lösung des Anfangswertproblems (AWP)

The leapfrog structure of the algorithm in time /
Die Bocksprung-Struktur des Algorithmus in der Zeit

$$\{\mathbf{B}\}^{(n_t)} = \{\mathbf{B}\}^{(n_t-1)} + \Delta t\{\dot{\mathbf{B}}\}^{(n_t-1/2)}$$

$$\{\mathbf{E}\}^{(n_t+1/2)} = \{\mathbf{E}\}^{(n_t-1/2)} + \Delta t\{\dot{\mathbf{E}}\}^{(n_t)}$$



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3-D FIT – ... Solution of the Initial Value Problem (IVP) / 3D-FIT – Lösung des Anfangswertproblems (AWP)

Electromagnetic grid equations (EMGE) of the so-called
Electromagnetic Finite Integration Technique (EMFIT) algorithm /
Elektromagnetische Gittergleichungen (EMGG) des so genannten
Elektromagnetischen Finite Integrationstechnik (EMFIT) Algorithmus

Faraday's induction grid equation / Faradaysche Induktionsgittergleichung

$$\{\dot{\mathbf{B}}\}^{(n_t-1/2)} = -[\mathbf{S}]^{-1}[\mathbf{curl}][\mathbf{R}]\{\mathbf{E}\}^{(n_t-1/2)} - \{\mathbf{J}_{\mathrm{m}}\}^{(n_t-1/2)}$$

Time integration / Zeitintegration

$$\{\mathbf{B}\}^{(n_t)} = \{\mathbf{B}\}^{(n_t-1)} + \Delta t\{\dot{\mathbf{B}}\}^{(n_t-1/2)}$$

Ampère-Maxwell's circuital grid equation / Ampère-Maxwellsche Durchflutungsgittergleichung

$$\{\dot{\mathbf{E}}\}^{(n_t} = [\widetilde{\mathbf{S}}]^{-1}[\widetilde{\boldsymbol{\varepsilon}}]^{-1}[\widetilde{\mathbf{curl}}][\widetilde{\mathbf{v}}][\widetilde{\mathbf{R}}]\{\mathbf{B}\}^{(n_t)} - [\widetilde{\boldsymbol{\varepsilon}}]^{-1}\{\mathbf{J}_{\mathrm{e}}\}^{(n_t)}$$

Time integration / Zeitintegration

$$\{\mathbf{E}\}^{(n_t+1/2)} = \{\mathbf{E}\}^{(n_t-1/2)} + \Delta t\{\dot{\mathbf{E}}\}^{(n_t)}$$

3-D FIT – ... Solution of the Initial Value Problem (IVP) / 3D-FIT – Lösung des Anfangswertproblems (AWP)

Electromagnetic grid equations (EMGE) of the so-called EMFIT algorithm /
Elektromagnetische Gittergleichungen (EMGG) des so genannten EMFIT-Algorithmus

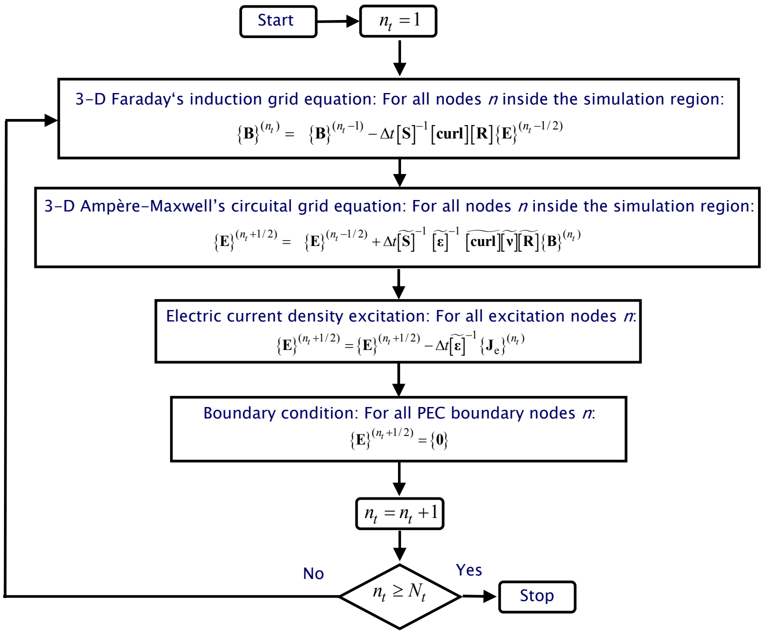
Time-integrated Faraday's induction grid equation /
Zeitlich integrierte Faradaysche Induktionsgittergleichung

