Numerical Methods of Electromagnetic Field Theory II (NFT II) Numerische Methoden der Elektromagnetischen Feldtheorie II (NFT II) /

1st Lecture / 1. Vorlesung

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- Finite Difference Time Domain (FDTD) Method / Methode der Finiten Differenzen im Zeitbereich
- Finite Element (FE) Method / Finite Elemente (FE) Methode
- Finite Volume (FV) Method / Finite Volumen (FV) Methode
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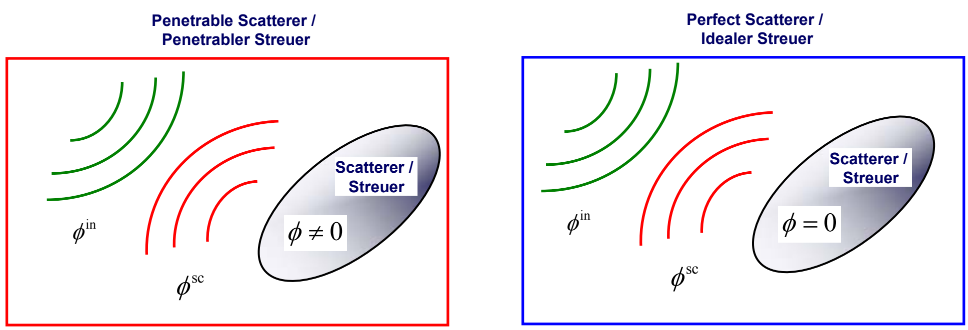
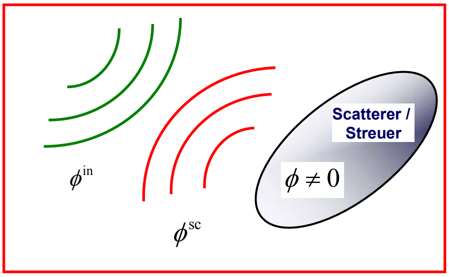


$$\phi = \phi^{\mathrm{in}} + \phi^{\mathrm{sc}}$$

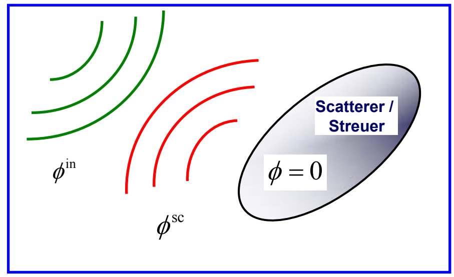
The Total Wavefield must Satisfy at the Boundary of the Scatterer:
Transition Conditions for a Penetrable Scatterer
Boundary Conditions for a Perfect Scatterer /
Das Gesamtwellenfeld muss die folgend Bedingungen am Streuerrand erfüllen:
Übergangsbedingungen für einen penetrablen Streuer
Randbedingungen für ein idealen Streuer

Scalar Scattering Problem (Outline) – Boundary and Transition Conditions / Skalares Streuproblem (Überblick) – Rand- und Übergangsbedingungen

Penetrable Scatterer / Penetrabler Streuer



Perfect Scatterer / Idealer Streuer



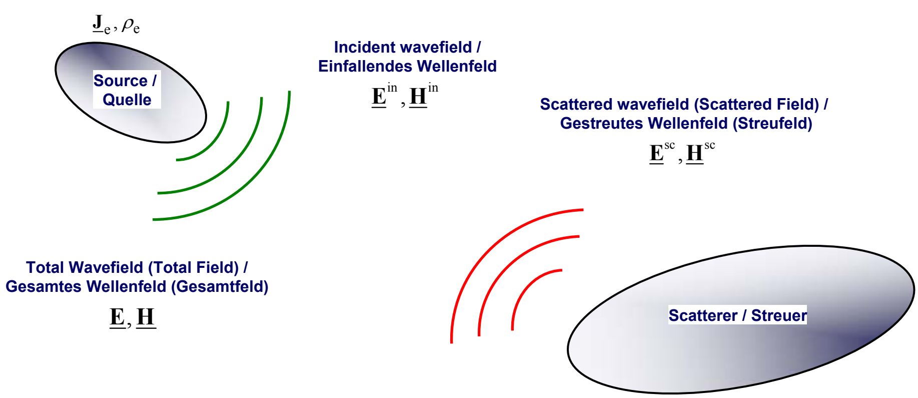
$$\phi = \begin{cases} \text{continuous /} \\ \text{stetig} \end{cases}$$

$$\frac{\partial}{\partial n}\phi = \begin{cases} \text{continuous /} \\ \text{stetig} \end{cases}$$

$$\phi = 0$$ Dirichlet Boundary Condition / Dirichlet-Randbedingung

$$\frac{\partial}{\partial n}\phi = 0$$ Neumann Boundary Condition / Neumann-Randbedingung

Electromagnetic (EM) Scattering Problem / Elektromagnetisches (EM) Streuproblem



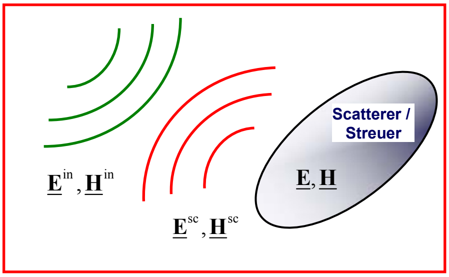
$$\underline{\mathbf{E}} = \underline{\mathbf{E}}^{\mathrm{in}} + \underline{\mathbf{E}}^{\mathrm{sc}}$$

$$\underline{\mathbf{H}} = \underline{\mathbf{H}}^{\mathrm{in}} + \underline{\mathbf{H}}^{\mathrm{sc}}$$

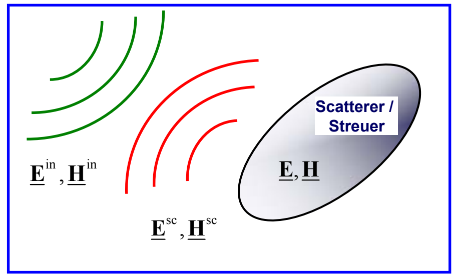
Superposition of Incident and Scattered Wavefield / Superposition von einfallenden und gestreutem Wellenfeld

Electromagnetic Scattering Problem (Outline) – Boundary and Transition Conditions / Electromagnetic Streuproblem (Überblick) – Rand- und Übergangsbedingungen

