Numerical Methods of Electromagnetic Field Theory II (NFT II) Numerische Methoden der Elektromagnetischen Feldtheorie II (NFT II) /

5th Lecture / 5. Vorlesung

Dr.-Ing. René Marklein

marklein@uni-kassel.de

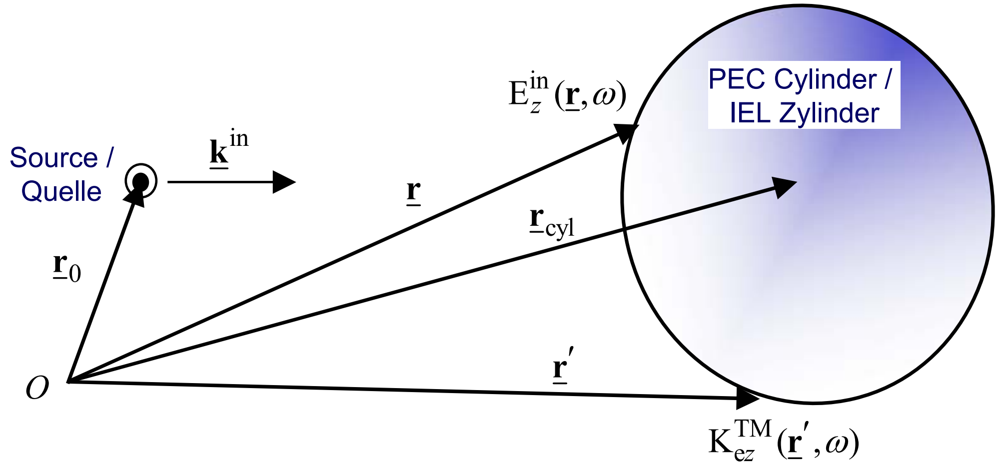
http://www.tet.e-technik.uni-kassel.de

http://www.uni-kassel.de/fb16/tet/marklein/index.html

Universität Kassel
Fachbereich Elektrotechnik / Informatik (FB 16)
Fachgebiet Theoretische Elektrotechnik (FG TET)
Wilhelmshöher Allee 71
Büro: Raum 2113 / 2115
D-34121 Kassel

University of Kassel
Dept. Electrical Engineering / Computer Science (FB 16)
Electromagnetic Field Theory (FG TET)
Wilhelmshöher Allee 71
Office: Room 2113 / 2115
D-34121 Kassel

EM Scattering by a Perfectly Electrically Conducting Cylinder: EFIE Discretized in the 2-D TM Case with Pulse Basis and Delta Testing Functions / EM-Streuung an einem ideal elektrisch leitendem Zylinder: EFIE diskretisiert im 2D-TM-Fall mit Impuls-Basisfunktionen und Delta-Testfunktionen



2-D Case / 2D-Fall

$$\underline{\mathbf{R}} = \underbrace{r\underline{\mathbf{e}}_r(\varphi)}_{=\underline{\mathbf{r}}} + \underbrace{z\underline{\mathbf{e}}_r(\varphi)}_{=\underline{\mathbf{0}}} = \underline{\mathbf{r}}$$

2-D PEC TM EFIE / 2D-IEL-TM-EFIE

$$-j\omega\mu_0 \oint_{\underline{\mathbf{r}}'\in C_{sc}=\partial S_{sc}} K_{ez}^{TM}(\underline{\mathbf{r}}',\omega)\, G\left(\underline{\mathbf{r}}-\underline{\mathbf{r}}',\omega\right) d\underline{\mathbf{r}}' = E_z^{in}(\underline{\mathbf{r}},\omega), \quad \underline{\mathbf{r}} \in C_{sc}$$

This is a *Fredholm integral equation of the 1. kind* in form of a *closed line integral* for the *unknown* electric surface current density for a *known* incident field. /
Dies ist eine *Fredholmsche Integralgleichung 1. Art* in Form eines *geschlossenen Linienintegrals* für die *unbekannte* elektrische Flächenladungsdichte für ein *bekanntes* einfallendes Feld.

$$G(\underline{\mathbf{r}}-\underline{\mathbf{r}}',\omega) = \frac{j}{4} H_0^{(1)}\left(k_0 \left|\underline{\mathbf{r}}-\underline{\mathbf{r}}'\right|\right)$$

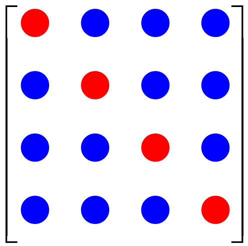
EM Scattering by a Perfectly Electrically Conducting Cylinder: EFIE Discretized in the 2-D TM Case with Pulse Basis and Delta Testing Functions / EM-Streuung an einem ideal elektrisch leitendem Zylinder: EFIE diskretisiert im 2D-TM-Fall mit Impuls-Basisfunktionen und Delta-Testfunktionen

2-D PEC TM EFIE / 2D-IEL-TM-EFIE

$$-\mathrm{j}\omega\mu_0 \oint_{\underline{\mathbf{r}}' \in C_{\mathrm{sc}} = \partial S_{\mathrm{sc}}} \mathrm{K}_{\mathrm{e}z}^{\mathrm{TM}}(\underline{\mathbf{r}}',\omega)\, G\left(\underline{\mathbf{r}} - \underline{\mathbf{r}}',\omega\right) \mathrm{d}\underline{\mathbf{r}}' = \mathrm{E}_z^{\mathrm{in}}(\underline{\mathbf{r}},\omega), \quad \underline{\mathbf{r}} \in C_{\mathrm{sc}}$$

We have to Consider Two Different Cases for the Elements of the Impedance Matrix /
Man unterscheidet zwei Verschiedene Fälle für die Elemente der Impedanzmatrix

$$\begin{bmatrix} Z_{11}(\omega) & Z_{12}(\omega) & \dots & Z_{1N}(\omega) \\ Z_{21}(\omega) & Z_{22}(\omega) & \dots & Z_{2N}(\omega) \\ \vdots & \vdots & \ddots & \vdots \\ Z_{N1}(\omega) & Z_{N2}(\omega) & \dots & Z_{NN}(\omega) \end{bmatrix} \Leftrightarrow$$



- Main Diagonal Elements / Hauptdiagonalelemente

  For / Für: $\underline{\mathbf{r}} = \underline{\mathbf{r}}'$ (Self Cell / Eigenzelle)

- Off Diagonal Elements / Nebendiagonalelemente

  For / Für $\underline{\mathbf{r}} \neq \underline{\mathbf{r}}'$

Main Diagonal Elements / Hauptdiagonalelemente

1. Flat Cell Approximation / Ebene-Zelle-Approximation
2. Power Series Expansion of the Hankel Function for Small Arguments / Potenzreihen-Approximation der Hankel-Funktion für kleine Argumente

Off Diagonal Elements / Nebendiagonalelemente

1. Flat Cell Approximation / Ebene-Zelle-Approximation
2. Application of the Midpoint Rule / Anwendung der Mittelpunktsregel

EM Scattering by a Perfectly Electrically Conducting Cylinder: EFIE Discretized in the 2-D TM Case with Pulse Basis and Delta Testing Functions / EM-Streuung an einem ideal elektrisch leitendem Zylinder: EFIE diskretisiert im 2D-TM-Fall mit Impuls-Basisfunktionen und Delta-Testfunktionen

Elements of the Impedance Matrix /
Elemente der Impedanzmatrix

$$Z_{mn}(\omega) = \frac{\omega\mu_0}{4}\Delta^{(n)} \begin{cases} 1 + \mathrm{j}\frac{2}{\pi}\left[\ln\left(\frac{k}{4}\Delta^{(n)}\right) + \gamma - 1\right] & m = n \\ \mathrm{H}_0^{(1)}(k\, r_{mn}) & m \neq n \end{cases}$$

Matrix Equation / Matrixgleichung

$$\underbrace{\left[Z\right]}_{=\mathrm{V/A}}(\omega)\underbrace{\left\{\mathrm{K}_{\mathrm{e}z}^{\mathrm{TM}}\right\}}_{=\mathrm{A/m}}(\omega) = \underbrace{\left\{\mathrm{E}_z^{\mathrm{in}}\right\}}_{=\mathrm{V/m}}(\omega)$$