$$\{\mathbf{B}\}^{(n_t)} = \{\mathbf{B}\}^{(n_t-1)} + \Delta t\left[-[\mathbf{S}]^{-1}[\mathbf{curl}][\mathbf{R}]\{\mathbf{E}\}^{(n_t-1/2)} - \{\mathbf{J}_{\mathrm{m}}\}^{(n_t-1/2)}\right]$$

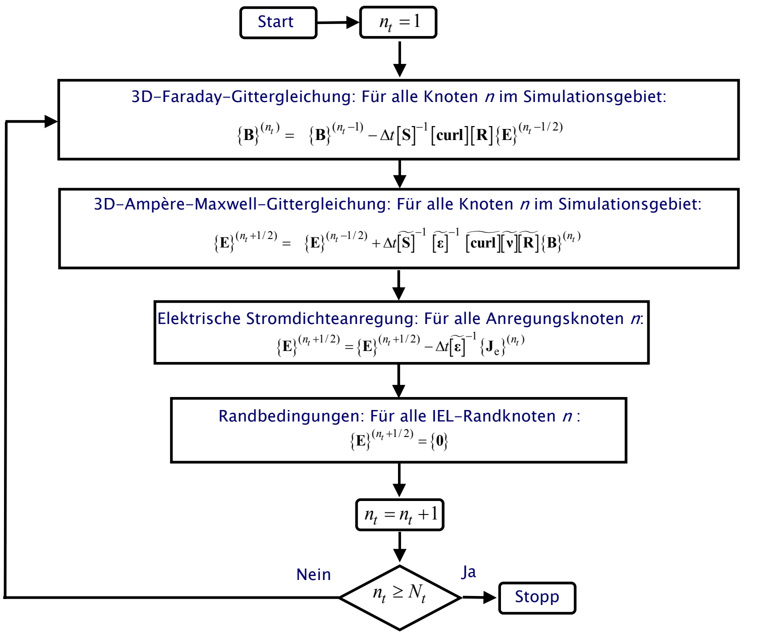
Time-integrated Ampère-Maxwell's circuital grid equation /
Zeitlich integrierte Ampère-Maxwellsche Durchflutungsgittergleichung

$$\{\mathbf{E}\}^{(n_t+1/2)} = \{\mathbf{E}\}^{(n_t-1/2)} + \Delta t\left[[\widetilde{\mathbf{S}}]^{-1}[\widetilde{\boldsymbol{\varepsilon}}]^{-1}[\widetilde{\mathbf{curl}}][\widetilde{\mathbf{v}}][\widetilde{\mathbf{R}}]\{\mathbf{B}\}^{(n_t)} - [\widetilde{\boldsymbol{\varepsilon}}]^{-1}\{\mathbf{J}_{\mathrm{e}}\}^{(n_t)}\right]$$

3-D FIT Algorithm – Flow Chart / 3D-FIT-Algorithmus – Flussdiagramm



3-D FIT Algorithm – Flow Chart / 3D-FIT-Algorithmus – Flussdiagramm



3-D FIT – ... Normalized ... Grid Equations / 3D-FIT – ... normierte ... Gittergleichungen