Penetrable Scatterer / Penetrabler Streuer



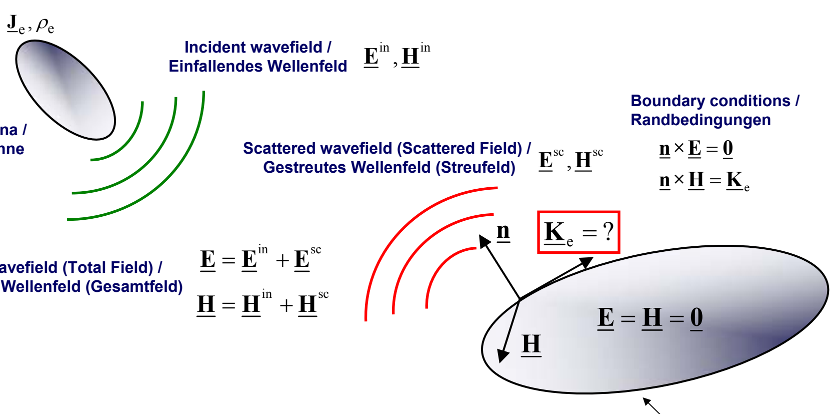
Perfect Scatterer / Idealer Streuer



$$\underline{\mathbf{E}} = \underline{\mathbf{E}}^{\text{in}} + \underline{\mathbf{E}}^{\text{sc}}$$
$$\underline{\mathbf{H}} = \underline{\mathbf{H}}^{\text{in}} + \underline{\mathbf{H}}^{\text{sc}}$$

The Total Wavefield must Satisfy at the Boundary of the Scatterer:
Transition Conditions for a Penetrable Scatterer
Boundary Conditions for a Perfect Scatterer /
Das Gesamtwellenfeld muss die folgend Bedingungen am Streuerrand erfüllen:
Übergangsbedingungen für einen penetrablen Streuer
Randbedingungen für ein idealen Streuer

Electromagnetic (EM) Scattering Problem / Elektromagnetisches (EM) Streuproblem



Boundary conditions / Randbedingungen

$$\underline{\mathbf{n}} \times \underline{\mathbf{E}} = \mathbf{0}$$
$$\underline{\mathbf{n}} \times \underline{\mathbf{H}} = \underline{\mathbf{K}}_{\text{e}}$$

Total Wavefield (Total Field) / Gesamtes Wellenfeld (Gesamtfeld)

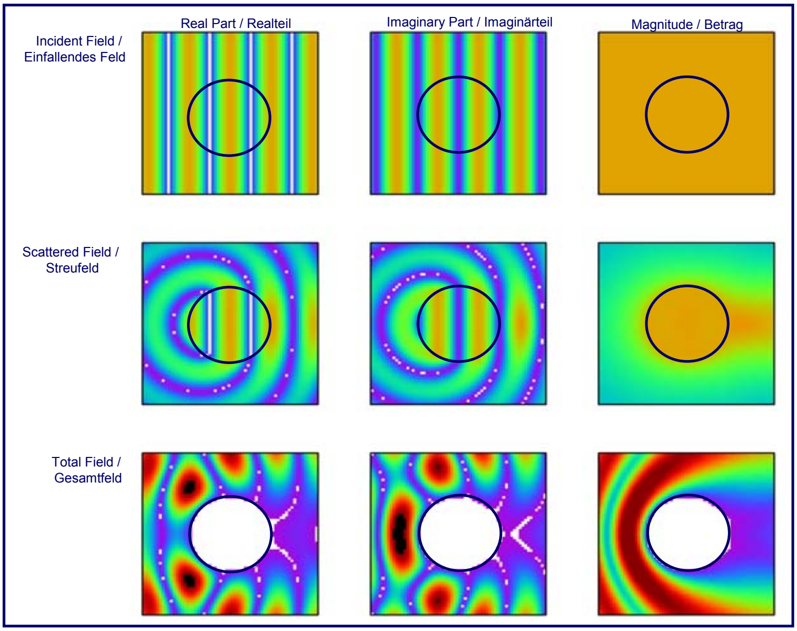
$$\underline{\mathbf{E}} = \underline{\mathbf{E}}^{\text{in}} + \underline{\mathbf{E}}^{\text{sc}}$$
$$\underline{\mathbf{H}} = \underline{\mathbf{H}}^{\text{in}} + \underline{\mathbf{H}}^{\text{sc}}$$

$\underline{\mathbf{n}} \times \underline{\mathbf{H}} = \underline{\mathbf{K}}_{\text{e}} = ?$ ← **Unknown induced electric surface current density / Unbekannten induzierten elektrischer Flächenstrom**

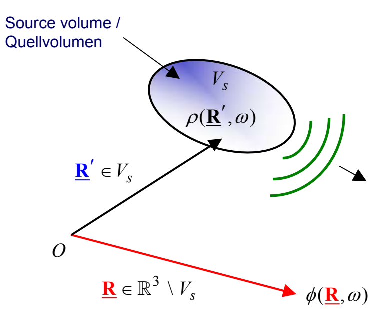
$\sigma_{\text{e}} \to \infty$

PEC: perfectly electrically conducting / IEL: ideal elektrisch leitend

Diffraction of an EM Plane Wave on a Circular PEC Cylinder – TM Case / Beugung einer EM Ebenen Welle an einem kreisrunden IEL-Zylinder – TM-Fall



Scalar Huygens' Principle – Scalar Wave Fields / Skalares Huygenssches Prinzip – Skalare Wellenfelder



Helmholtz Equation – Reduced Wave Equation /
Helmholtz-Gleichung – Reduzierte Wellengleichung

$$(\Delta + k_0^2)\,\phi(\mathbf{R},\omega) = -\frac{1}{\varepsilon_0}\,\rho_e(\mathbf{R},\omega)$$

Wave Number / Wellenzahl $\quad k_0 = \dfrac{\omega}{c_0}$

Sommerfeld's Radiation Condition /
Sommerfeldsche Ausstrahlungsbedingung

$$\lim_{R\to\infty}\left[\frac{\partial}{\partial R}\phi(\mathbf{R},\omega) + \mathrm{j}k_0\phi(\mathbf{R},\omega)\right] = 0$$

$$\phi(\mathbf{R},\omega) = \frac{1}{\varepsilon_0}\iiint_{\mathbf{R}'\in V_S} G(\mathbf{R}-\mathbf{R}',\omega)\,\rho_e(\mathbf{R}',\omega)\,\mathrm{d}^3\mathbf{R}'$$

Scalar 3-D Green's function of free-space /
Skalare 3D-Greensche Funktion des Freiraumes