Problem: Large Impedance Matrix /
Problem: Große Impedanzmatrix **!**

⇩

Iterative Solution via Conjugate Gradient (CG) Method /
Iterative Lösung durch Konjugierte Gradienten (KG) Methode

Solution of the Matrix Equation / Lösung der Matrixgleichung

$$\underbrace{\left\{\mathrm{K}_{\mathrm{e}z}^{\mathrm{TM}}\right\}}_{=\mathrm{A/m}}(\omega) = \underbrace{\left[Z\right]^{-1}}_{=\mathrm{A/V}}(\omega)\underbrace{\left\{\mathrm{E}_z^{\mathrm{in}}\right\}}_{=\mathrm{V/m}}(\omega)$$

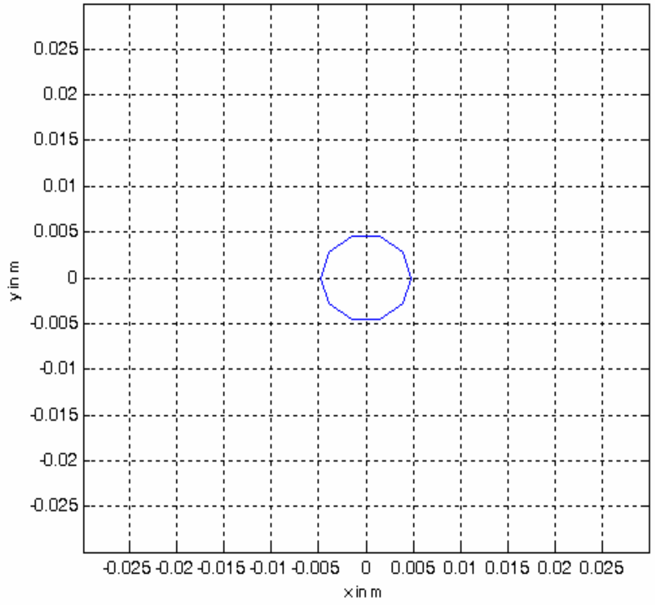
MATLAB Implementation / MATLAB-Implementierung

MATLAB Program to Generate the Geometry of a Circular Cylinder /
MATLAB-Programm zur Generierung der Geometrie eines kreisförmigen Zylinders

$$\underline{\mathbf{R}} = a\cos\varphi\,\underline{\mathbf{e}}_x + a\sin\varphi\,\underline{\mathbf{e}}_y \qquad 0 \le \varphi \le 2\pi$$

```
sca_grid.nodes = zeros(N+1,3);

sca_grid.number_of_nodes = N;

for j=1:N+1

  phi_m = 2.0*M_PI*real((j-1))/real(N);

  sca_grid.nodes(j,1) = a * cos( phi_m ); % x component
  sca_grid.nodes(j,2) = a * sin( phi_m );  % y component
  sca_grid.nodes(j,3) = 0;                    % z component

end

circumference = 2.0*M_PI*a;
```

Geometry of a Circular Cylinder /
Geometrie des kreisförmigen Zylinders



EM Scattering by a Perfectly Electrically Conducting Cylinder: EFIE Discretized in the 2-D TM Case with Pulse Basis and Delta Testing Functions / EM-Streuung an einem ideal elektrisch leitendem Zylinder: EFIE diskretisiert im 2D-TM-Fall mit Impuls-Basisfunktionen und Delta-Testfunktionen

Elements of the Impedance Matrix /
Elemente der Impedanzmatrix

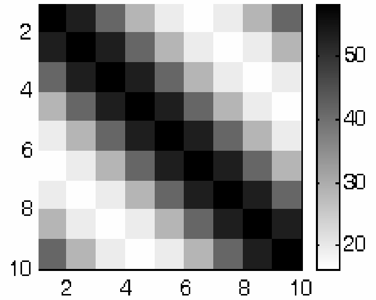
$$Z_{mn}(\omega) = \frac{\omega\mu_0}{4}\Delta^{(n)} \begin{cases} 1 + \mathrm{j}\dfrac{2}{\pi}\left[\ln\left(\dfrac{k}{4}\Delta^{(n)}\right) + \gamma - 1\right] & m = n \\ \mathrm{H}_0^{(1)}(k\, r_{mn}) & m \neq n \end{cases}$$

MATLAB Program to Compute the Impedance Matrix / MATLAB-Programm zur Berechnung der Impedanzmatrix

```
for j=1:N %___loop for r_m the obervation point____*/

 %/*___Coordinates of the observation point r_m______*/
 vrm(1) = ( sca_grid.nodes(j,1) + sca_grid.nodes(j+1,1) )/2;
 vrm(2) = ( sca_grid.nodes(j,2) + sca_grid.nodes(j+1,2) )/2;
 vrm(3) = ( sca_grid.nodes(j,3) + sca_grid.nodes(j+1,3) )/2;

 for i=1:N

  %/*___vrpn is the phase center of the ith element____*/
  vrn(1) = ( sca_grid.nodes(i,1) + sca_grid.nodes(i+1,1) )/2;
  vrn(2) = ( sca_grid.nodes(i,2) + sca_grid.nodes(i+1,2) )/2;
  vrn(3) = ( sca_grid.nodes(i,3) + sca_grid.nodes(i+1,3) )/2;

  %/*___Difference vector vd of the ith element________*/
  vd(1) = sca_grid.nodes(i+1,1) - sca_grid.nodes(i,1);
  vd(2) = sca_grid.nodes(i+1,2) - sca_grid.nodes(i,2);
  vd(3) = sca_grid.nodes(i+1,3) - sca_grid.nodes(i,3);

  Delta = norm(vd);

  if j == i

    Z(i,j) = 0.25*omega*mu0*Delta
          * complex(1.0,2.0/M_PI *( log(0.25*k*Delta) + M_GAMMA-1.0
));

  else %/*___Calculate off-diagonal_*/

    vrmn(1) = vrm(1) - vrn(1);
    vrmn(2) = vrm(2) - vrn(2);
    vrmn(3) = vrm(3) - vrn(3);

    rmn = norm(vrmn);

    %/*___Calculate Hankel function: H^1_0(z) _______________*/
    k_rmn = k * rmn;
    z       = complex( k_rmn, 0);      %/*___Complex argument___*/

    nu   = 0; %/*__initial order: n=0_*/
    kind = 1; %/*__compute 1st kind___*/

    [H10, ierr] = besselh(nu,kind,z);

    Z(i,j) = 0.25 * omega * mu0 * Delta * H10;

  end
 end
end
```

EM Scattering by a Perfectly Electrically Conducting Cylinder: EFIE Discretized in the 2-D TM Case with Pulse Basis and Delta Testing Functions / EM-Streuung an einem ideal elektrisch leitendem Zylinder: EFIE diskretisiert im 2D-TM-Fall mit Impuls-Basisfunktionen und Delta-Testfunktionen

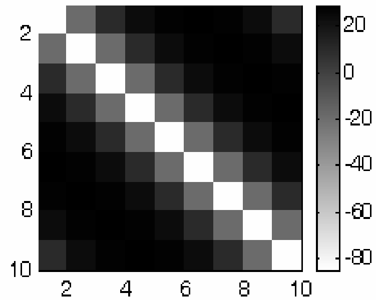
$\mathrm{Re}\{[\mathbf{Z}](\omega)\}$



Elements of the Impedance Matrix / Elemente der Impedanzmatrix

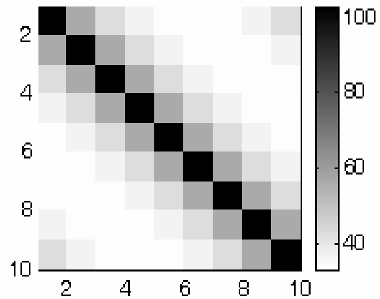
$$Z_{mn}(\omega) = \frac{\omega\mu_0}{4}\Delta^{(n)} \begin{cases} 1 + \mathrm{j}\frac{2}{\pi}\left[\ln\left(\frac{k}{4}\Delta^{(n)}\right) + \gamma - 1\right] & m = n \\ \mathrm{H}_0^{(1)}(k\, r_{mn}) & m \neq n \end{cases}$$