Normalized electromagnetic grid equations (EMGE) of the so-called EMFIT algorithm /
Normierte elektromagnetische Gittergleichungen (EMGG) des so genannten EMFIT-Algorithmus

Normalized time-integrated Faraday's induction grid equation /
Normierte zeitlich integrierte Faradaysche Induktionsgittergleichung

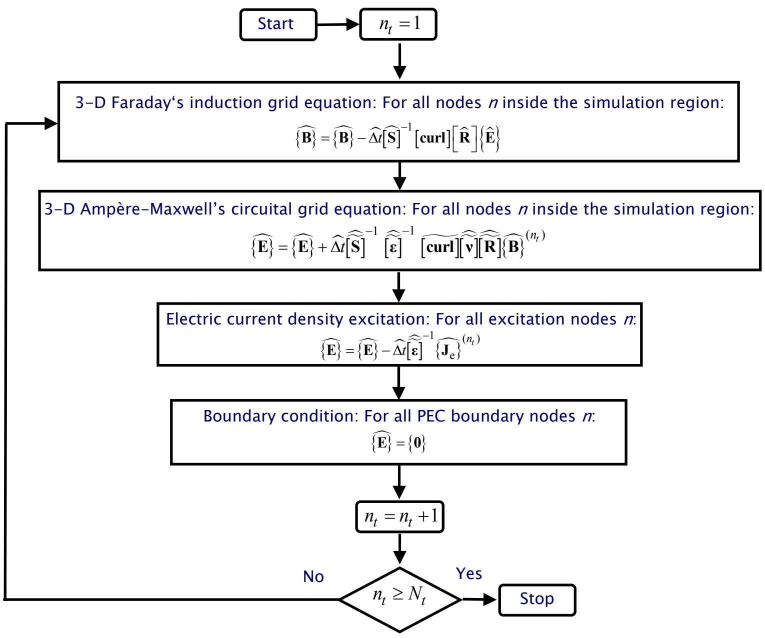
$$\widehat{\{\mathbf{B}\}}^{(n_t)} = \widehat{\{\mathbf{B}\}}^{(n_t-1)} + \widehat{\Delta t}\left[-\widehat{[\mathbf{S}]}^{-1}[\mathbf{curl}]\left[\widehat{\mathbf{R}}\right]\left\{\widehat{\mathbf{E}}\right\}^{(n_t-1/2)} - \widehat{\{\mathbf{J}_\mathrm{m}\}}^{(n_t-1/2)}\right]$$

Normalized time-integrated Ampère-Maxwell's circuital grid equation /
Normierte zeitlich integrierte Ampère-Maxwellsche Durchflutungsgittergleichung

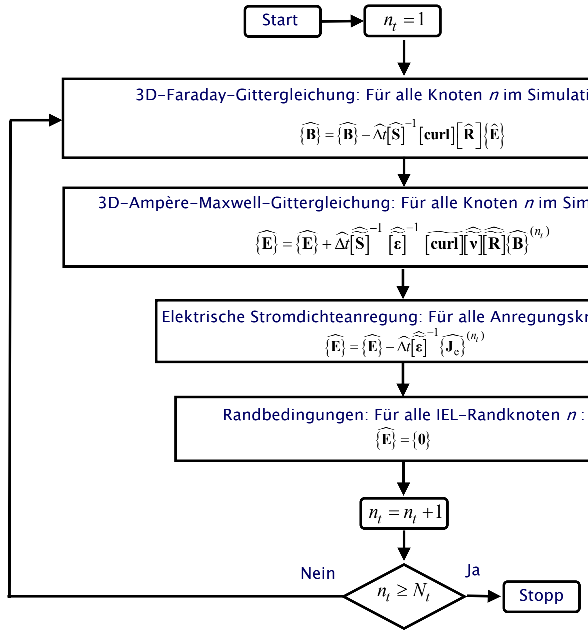
$$\widehat{\{\mathbf{E}\}}^{(n_t+1/2)} = \widehat{\{\mathbf{E}\}}^{(n_t-1/2)} + \widehat{\Delta t}\left[\widehat{\widetilde{[\mathbf{S}]}}^{-1}\widehat{\widetilde{[\boldsymbol{\varepsilon}]}}^{-1}\widetilde{[\mathbf{curl}]}\widehat{\widetilde{[\boldsymbol{\nu}]}}\widehat{\widetilde{[\mathbf{R}]}}\widehat{\{\mathbf{B}\}}^{(n_t)} - \widehat{\widetilde{[\boldsymbol{\varepsilon}]}}^{-1}\widehat{\{\mathbf{J}_\mathrm{e}\}}^{(n_t)}\right]$$