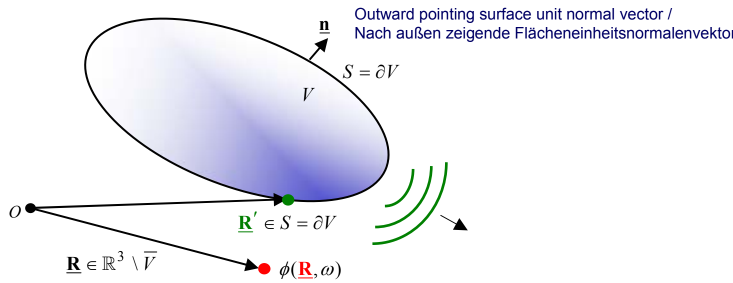
$$G(\mathbf{R}-\mathbf{R}',\omega) = \frac{\mathrm{e}^{\mathrm{j}k_0|\mathbf{R}-\mathbf{R}'|}}{4\pi|\mathbf{R}-\mathbf{R}'|}$$

$$(\Delta + k_0^2)\,G(\mathbf{R}-\mathbf{R}',\omega) = -\delta(\mathbf{R}-\mathbf{R}')$$

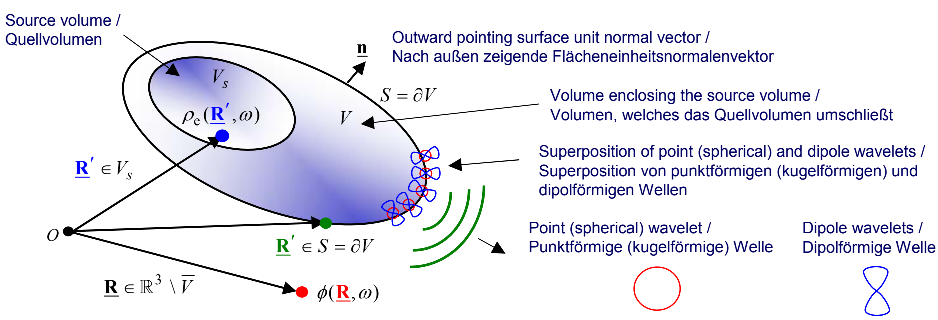
Scalar Huygens' Principle – Helmholtz Integral / Skalares Huygenssches Prinzip – Helmholtz-Integral

$$\phi(\underline{\mathbf{R}},\omega) = \underbrace{\oint\!\!\!\oint_{\underline{\mathbf{R}}'\in S=\partial V}\left[\phi(\underline{\mathbf{R}}',\omega)\nabla' G(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega) - G(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega)\nabla'\phi(\underline{\mathbf{R}}',\omega)\right]\bullet\underline{\mathbf{n}}'\,\mathrm{d}S'}_{=H_{S=\partial V}(\underline{\mathbf{R}},\omega)}$$

$$= \underbrace{H_{S=\partial V}(\underline{\mathbf{R}},\omega)}_{\text{Helmholtz Integral / Helmholtz-Integral}}$$



Scalar Huygens' Principle – Representation Theorem / Skalares Huygenssches Prinzip – Repräsentationstheorem



For / Für $\underline{\mathbf{R}}\in\mathbb{R}^3\setminus\overline{V}$ we obtain the so-called representations theorem / erhalten wir das so genannte Repräsentationstheorem

$$\phi(\underline{\mathbf{R}},\omega) = \begin{cases} \dfrac{1}{\varepsilon_0}\iiint_{\underline{\mathbf{R}}'\in V_s} G(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega)\,\rho_e(\underline{\mathbf{R}}',\omega)\,\mathrm{d}^3\underline{\mathbf{R}}' \\ \oint\!\!\!\oint_{\underline{\mathbf{R}}'\in S=\partial V}\left[\phi(\underline{\mathbf{R}}',\omega)\nabla' G(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega) - G(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega)\nabla'\phi(\underline{\mathbf{R}}',\omega)\right]\bullet\underline{\mathbf{n}}'\,\mathrm{d}S' \end{cases}$$

Scalar 3-D Green's function of free-space / Skalare 3D-Greensche Funktion des Freiraumes

$$G(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega) = \frac{\mathrm{e}^{\mathrm{j}k_0|\underline{\mathbf{R}}-\underline{\mathbf{R}}'|}}{4\pi|\underline{\mathbf{R}}-\underline{\mathbf{R}}'|}$$

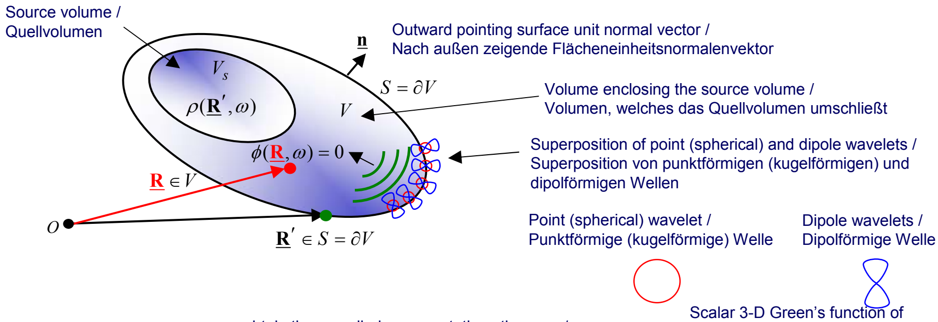
ϕ: Density of double-layer potential / Dichte des Doppelschichtpotentials

$\nabla' G$: Dipole wavelet / Dipolförmige Welle

G: Spherical wavelet / Kugelförmige Welle

$\nabla'\phi$: Density of single-layer potential / Dichte des Einfachschichtpotentials

Scalar Huygens' Principle – Extinction Theorem / Skalares Huygenssches Prinzip – (Aus)Löschungsstheorem



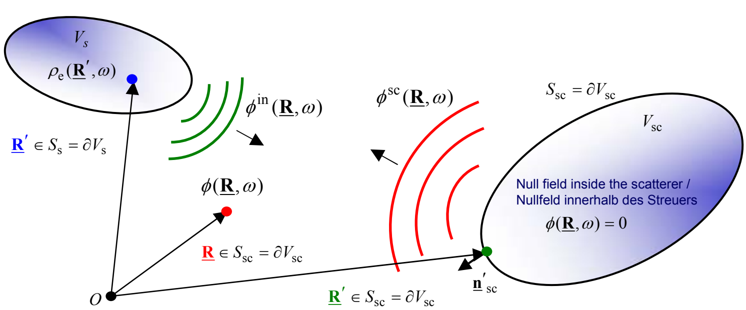
Scalar 3-D Green's function of free-space / Skalare 3D-Greensche Funktion des Freiraumes