$\mathrm{Im}\{[\mathbf{Z}](\omega)\}$



$ka = 1, \qquad N = 10$

$\left|[\mathbf{Z}](\omega)\right|$



Iterative Methods for the Solution of Discrete Integral Equations / Iterative Methode zur Lösung von diskreten Integralgleichungen

CG Method – Conjugate Gradient (CG) Method

M. R. Hestenes & E. Stiefel, 1952

BiCG Method – Biconjugate Gradient (BiCG) Method

C. Lanczos, 1952
D. A. H. Jacobs, 1981
C. F. Smith et al., 1990
R. Barret et al., 1994

CGS Method – Conjugate Gradient Squared (CGS) Method (MATLAB Function)

P. Sonneveld, 1989

GMRES Method – Generalized Minimal – Residual (GMRES) Method

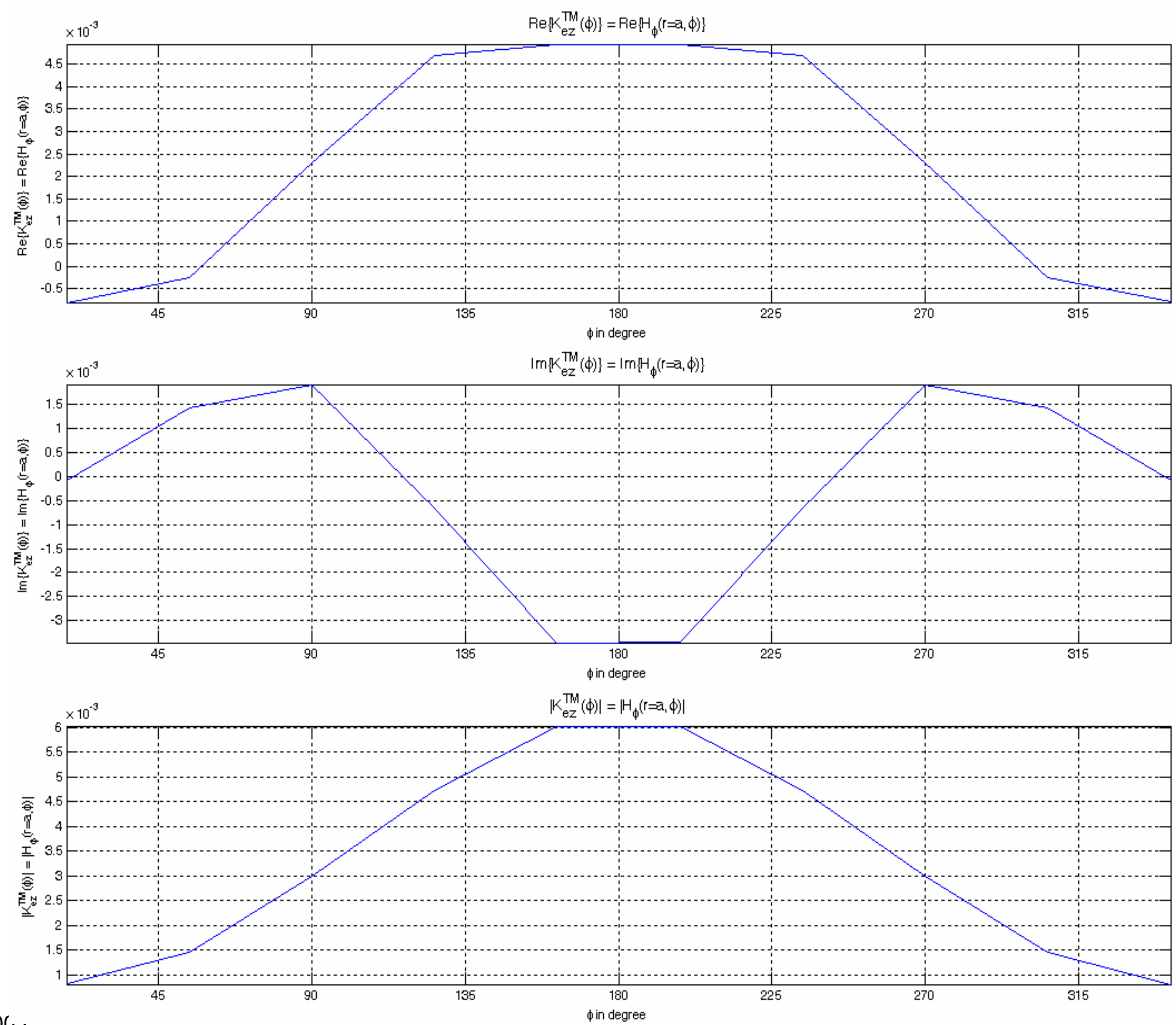
Y. Saad & M. H. Schultz, 1986
R. Barret et al., 1994
Y. Saad, 1996

QMR Method – Quasi–Minimal–Residual (QMR) Method

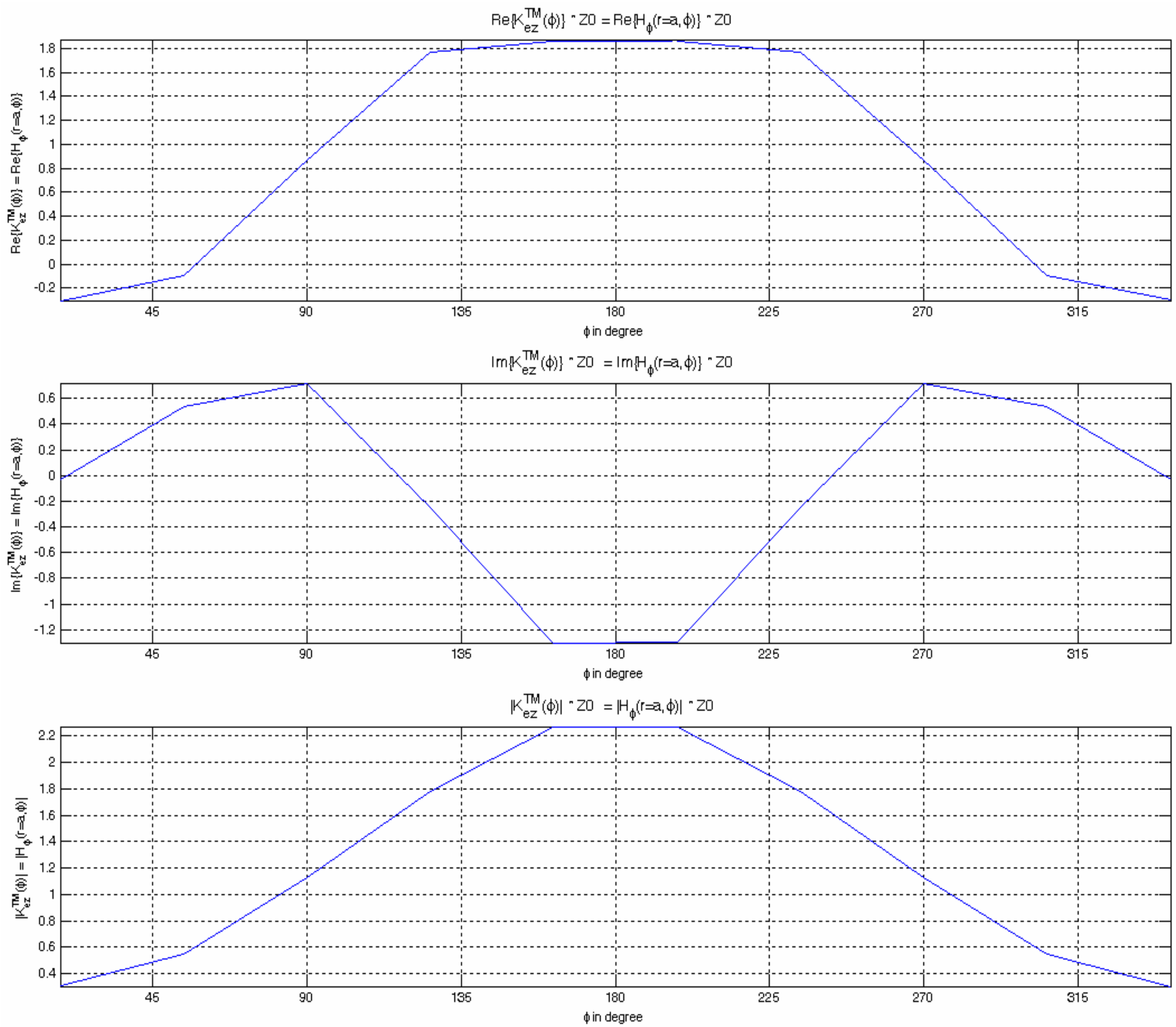
R. Freund & N. Nachtigal, 1990
N. Nachtigal, 1991
R. Barret et al., 1994
Y. Saad, 1996