In a computer implementation we can neglect the integer time step counter n_t. /
In der Rechnerimplementierung kann der ganzzahlige Zeitschrittzähler n_t unterdrückt werden.

3-D FIT Algorithm – Flow Chart / 3D-FIT-Algorithmus – Flussdiagramm



3-D FIT Algorithm – Flow Chart / 3D-FIT-Algorithmus – Flussdiagramm



FIT Discretization of the 3rd and 4th Maxwell's Equation / FIT-Diskretisierung der 3. und 4. Maxwellschen Gleichung

Maxwell's equations in integral form / Maxwellsche Gleichungen in Integralform

FIT
Maxwell's grid equations / Maxwellsche Gittergleichungen

$$\frac{\mathrm{d}}{\mathrm{d}t}\iint_S \underline{\mathbf{B}}(\underline{\mathbf{R}},t)\bullet\underline{\mathbf{dS}} = -\oint_{C=\partial S}\underline{\mathbf{E}}(\underline{\mathbf{R}},t)\bullet\underline{\mathbf{dR}} - \iint_S \underline{\mathbf{J}}_\mathrm{m}(\underline{\mathbf{R}},t)\bullet\underline{\mathbf{dS}}$$

$$[\mathbf{S}]\frac{\mathrm{d}}{\mathrm{d}t}\{\mathbf{B}\}(t) = -[\mathbf{curl}][\mathbf{R}]\{\mathbf{E}\}(t) \quad -[\mathbf{S}]\{\mathbf{J}_\mathrm{m}\}(t)$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\iint_S \underline{\mathbf{D}}(\underline{\mathbf{R}},t)\bullet\underline{\mathbf{dS}} = \oint_{C=\partial S}\underline{\mathbf{H}}(\underline{\mathbf{R}},t)\bullet\underline{\mathbf{dR}} - \iint_S \underline{\mathbf{J}}_\mathrm{e}(\underline{\mathbf{R}},t)\bullet\underline{\mathbf{dS}}$$

$$\widetilde{[\boldsymbol{\varepsilon}]}\widetilde{[\mathbf{S}]}\frac{\mathrm{d}}{\mathrm{d}t}\{\mathbf{E}\}(t) = \widetilde{[\mathbf{curl}]}\widetilde{[\mathbf{v}]}\widetilde{[\mathbf{R}]}\{\mathbf{B}\}(t) - \widetilde{[\mathbf{S}]}\{\mathbf{J}_\mathrm{e}\}(t)$$

$$\left.\begin{aligned}&\oiint_{S=\partial V}\underline{\mathbf{D}}(\underline{\mathbf{R}},t)\bullet\underline{\mathbf{dS}} = \iiint_V \rho_\mathrm{e}(\underline{\mathbf{R}},t)\,dV\\&\oiint_{S=\partial V}\underline{\mathbf{B}}(\underline{\mathbf{R}},t)\bullet\underline{\mathbf{dS}} = \iiint_V \rho_\mathrm{m}(\underline{\mathbf{R}},t)\,dV\end{aligned}\right\} \ ?$$

End of Lecture 9 /
Ende der 9. Vorlesung