For / Für $\mathbf{\underline{R}} \in \mathbb{R}^3 \setminus \overline{V}$ we obtain the so-called representations theorem / erhalten wir das so genannte Repräsentationstheorem

$$\oint\!\!\oint_{\mathbf{\underline{R}}' \in S = \partial V} \left[\phi(\mathbf{\underline{R}}',\omega) \nabla' G(\mathbf{\underline{R}} - \mathbf{\underline{R}}',\omega) - G(\mathbf{\underline{R}} - \mathbf{\underline{R}}',\omega) \nabla' \phi(\mathbf{\underline{R}}',\omega) \right] \cdot \mathbf{\underline{n}}' \, \mathrm{d}S' = \phi(\mathbf{\underline{R}},\omega) = 0 \qquad G(\mathbf{\underline{R}} - \mathbf{\underline{R}}',\omega) = \frac{\mathrm{e}^{\mathrm{j}k_0 |\mathbf{\underline{R}} - \mathbf{\underline{R}}'|}}{4\pi |\mathbf{\underline{R}} - \mathbf{\underline{R}}'|}$$

This means, that inside the volume V the Huygens wavelets interfere to zero. This zero wave field is called a null field (null field method) / Dies bedeutet, dass innerhalb des Volumens V die Huygens-Wellen (Wavelets) zu null interferieren. Dieses Null-Wellenfeld wird Nullfeld genannt (Nullfeld-Methode).

Scalar Huygens' Principle – Direct Scattering Problem / Skalares Huygenssches Prinzip – Direktes Streuproblem



$$\phi^{\mathrm{in}}(\mathbf{\underline{R}},\omega) = \frac{1}{\varepsilon_0} \iiint_{\mathbf{\underline{R}}' \in V_s} G(\mathbf{\underline{R}} - \mathbf{\underline{R}}',\omega)\, \rho_e(\mathbf{\underline{R}}',\omega) \, \mathrm{d}V(\mathbf{\underline{R}}')$$

$$\phi^{\mathrm{sc}}(\mathbf{\underline{R}},\omega) = \oint\!\!\oint_{\mathbf{\underline{R}}' \in S = \partial V} \left[\phi(\mathbf{\underline{R}}',\omega) \nabla' G(\mathbf{\underline{R}} - \mathbf{\underline{R}}',\omega) - G(\mathbf{\underline{R}} - \mathbf{\underline{R}}',\omega) \nabla' \phi(\mathbf{\underline{R}}',\omega) \right] \cdot \mathbf{\underline{n}}' \, \mathrm{d}S'(\mathbf{\underline{R}}')$$

$$\phi(\mathbf{\underline{R}},\omega) = \phi^{\mathrm{in}}(\mathbf{\underline{R}},\omega) + \phi^{\mathrm{sc}}(\mathbf{\underline{R}},\omega)$$

$$G(\mathbf{\underline{R}} - \mathbf{\underline{R}}',\omega) = \frac{\mathrm{e}^{\mathrm{j}k_0 |\mathbf{\underline{R}} - \mathbf{\underline{R}}'|}}{4\pi |\mathbf{\underline{R}} - \mathbf{\underline{R}}'|}$$

Scalar Huygens' Principle – Direct Scattering Problem / Skalares Huygenssches Prinzip – Direktes Streuproblem

$$\phi^{\mathrm{sc}}(\underline{\mathbf{R}},\omega) = \oiint_{\underline{\mathbf{R}}'\in S=\partial V}\Big[\phi(\underline{\mathbf{R}}',\omega)\nabla' G(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega) - G(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega)\nabla'\phi(\underline{\mathbf{R}}',\omega)\Big]\bullet\underline{\mathbf{n}}'\,\mathrm{d}S'$$

$$= \oiint_{\underline{\mathbf{R}}'\in S=\partial V}\Big[\phi(\underline{\mathbf{R}}',\omega)\nabla' G(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega)\bullet\underline{\mathbf{n}}' - G(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega)\nabla'\phi(\underline{\mathbf{R}}',\omega)\bullet\underline{\mathbf{n}}'\Big]\mathrm{d}S'$$

$$= \oiint_{\underline{\mathbf{R}}'\in S=\partial V}\Big\{\phi(\underline{\mathbf{R}}',\omega)\Big[\nabla' G(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega)\Big]\bullet\underline{\mathbf{n}}' - G(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega)\Big[\nabla'\phi(\underline{\mathbf{R}}',\omega)\Big]\bullet\underline{\mathbf{n}}'\Big\}\mathrm{d}S'$$

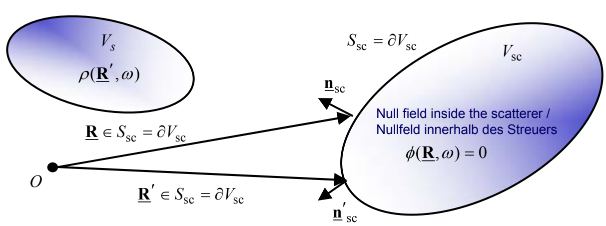
$$= \oiint_{\underline{\mathbf{R}}'\in S=\partial V}\Big\{\phi(\underline{\mathbf{R}}',\omega)\underline{\mathbf{n}}'\bullet\Big[\nabla' G(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega)\Big] - G(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega)\underline{\mathbf{n}}'\bullet\Big[\nabla'\phi(\underline{\mathbf{R}}',\omega)\Big]\Big\}\mathrm{d}S'$$

$$\underline{\mathbf{n}}'\bullet\nabla' = \frac{\partial}{\partial n'}$$

Normal Derivative /
Normalenableitung

$$\phi^{\mathrm{sc}}(\underline{\mathbf{R}},\omega) = \oiint_{\underline{\mathbf{R}}'\in S=\partial V}\Big[\phi(\underline{\mathbf{R}}',\omega)\frac{\partial}{\partial n'}G(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega) - G(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega)\frac{\partial}{\partial n'}\phi(\underline{\mathbf{R}}',\omega)\Big]\mathrm{d}S'$$

Scalar Integral Equations of the 1st and 2nd Kind / Skalare Integralgleichungen der 1. und 2. Art