MATLAB Function CGS – Conjugate Gradient Squared / MATLAB-Funktion CGS – Konjugierte Gradienten Quadriert

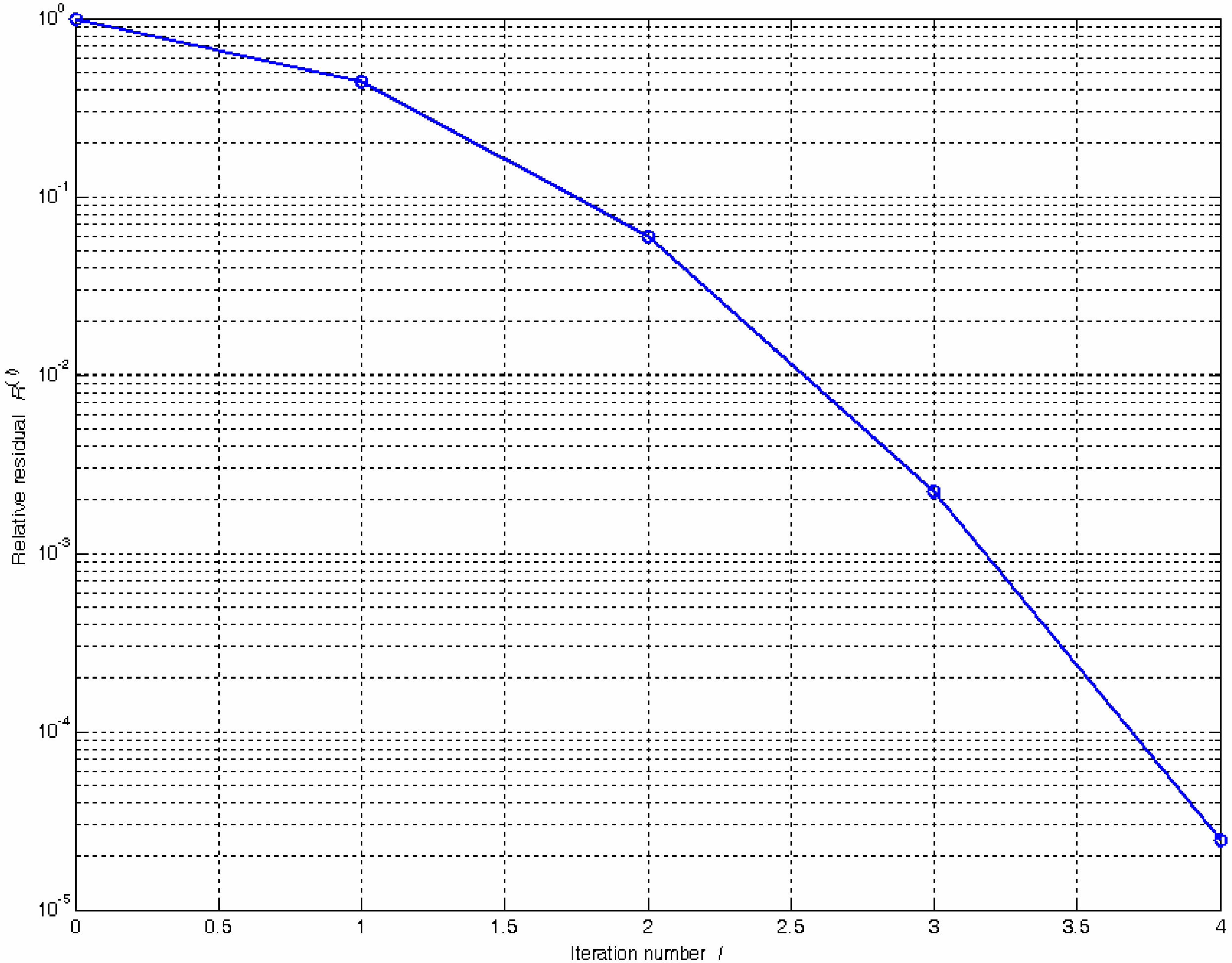
```
cgs Conjugate Gradients Squared method Syntaxx =
cgs(A,b)

cgs(A,b,tol)
cgs(A,b,tol,maxit)
cgs(A,b,tol,maxit,M)
cgs(A,b,tol,maxit,M1,M2)
cgs(A,b,tol,maxit,M1,M2,x0)
cgs(afun,b,tol,maxit,m1fun,m2fun,x0,p1,p2,...)

[x,flag] = cgs(A,b,...)
[x,flag,relres] = cgs(A,b,...)
[x,flag,relres,iter] = cgs(A,b,...)
[x,flag,relres,iter,resvec] = cgs(A,b,...)
```



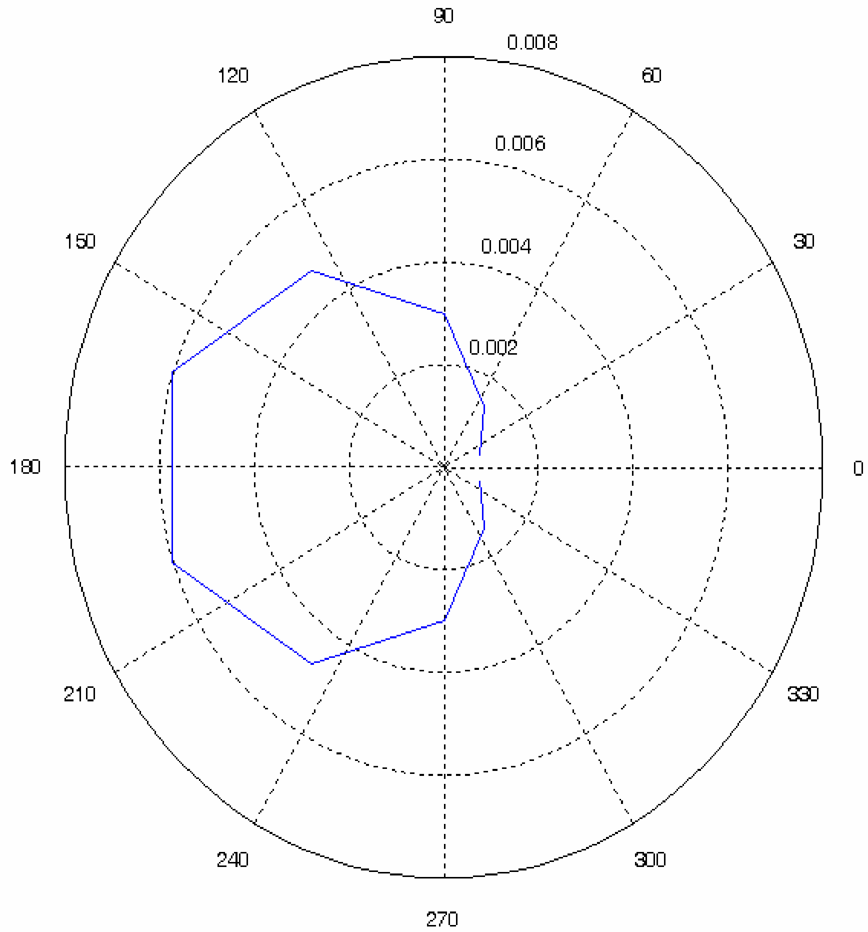
Re{$K_{ez}^{TM}(\phi)$} = Re{$H_\phi(r=a,\phi)$}
× 10⁻³
ϕ in degree
Im{$K_{ez}^{TM}(\phi)$} = Im{$H_\phi(r=a,\phi)$}
|$K_{ez}^{TM}(\phi)$| = |$H_\phi(r=a,\phi)$|