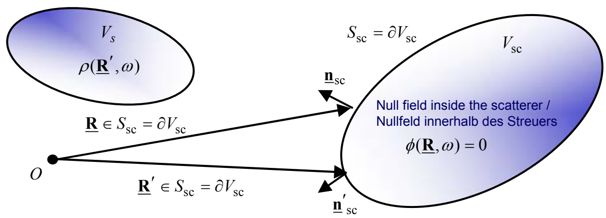
$$\lim_{\underline{\mathbf{R}}\to S_{\mathrm{sc}}}\left\{\phi(\underline{\mathbf{R}},\omega) = \phi^{\mathrm{in}}(\underline{\mathbf{R}},\omega) + \underbrace{\oiint_{\underline{\mathbf{R}}'\in S=\partial V}\Big[\phi(\underline{\mathbf{R}}',\omega)\frac{\partial}{\partial n'}G(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega) - G(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega)\frac{\partial}{\partial n'}\phi(\underline{\mathbf{R}}',\omega)\Big]\mathrm{d}S'}_{=\phi^{\mathrm{sc}}(\underline{\mathbf{R}},\omega)}\right\}$$

$$\lim_{\underline{\mathbf{R}}\to S_{\mathrm{sc}}}\left\{\oiint_{\underline{\mathbf{R}}'\in S=\partial V}\Big[\phi(\underline{\mathbf{R}}',\omega)\frac{\partial}{\partial n'}G(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega) - G(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega)\frac{\partial}{\partial n'}\phi(\underline{\mathbf{R}}',\omega)\Big]\mathrm{d}S'\right\}$$

$$= \frac{1}{2}\phi(\underline{\mathbf{R}},\omega) + \oiint_{\underline{\mathbf{R}}'\in S=\partial V}\Big[\phi(\underline{\mathbf{R}}',\omega)\frac{\partial}{\partial n'}G(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega) - G(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega)\frac{\partial}{\partial n'}\phi(\underline{\mathbf{R}}',\omega)\Big]\mathrm{d}S' \quad \underline{\mathbf{R}}'\in S_{\mathrm{sc}}$$

$$\frac{1}{2}\phi(\underline{\mathbf{R}},\omega) = \phi^{\mathrm{in}}(\underline{\mathbf{R}},\omega) + \underbrace{\oiint_{\underline{\mathbf{R}}'\in S=\partial V}\Big[\phi(\underline{\mathbf{R}}',\omega)\frac{\partial}{\partial n'}G(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega) - G(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega)\frac{\partial}{\partial n'}\phi(\underline{\mathbf{R}}',\omega)\Big]\mathrm{d}S'}_{=\phi^{\mathrm{sc}}(\underline{\mathbf{R}},\omega)} \qquad \underline{\mathbf{R}}\in S_{\mathrm{sc}}$$

Scalar Integral Equations of the 1st and 2nd Kind / Skalare Integralgleichungen der 1. und 2. Art



$$\frac{1}{2}\phi(\underline{\mathbf{R}},\omega) = \phi^{\mathrm{in}}(\underline{\mathbf{R}},\omega) + \underbrace{\oint\!\!\!\oint_{\underline{\mathbf{R}}'\in S=\partial V}\left[\phi(\underline{\mathbf{R}}',\omega)\frac{\partial}{\partial n'}G(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega) - G(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega)\frac{\partial}{\partial n'}\phi(\underline{\mathbf{R}}',\omega)\right]\mathrm{d}S'}_{=\phi^{\mathrm{sc}}(\underline{\mathbf{R}},\omega)} \qquad \underline{\mathbf{R}} \in S_{\mathrm{sc}}$$

For Perfect (Non-Penetrable) Scatterer we can prescribe one of the following Boundary Conditions: /
Für einen ideal (nicht penetrablen) Streuer können wir eines der beiden folgenden Randbedingungen vorgeben:

1. Dirichlet Boundary Condition / Dirichlet-Randbedingung $\qquad \phi(\underline{\mathbf{R}}',\omega) = 0 \quad \underline{\mathbf{R}}' \in S_{\mathrm{sc}}$
2. Neumann Boundary Condition / Neumann-Randbedingung $\qquad \underline{\mathbf{n}}' \bullet \nabla'\phi(\underline{\mathbf{R}}',\omega) = \frac{\partial}{\partial n'}\phi(\underline{\mathbf{R}}',\omega) = 0 \quad \underline{\mathbf{R}}' \in S_{\mathrm{sc}}$

Scalar Fredholm Integral Equation of 1st Kind / Skalare Fredholm Integralgleichung der 1. Art

$$\frac{1}{2}\phi(\underline{\mathbf{R}},\omega) = \phi^{\mathrm{in}}(\underline{\mathbf{R}},\omega) + \underbrace{\oint\!\!\!\oint_{\underline{\mathbf{R}}'\in S=\partial V}\left[\phi(\underline{\mathbf{R}}',\omega)\frac{\partial}{\partial n'}G(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega) - G(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega)\frac{\partial}{\partial n'}\phi(\underline{\mathbf{R}}',\omega)\right]\mathrm{d}S'}_{=\phi^{\mathrm{sc}}(\underline{\mathbf{R}},\omega)} \qquad \underline{\mathbf{R}} \in S_{\mathrm{sc}}$$

1. Dirichlet Boundary Condition / Dirichlet-Randbedingung $\qquad \phi(\underline{\mathbf{R}}',\omega) = 0 \quad \underline{\mathbf{R}}' \in S_{\mathrm{sc}}$

$$\frac{1}{2}\phi(\underline{\mathbf{R}},\omega) = 0 = \phi^{\mathrm{in}}(\underline{\mathbf{R}},\omega) + \phi^{\mathrm{sc}}(\underline{\mathbf{R}},\omega)$$

$$\phi^{\mathrm{in}}(\underline{\mathbf{R}},\omega) = -\phi^{\mathrm{sc}}(\underline{\mathbf{R}},\omega)$$

$$= -\underbrace{\oint\!\!\!\oint_{\underline{\mathbf{R}}'\in S=\partial V}\left[\underbrace{\phi(\underline{\mathbf{R}}',\omega)}_{=0}\frac{\partial}{\partial n'}G(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega) - G(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega)\frac{\partial}{\partial n'}\phi(\underline{\mathbf{R}}',\omega)\right]\mathrm{d}S'}_{=\phi^{\mathrm{sc}}(\underline{\mathbf{R}},\omega)}$$

$$= \underbrace{\oint\!\!\!\oint_{\underline{\mathbf{R}}'\in S=\partial V} G(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega)\frac{\partial}{\partial n'}\phi(\underline{\mathbf{R}}',\omega)\,\mathrm{d}S'}_{=\phi^{\mathrm{sc}}(\underline{\mathbf{R}},\omega)}$$

$$\phi^{\mathrm{in}}(\underline{\mathbf{R}},\omega) = \underbrace{\oint\!\!\!\oint_{\underline{\mathbf{R}}'\in S=\partial V} G(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega)\underbrace{\frac{\partial}{\partial n'}\phi(\underline{\mathbf{R}}',\omega)}_{\text{Unknown / Unbekannt}}\,\mathrm{d}S'}_{=\phi^{\mathrm{sc}}(\underline{\mathbf{R}},\omega)} \qquad \underline{\mathbf{R}} \in S_{\mathrm{sc}}$$

Fredholm Integral Equation of the 1st Kind /
Fredholmsche Integralgleichung der 1. Art

Scalar Integral Equations of the 2nd Kind / Skalare Integralgleichungen der 2. Art

$$\frac{1}{2}\phi(\underline{\mathbf{R}},\omega) = \phi^{\mathrm{in}}(\underline{\mathbf{R}},\omega) + \underbrace{\oint\!\!\!\oint_{\underline{\mathbf{R}}'\in S=\partial V}\left[\phi(\underline{\mathbf{R}}',\omega)\frac{\partial}{\partial n'}G(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega) - G(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega)\frac{\partial}{\partial n'}\phi(\underline{\mathbf{R}}',\omega)\right]\mathrm{d}S'}_{=\phi^{\mathrm{sc}}(\underline{\mathbf{R}},\omega)} \qquad \underline{\mathbf{R}}\in S_{\mathrm{sc}}$$

2. Neumann Boundary Condition / Neumann-Randbedingung

$$\underline{\mathbf{n}}'\bullet\nabla'\phi(\underline{\mathbf{R}}',\omega) = \frac{\partial}{\partial n'}\phi(\underline{\mathbf{R}}',\omega) = 0 \quad \underline{\mathbf{R}}'\in S_{\mathrm{sc}}$$

$$\frac{1}{2}\underbrace{\phi(\underline{\mathbf{R}},\omega)}_{\text{Unknown / Unbekannt}} = \phi^{\mathrm{in}}(\underline{\mathbf{R}},\omega) + \underbrace{\oint\!\!\!\oint_{\underline{\mathbf{R}}'\in S=\partial V}\underbrace{\phi(\underline{\mathbf{R}}',\omega)}_{\text{Unknown / Unbekannt}}\frac{\partial}{\partial n'}G(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega)\,\mathrm{d}S'}_{=\phi^{\mathrm{sc}}(\underline{\mathbf{R}},\omega)} \qquad \underline{\mathbf{R}}'\in S_{\mathrm{sc}}$$

Fredholm Integral Equation of the 2nd Kind /
Fredholmsche Integralgleichung der 2. Art

Unknown /
Unbekannt

Unknown /
Unbekannt

The Unknown Field Appears Inside and Outside the Integral /
Das unbekannte Feld steht außerhalb und innerhalb des Integrals

Scalar Integral Equations of the 1st and 2nd Kind / Skalare Integralgleichungen der 1. und 2. Art

Fredholm Integral Equation of the 1st Kind /
Fredholmsche Integralgleichung der 1. Art

$$\oint\!\!\!\oint_{\underline{\mathbf{R}}'\in S=\partial V} G(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega)\underbrace{\frac{\partial}{\partial n'}\phi(\underline{\mathbf{R}}',\omega)}_{\text{Unknown / Unbekannt}}\,\mathrm{d}S' = \phi^{\mathrm{in}}(\underline{\mathbf{R}},\omega) \qquad \underline{\mathbf{R}}\in S_{\mathrm{sc}}$$

Fredholm Integral Equation of the 2nd Kind /
Fredholmsche Integralgleichung der 2. Art

$$\frac{1}{2}\underbrace{\phi(\underline{\mathbf{R}},\omega)}_{\text{Unknown / Unbekannt}} - \oint\!\!\!\oint_{\underline{\mathbf{R}}'\in S=\partial V}\left[\frac{\partial}{\partial n'}G(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega)\right]\underbrace{\phi(\underline{\mathbf{R}}',\omega)}_{\text{Unknown / Unbekannt}}\,\mathrm{d}S' = \phi^{\mathrm{in}}(\underline{\mathbf{R}},\omega) \qquad \underline{\mathbf{R}}\in S_{\mathrm{sc}}$$

$$\overbrace{\phi(\underline{\mathbf{R}},\omega) = \phi^{\mathrm{in}}(\underline{\mathbf{R}},\omega) + \phi^{\mathrm{sc}}(\underline{\mathbf{R}},\omega)}$$

Solution of the Scalar Integral Equations of the 1st and 2nd Kind / Lösung der skalaren Integralgleichungen der 1. und 2. Art

Fredholm Integral Equation of the 1st Kind /
Fredholmsche Integralgleichung der 1. Art

$$\oiint_{\underline{\mathbf{R}}' \in S=\partial V} G(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega)\frac{\partial}{\partial n'}\phi(\underline{\mathbf{R}}',\omega)\,\mathrm{d}S' = \phi^{\mathrm{in}}(\underline{\mathbf{R}},\omega) \qquad \underline{\mathbf{R}} \in S_{\mathrm{sc}}$$

Discretization (Method of Moments) /
Diskretisierung (Momenten-Methode)

$$[G_{\mathrm{F1}}]\{\phi\}(\omega) = \{\phi^{\mathrm{in}}\}(\omega) \quad \Rightarrow \quad \{\phi\}(\omega) = [G_{\mathrm{F1}}]^{-1}\{\phi^{\mathrm{in}}\}(\omega)$$

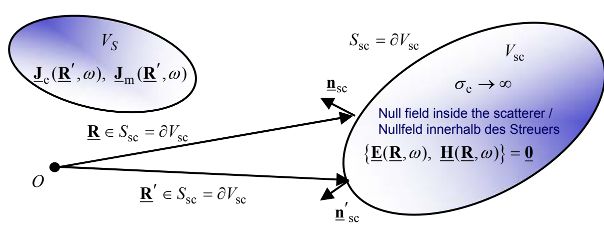
Fredholm Integral Equation of the 2nd Kind /
Fredholmsche Integralgleichung der 2. Art

$$\frac{1}{2}\phi(\underline{\mathbf{R}},\omega) - \oiint_{\underline{\mathbf{R}}' \in S=\partial V}\left[\frac{\partial}{\partial n'}G(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega)\right]\phi(\underline{\mathbf{R}}',\omega)\,\mathrm{d}S' = \phi^{\mathrm{in}}(\underline{\mathbf{R}},\omega) \qquad \underline{\mathbf{R}} \in S_{\mathrm{sc}}$$