$Re\{K_{ez}^{TM}(\phi)\} * Z0 = Re\{H_\phi(r=a,\phi)\} * Z0$
$Re\{K_{ez}^{TM}(\phi)\} = Re\{H_\phi(r=a,\phi)\}$
ϕ in degree
$Im\{K_{ez}^{TM}(\phi)\} * Z0 = Im\{H_\phi(r=a,\phi)\} * Z0$
$Im\{K_{ez}^{TM}(\phi)\} = Im\{H_\phi(r=a,\phi)\}$
ϕ in degree
$|K_{ez}^{TM}(\phi)| * Z0 = |H_\phi(r=a,\phi)| * Z0$
$|K_{ez}^{TM}(\phi)| = |H_\phi(r=a,\phi)|$
ϕ in degree



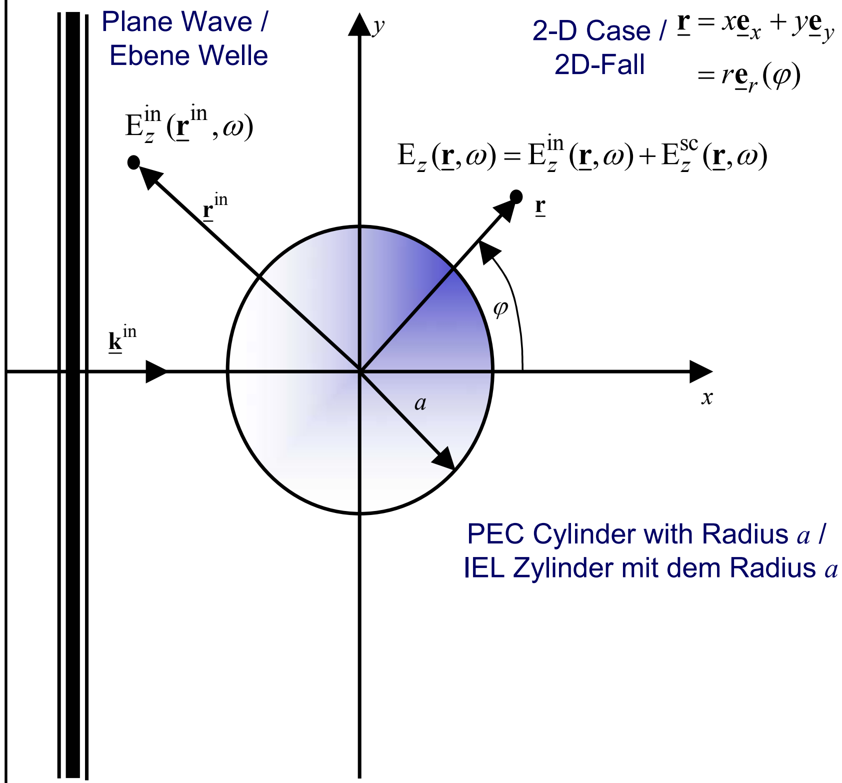
10^0
10^-1
10^-2
10^-3
10^-4
10^-5
0
0.5
1
1.5
2
2.5
3
3.5
4
Relative residual R^(l)
Iteration number l

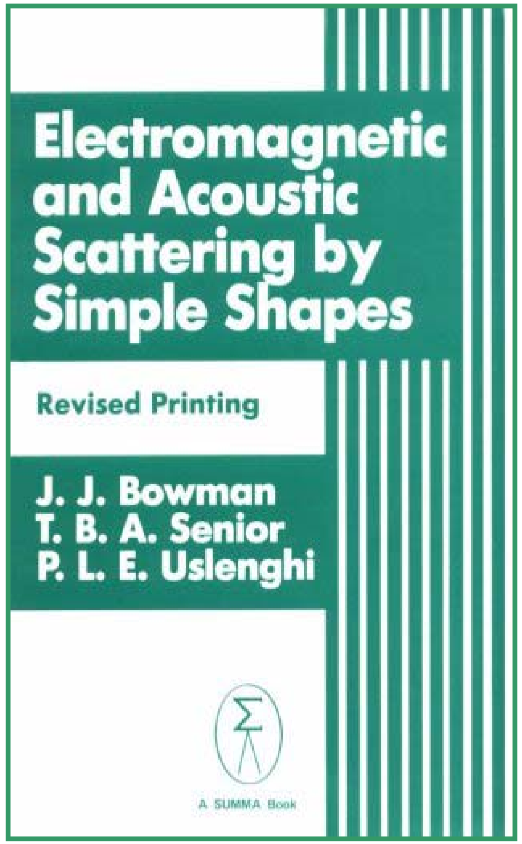

90
0.008
120
60
0.006
150
30
0.004
0.002
180
0
210
330
240
300
270

Diffraction of an EM Plane Wave on a Circular PEC Cylinder – TM Case / Beugung einer EM Ebenen Welle an einem kreisrunden IEL-Zylinder – TM-Fall

Separation of Variables
Analytic Solution in Terms of Eigenfunctions /
Separation der Variablen
Analytische Lösung in Form von Eigenfunktionen

J. J. Bowman, T. B. A. Senior, P. L. E. Uslenghi (Editors): *Electromagnetic and Acoustic Scattering by Simple Shapes*. Taylor & Francis Inc, New York, 1988.





Diffraction of an EM Plane Wave on a Circular PEC Cylinder – TM Case – Analytic Solution: Separation of Variables / Beugung einer EM Ebenen Welle an einem kreisrunden IEL-Zylinder – TM-Fall – Analytische Lösung: Separation der Variablen

Electric Field Strength of the Incident Wave /
Elektrische Feldstärke der einfallenden Welle

$$E_z^{\text{in}}(r,\varphi,\varphi_{\text{in}},\omega) = \underbrace{E_0(\omega)}_{=1\ \text{V/m}}\, \mathrm{e}^{\mathrm{j}\underline{\mathbf{k}}^{\text{in}}\bullet\underline{\mathbf{r}}}$$

Boundary Condition at the PEC Cylinder /
Randbedingung am IEL-Zylinder

$$E_z(r=a,\varphi,\varphi_{\text{in}},\omega) = E_z^{\text{in}}(r=a,\varphi,\varphi_{\text{in}},\omega) + E_z^{\text{sca}}(r=a,\varphi,\varphi_{\text{in}},\omega) = 0$$

Solution /
Lösung

Electric Field Strength of the Scattered Wave /
Elektrische Feldstärke der gestreuten Welle

$$E_z^{\text{sc}}(r,\varphi,\varphi_{\text{in}},\omega) = -\sum_{n=0}^{\infty} \varepsilon_n (-\mathrm{j})^n \frac{\mathrm{J}_n(ka)}{\mathrm{H}_n^{(1)}(ka)} \mathrm{H}_n^{(1)}(kr) \cos\left[n\left(\varphi-\varphi_{\text{in}}\right)\right]$$

Neumann Function /
Neumann-Funktion

$$\varepsilon_n = \begin{cases} 1 & n=0 \\ 2 & n=1,2,3,\ldots \end{cases}$$

Diffraction of an EM Plane Wave on a Circular PEC Cylinder – TM Case – Analytic Solution: Separation of Variables / Beugung einer EM Ebenen Welle an einem kreisrunden IEL-Zylinder – TM-Fall – Analytische Lösung: Separation der Variablen