Discretization (Method of Moments) /
Diskretisierung (Momenten-Methode)

$$[G_{\mathrm{F2}}]\{\phi\}(\omega) = \{\phi^{\mathrm{in}}\}(\omega) \quad \Rightarrow \quad \{\phi\}(\omega) = [G_{\mathrm{F2}}]^{-1}\{\phi^{\mathrm{in}}\}(\omega)$$

PEC Scatterer: – Franz, Stratton-Chu, and Franz-Lamor Version of EFIE and MFIE / IEL Streuer: Franz, Stratton-Chu und Franz-Lamor Version von EFIE und MFIE



Boundary condition for $\underline{\mathbf{R}} \in S_{\mathrm{sc}}$
Randbedingung für $\underline{\mathbf{R}} \in S_{\mathrm{sc}}$

$$\underline{\mathbf{n}}_{\mathrm{sc}} \times \underline{\mathbf{E}}(\underline{\mathbf{R}},\omega) = \underline{\mathbf{0}}$$
$$\to \underline{\mathbf{K}}_{\mathrm{m}}(\underline{\mathbf{R}},\omega) = \underline{\mathbf{0}}$$

Direct scattering problem for PEC scatterer /
Direktes Streuproblem für IEL Streuer

Different versions of EFIE and MFIE (for $\underline{\mathbf{R}} \in S_{\mathrm{sc}}$) / Verschiedene Versionen von EFIE und MFIE (für $\underline{\mathbf{R}} \in S_{\mathrm{sc}}$):

Franz version / Franz-Version:

$$\mathrm{j}\omega\mu_0 \mathrm{PV}_{\varepsilon}\,\underline{\mathbf{n}}_{\mathrm{sc}} \times \oiint_{\underline{\mathbf{R}}' \in S_{\mathrm{sc}}=\partial V_{\mathrm{sc}}} \underline{\mathbf{K}}_{\mathrm{e}}(\underline{\mathbf{R}}',\omega)\bullet\underline{\underline{\mathbf{G}}}(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega)\,\mathrm{d}^2\underline{\mathbf{R}}' = -\underline{\mathbf{n}}_{\mathrm{sc}} \times \underline{\mathbf{E}}^{\mathrm{in}}(\underline{\mathbf{R}},\omega)$$

$$\frac{1}{2}\underline{\mathbf{K}}_{\mathrm{e}}(\underline{\mathbf{R}},\omega) + \underline{\mathbf{n}}_{\mathrm{sc}} \times \oiint_{\underline{\mathbf{R}}' \in S_{\mathrm{sc}}=\partial V_{\mathrm{sc}}} \underline{\mathbf{K}}_{\mathrm{e}}(\underline{\mathbf{R}}',\omega)\bullet\underline{\underline{\mathbf{G}}}(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega)\,\mathrm{d}^2\underline{\mathbf{R}}' = \underline{\mathbf{n}}_{\mathrm{sc}} \times \underline{\mathbf{H}}^{\mathrm{in}}(\underline{\mathbf{R}},\omega)$$

Stratton-Chu version / Stratton-Chu-Version:

$$\underline{\mathbf{n}}_{\mathrm{sc}} \times \oiint_{\underline{\mathbf{R}}' \in S_{\mathrm{sc}}=\partial V_{\mathrm{sc}}}\left[\mathrm{j}\omega\mu_0\underline{\mathbf{K}}_{\mathrm{e}}(\underline{\mathbf{R}}',\omega)\,G(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega) + \tfrac{1}{\mathrm{j}\omega\varepsilon_0}\nabla'\bullet\underline{\mathbf{K}}_{\mathrm{e}}(\underline{\mathbf{R}}',\omega)\nabla' G(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega)\right]\mathrm{d}^2\underline{\mathbf{R}}' = -\underline{\mathbf{n}}_{\mathrm{sc}} \times \underline{\mathbf{E}}^{\mathrm{in}}(\underline{\mathbf{R}},\omega)$$

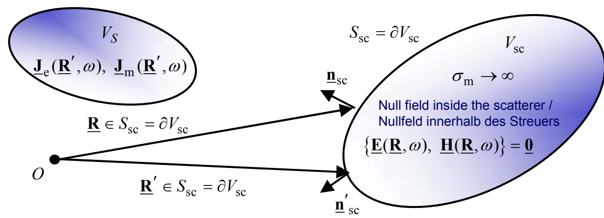
$$\frac{1}{2}\underline{\mathbf{K}}_{\mathrm{e}}(\underline{\mathbf{R}},\omega) + \underline{\mathbf{n}}_{\mathrm{sc}} \times \nabla \times \oiint_{\underline{\mathbf{R}}' \in S_{\mathrm{sc}}=\partial V_{\mathrm{sc}}} \underline{\mathbf{K}}_{\mathrm{e}}(\underline{\mathbf{R}}',\omega)\times\nabla' G(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega)\,\mathrm{d}^2\underline{\mathbf{R}}' = \underline{\mathbf{n}}_{\mathrm{sc}} \times \underline{\mathbf{H}}^{\mathrm{in}}(\underline{\mathbf{R}},\omega)$$

Franz-Larmor version / Franz-Larmor-Version:

$$\frac{1}{\mathrm{j}\omega\varepsilon_0}\underline{\mathbf{n}}_{\mathrm{sc}} \times \nabla \times \nabla \times \oiint_{\underline{\mathbf{R}}' \in S_{\mathrm{sc}}=\partial V_{\mathrm{sc}}} \underline{\mathbf{K}}_{\mathrm{e}}(\underline{\mathbf{R}}',\omega)\,G(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega)\,\mathrm{d}^2\underline{\mathbf{R}}' = -\underline{\mathbf{n}}_{\mathrm{sc}} \times \underline{\mathbf{E}}^{\mathrm{in}}(\underline{\mathbf{R}},\omega)$$

$$\frac{1}{2}\underline{\mathbf{K}}_{\mathrm{e}}(\underline{\mathbf{R}},\omega) + \underline{\mathbf{n}}_{\mathrm{sc}} \times \nabla \times \oiint_{\underline{\mathbf{R}}' \in S_{\mathrm{sc}}=\partial V_{\mathrm{sc}}} \underline{\mathbf{K}}_{\mathrm{e}}(\underline{\mathbf{R}}',\omega)\,G(\underline{\mathbf{R}}-\underline{\mathbf{R}}',\omega)\,\mathrm{d}^2\underline{\mathbf{R}}' = \underline{\mathbf{n}}_{\mathrm{sc}} \times \underline{\mathbf{H}}^{\mathrm{in}}(\underline{\mathbf{R}},\omega)$$