Boundary Condition at the PEC Cylinder /
Randbedingung am IEL-Zylinder

$$\underline{\mathbf{n}} \times \underline{\mathbf{E}}(r = a, \varphi, \varphi_{\text{in}}, \omega) = \underline{\mathbf{0}}$$

$$E_z(r = a, \varphi, \varphi_{\text{in}}, \omega) = 0$$

Induced Electric Surface Current Density at /
Induzierte elektrische Flächenstromdichte bei $r = a$

$$\underline{\mathbf{n}} \times \underline{\mathbf{H}}(r = a, \varphi, \varphi_{\text{in}}, \omega) = \underline{\mathbf{K}}_{\text{e}}(r = a, \varphi, \varphi_{\text{in}}, \omega), \qquad \underline{\mathbf{n}} = \underline{\mathbf{e}}_R$$

$$\begin{aligned} K_{\text{e}z}^{\text{TM}}(\varphi, \varphi_{\text{in}}, \omega) &= H_\varphi(r = a, \varphi, \varphi_{\text{in}}, \omega) \\ &= H_\varphi^{\text{in}}(r = a, \varphi, \varphi_{\text{in}}, \omega) + H_\varphi^{\text{sc}}(r = a, \varphi, \varphi_{\text{in}}, \omega) \\ &= 2\frac{Y_0}{\pi}\frac{1}{ka}\sum_{n=0}^{\infty} \varepsilon_n \frac{(-\text{j})^n}{\text{H}_n^{(1)}(ka)} \cos\left[n\left(\varphi - \varphi_{\text{in}}\right)\right] \end{aligned}$$

$$K_{\text{e}z}^{\text{TM}}(\varphi, \varphi_{\text{in}}, \omega) = 2\frac{Y_0}{\pi}\frac{1}{ka}\sum_{n=0}^{\infty} \varepsilon_n \frac{(-\text{j})^n}{\text{H}_n^{(1)}(ka)} \cos\left[n\left(\varphi - \varphi_{\text{in}}\right)\right]$$

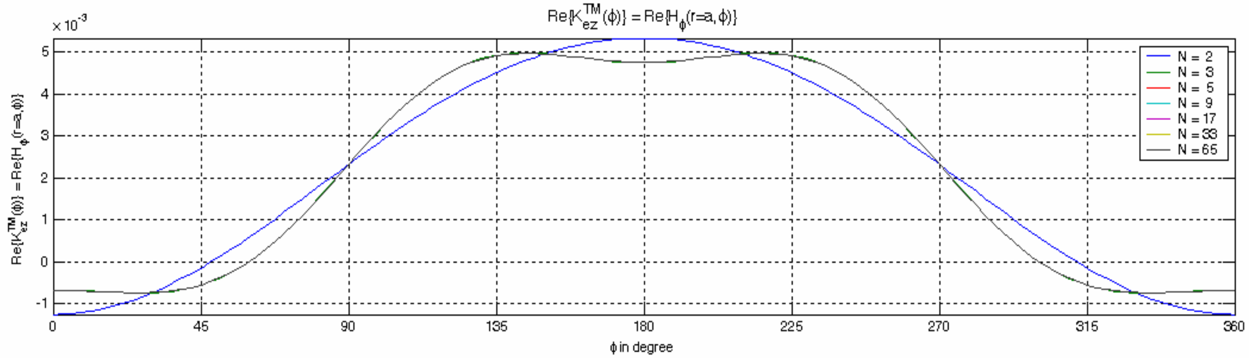
MATLAB Programme / MATLAB-Programm

MATLAB Program / MATLAB-Programm

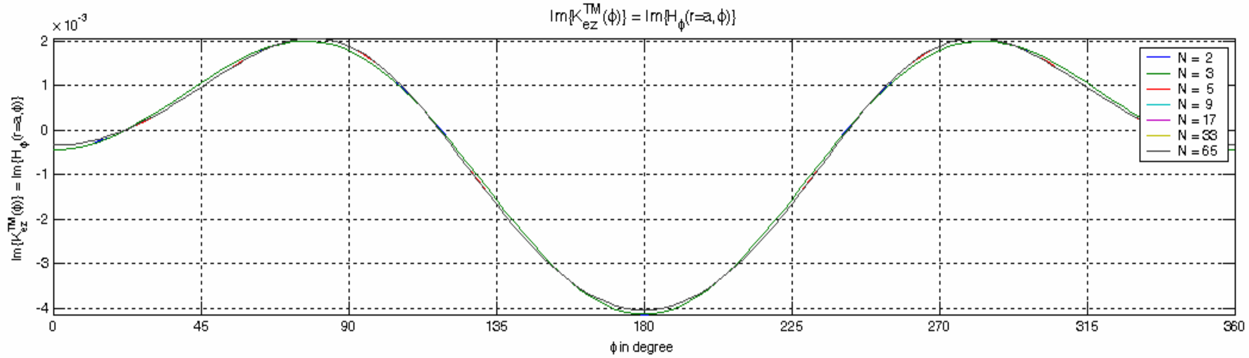
```
for nka=1:N_max_kas

  ka = max_kas(nka);  % max_kas = {1, 5, 10}

  a = ka / k;

  legend_matrix(nka,:) = sprintf('ka = %2d',ka)

  for nphi=1:Nphi

    phi(nphi)         = (nphi-1)*2.0*pi/(Nphi-1);
    phi_deg(nphi)  = (nphi-1)*2.0*pi/(Nphi-1)*180.0/pi;

    Hphi(nphi,nka) = 0.0;

    for n = 0:N
      Hphi(nphi,nka) = Hphi(nphi,nka) + epsilon_n(n+1) * (complex(0,-1))^n / besselh(n,1,ka) * cos(n*(phi(nphi)-phi_in));
    end

    % Magnetic field strength component / Magnetische Feldstärkekomponente
    Hphi(nphi,nka) = Hphi(nphi,nka) * 2.0/M_PI * Y0/ka;
    % Normalized magnetic field strength component / Normierte magnetische Feldstärkekomponente
    Hphi_Z0(nphi,nka) = Hphi(nphi,nka) * Z0;

end
end
```

Induced Electric Surface Current Density for Different Order N, $ka = 1$ / Induzierte elektrische Flächenstromdichte für verschiedene Ordnungen N, ka = 1

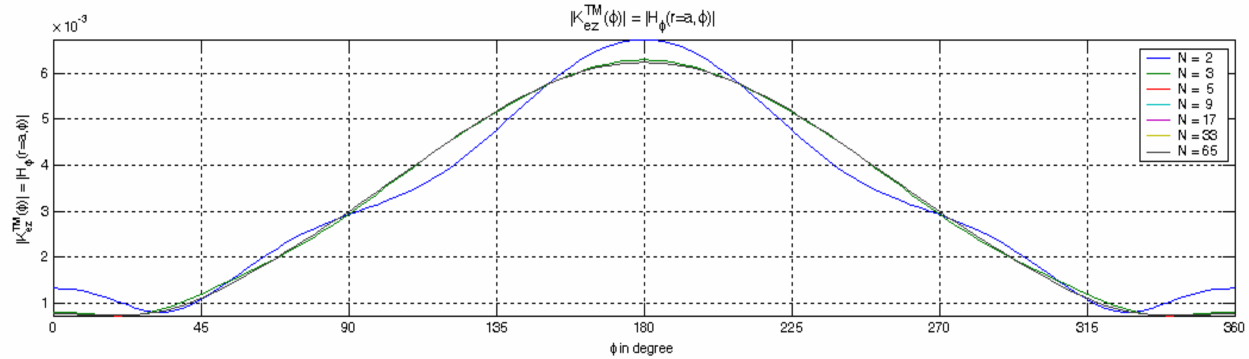
$\mathrm{Re}\left\{K_{\mathrm{ez}}^{\mathrm{TM}}(\varphi,\varphi_{\mathrm{in}}=0°,\omega)\right\}$



$\mathrm{Im}\left\{K_{\mathrm{ez}}^{\mathrm{TM}}(\varphi,\varphi_{\mathrm{in}}=0°,\omega)\right\}$