PMC Scatterer: – Franz, Stratton-Chu, and Franz-Lamor Version of EFIE and MFIE / IML Streuer: Franz, Stratton-Chu und Franz-Lamor Version von EFIE und MFIE



Boundary condition for $\mathbf{R} \in S_{sc}$
Randbedingung für $\mathbf{R} \in S_{sc}$

$$\mathbf{n}_{sc} \times \mathbf{H}(\mathbf{R},\omega) = \mathbf{0}$$
$$\to \mathbf{K}_e(\mathbf{R},\omega) = \mathbf{0}$$

Direct scattering problem for PEC scatterer / Direktes Streuproblem für IEL Streuer

Different versions of EFIE and MFIE (for $\mathbf{R} \in S_{sc}$) / Verschiedene Versionen von EFIE und MFIE (für $\mathbf{R} \in S_{sc}$):

Franz version / Franz-Version:

$$\frac{1}{2}\mathbf{K}_m(\mathbf{R},\omega) + \mathbf{n}_{sc} \times \oiint_{\mathbf{R}' \in S_{sc}=\partial V_{sc}} \mathbf{K}_m(\mathbf{R}',\omega) \cdot \underline{\underline{\mathbf{G}}}_m(\mathbf{R}-\mathbf{R}',\omega)\, d^2\mathbf{R}' = -\mathbf{n}_{sc} \times \mathbf{E}^{in}(\mathbf{R},\omega)$$

$$j\omega\varepsilon_0 \mathrm{PV}_\varepsilon \mathbf{n}_{sc} \times \oiint_{\mathbf{R}' \in S_{sc}=\partial V_{sc}} \mathbf{K}_m(\mathbf{R}',\omega) \cdot \underline{\underline{\mathbf{G}}}(\mathbf{R}-\mathbf{R}',\omega)\, d^2\mathbf{R}' = -\mathbf{n}_{sc} \times \mathbf{H}^{in}(\mathbf{R},\omega)$$

Stratton-Chu version / Stratton-Chu-Version:

$$\frac{1}{2}\mathbf{K}_m(\mathbf{R},\omega) - \mathbf{n}_{sc} \times \nabla \times \oiint_{\mathbf{R}' \in S_{sc}=\partial V_{sc}} \mathbf{K}_m(\mathbf{R}',\omega) \times \nabla' G(\mathbf{R}-\mathbf{R}',\omega)\, d^2\mathbf{R}' = -\mathbf{n}_{sc} \times \mathbf{E}^{in}(\mathbf{R},\omega)$$

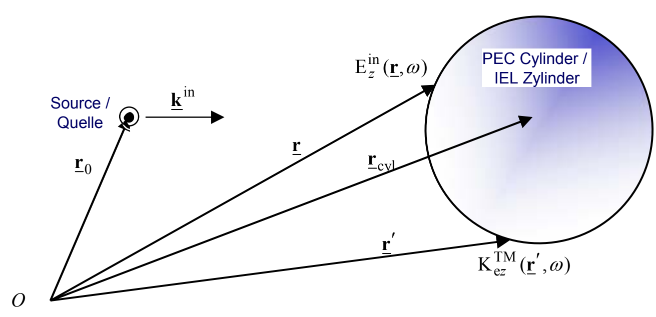
$$\mathbf{n}_{sc} \times \oiint_{\mathbf{R}' \in S_{sc}=\partial V_{sc}} \left[j\omega\varepsilon_0 \mathbf{K}_m(\mathbf{R}',\omega) G(\mathbf{R}-\mathbf{R}',\omega) + \frac{1}{j\omega\mu_0} \nabla' \cdot \mathbf{K}_m(\mathbf{R}',\omega) \nabla' G(\mathbf{R}-\mathbf{R}',\omega) \right] d^2\mathbf{R}' = -\mathbf{n}_{sc} \times \mathbf{H}^{in}(\mathbf{R},\omega)$$

Franz-Larmor version / Franz-Larmor-Version:

$$\frac{1}{2}\mathbf{K}_m(\mathbf{R},\omega) - \mathbf{n}_{sc} \times \nabla \times \oiint_{\mathbf{R}' \in S_{sc}=\partial V_{sc}} \mathbf{K}_m(\mathbf{R}',\omega) G(\mathbf{R}-\mathbf{R}',\omega)\, d^2\mathbf{R}' = -\mathbf{n}_{sc} \times \mathbf{E}^{in}(\mathbf{R},\omega)$$

$$\frac{1}{j\omega\mu_0}\mathbf{n}_{sc} \times \nabla \times \nabla \times \oiint_{\mathbf{R}' \in S_{sc}=\partial V_{sc}} \mathbf{K}_m(\mathbf{R}',\omega) G(\mathbf{R}-\mathbf{R}',\omega)\, d^2\mathbf{R}' = \mathbf{n}_{sc} \times \mathbf{H}^{in}(\mathbf{R},\omega)$$

EM Scattering by a Perfectly Electrically Conducting Cylinder: EFIE Discretized in the 2-D TM Case with Pulse Basis and Delta Testing Functions / EM-Streuung an einem ideal elektrisch leitendem Zylinder: EFIE diskretisiert im 2D-TM-Fall mit Impuls-Basisfunktionen und Delta-Testfunktionen



2-D Case / 2-D-Fall

$$\mathbf{R} = \underbrace{x\mathbf{e}_x + y\mathbf{e}_y}_{r\mathbf{e}_r(\varphi)} + z\mathbf{e}_z\Big|_{z=0} = \underbrace{r\mathbf{e}_r(\varphi)}_{=\mathbf{r}} + \underbrace{z\mathbf{e}_z}_{=\mathbf{0}} = \mathbf{r}$$

2-D TM EFIE / 2D-TM-EFIE

$$\underbrace{E_z^{in}(\mathbf{r},\omega)}_{\text{Known / Bekannt}} = -jkZ \int_{C_{sc}} \underbrace{G(\mathbf{r}-\mathbf{r}',\omega)}_{\text{Known / Bekannt}} \underbrace{K_{ez}^{TM}(\mathbf{r}',\omega)}_{\text{Unknown / Unbekannt}} d\mathbf{r}'$$

$$G(\mathbf{r}-\mathbf{r}',\omega) = \frac{j}{4} H_0^{(1)}(k\,|\mathbf{r}-\mathbf{r}'|)$$

Scalar 2-D Green's function of free-space / Skalare 2D-Greensche Funktion des Freiraumes

**End of Lecture 1 /
Ende der 1. Vorlesung**