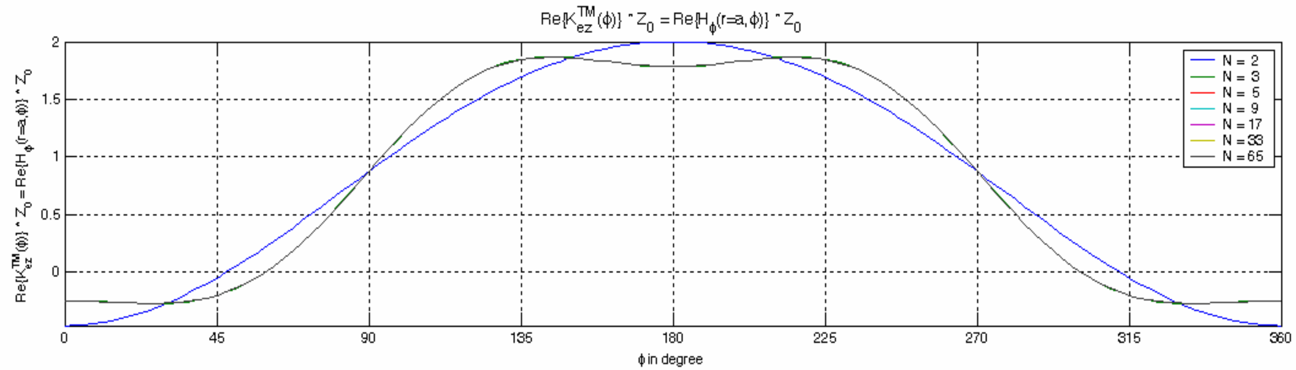


$\left|K_{\mathrm{ez}}^{\mathrm{TM}}(\varphi,\varphi_{\mathrm{in}}=0°,\omega)\right|$

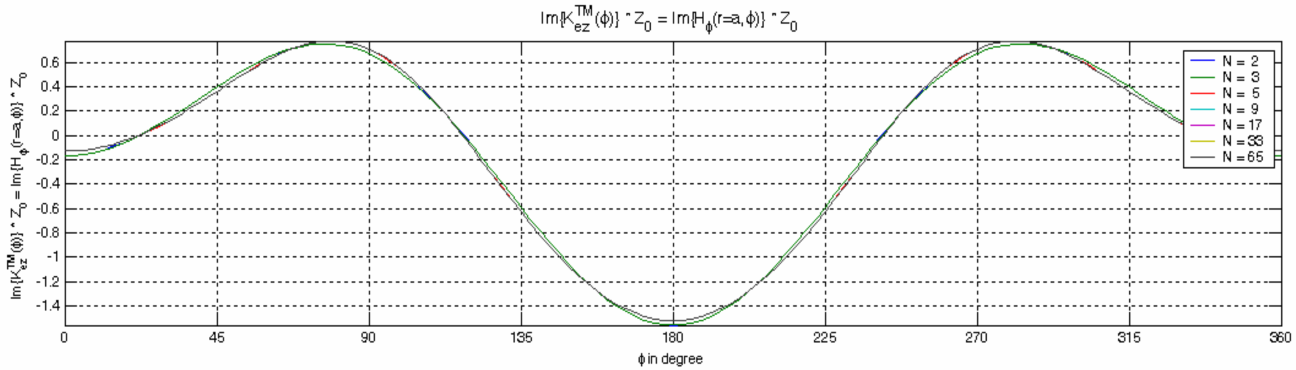


Induced Electric Surface Current for Different Order N, $ka = 1$ / Induzierte elektrische Flächenstrom für verschiedene Ordnungen N, ka = 1

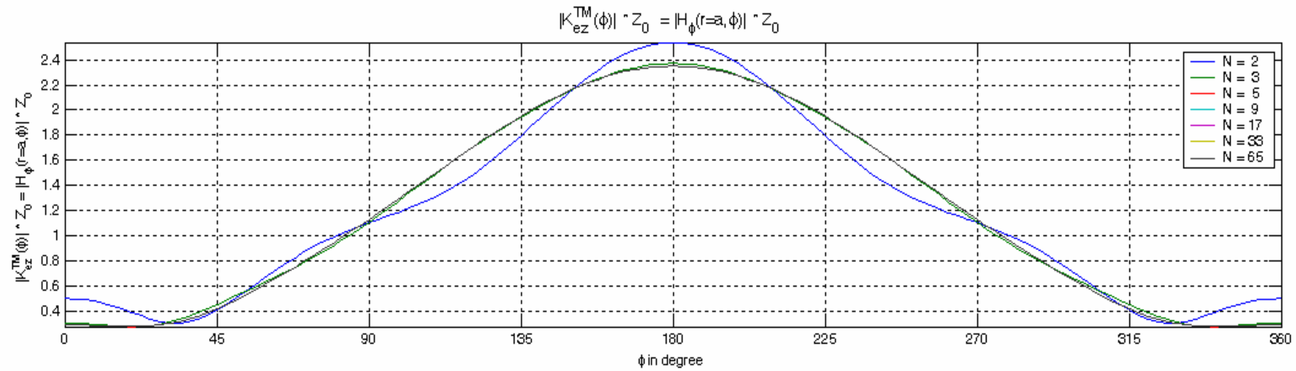
$$Z_0\,\mathrm{Re}\left\{K_{\mathrm{ez}}^{\mathrm{TM}}(\varphi,\varphi_{\mathrm{in}}=0°,\omega)\right\}$$



$$Z_0\,\mathrm{Im}\left\{K_{\mathrm{ez}}^{\mathrm{TM}}(\varphi,\varphi_{\mathrm{in}}=0°,\omega)\right\}$$

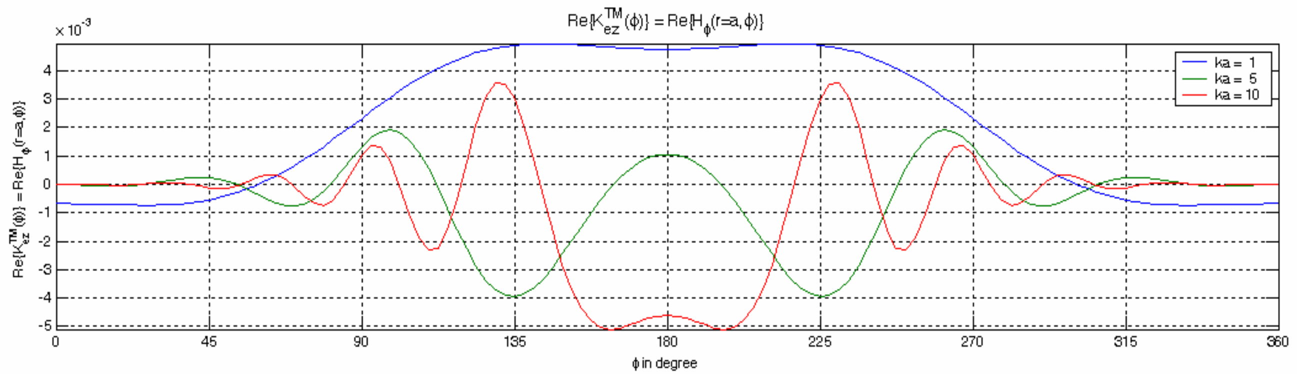


$$Z_0\left|K_{\mathrm{ez}}^{\mathrm{TM}}(\varphi,\varphi_{\mathrm{in}}=0°,\omega)\right|$$

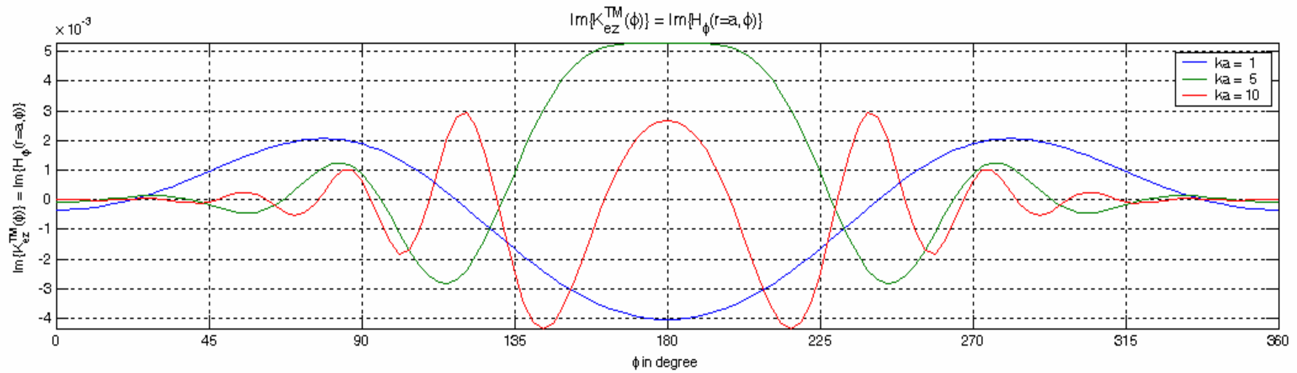


Induced Electric Surface Current for Different $ka = \{1, 5, 10\}$ and $N = 128$/ Induzierter elektrischer Flächenstrom für verschiedene $ka = \{1, 5, 10\}$ und $N = 128$

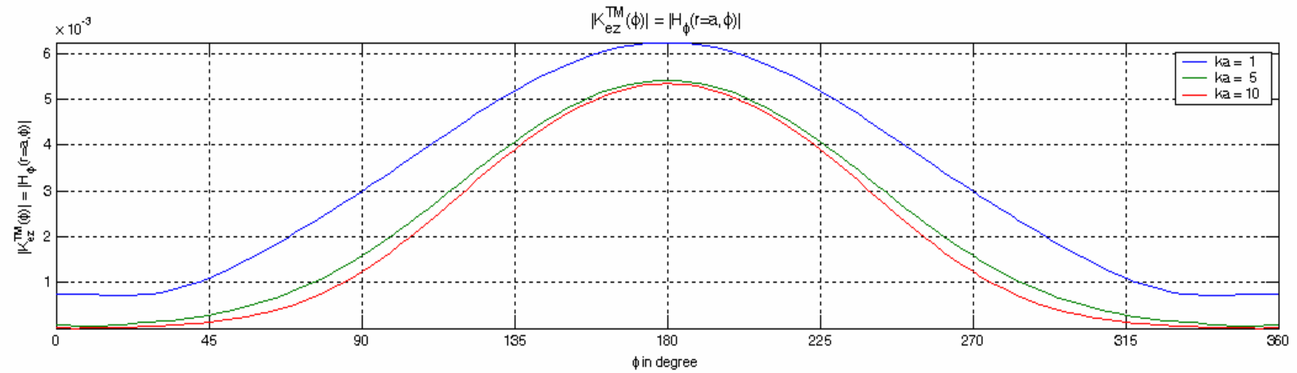
$$\mathrm{Re}\left\{K_{\mathrm{ez}}^{\mathrm{TM}}(\varphi, \varphi_{\mathrm{in}} = 0°, \omega)\right\}$$



$$\mathrm{Im}\left\{K_{\mathrm{ez}}^{\mathrm{TM}}(\varphi, \varphi_{\mathrm{in}} = 0°, \omega)\right\}$$



$$\left|K_{\mathrm{ez}}^{\mathrm{TM}}(\varphi, \varphi_{\mathrm{in}} = 0°, \omega)\right|$$

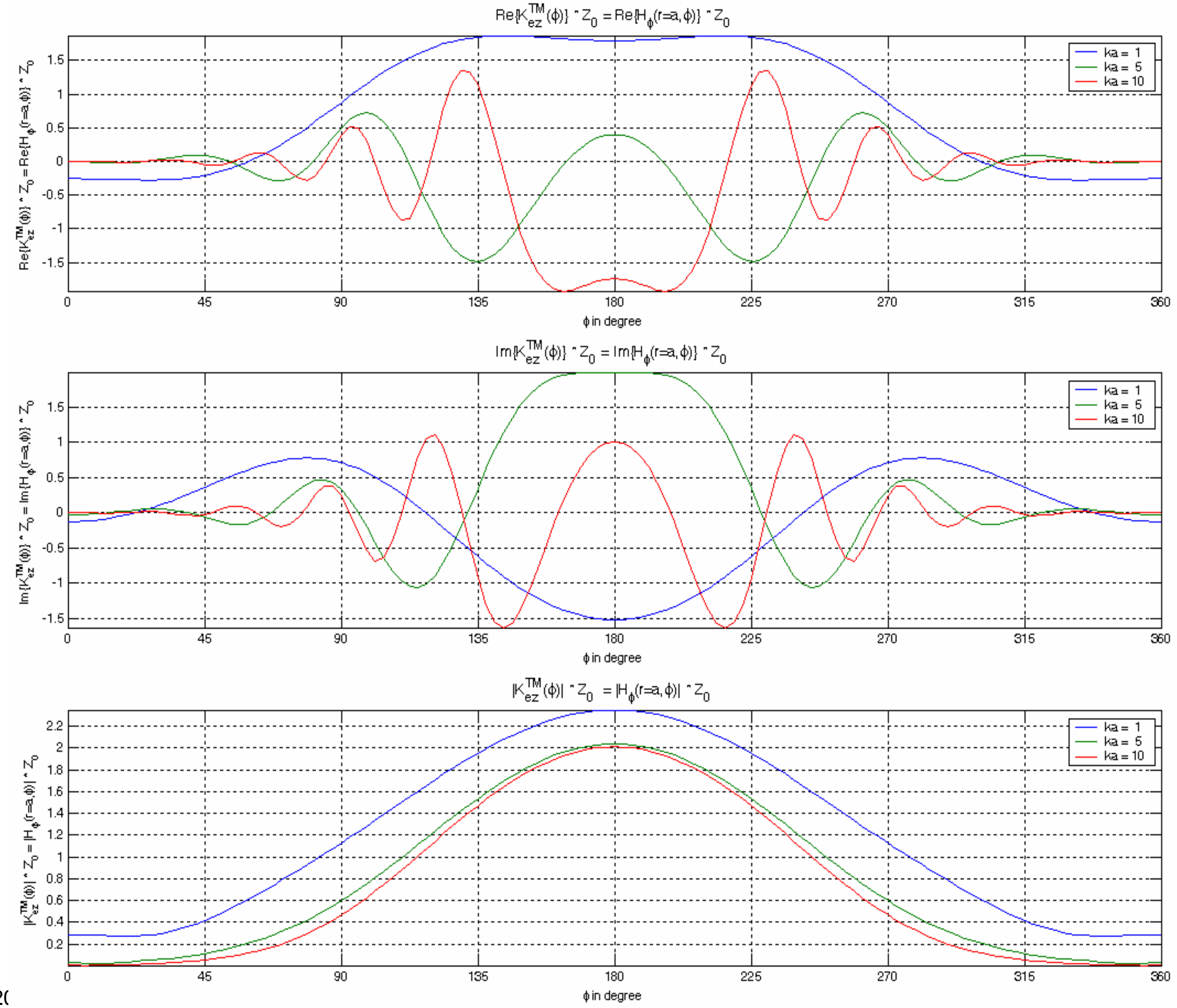


Induced Electric Surface Current for Different $ka = \{1, 5, 10\}$ and $N = 128$/ Induzierter elektrischer Flächenstrom für verschiedene $ka = \{1, 5, 10\}$ und $N = 128$

$$Z_0 \operatorname{Re}\left\{K_{ez}^{\mathrm{TM}}(\varphi, \varphi_{\mathrm{in}} = 0°, \omega)\right\}$$

$$Z_0 \operatorname{Im}\left\{K_{ez}^{\mathrm{TM}}(\varphi, \varphi_{\mathrm{in}} = 0°, \omega)\right\}$$

$$Z_0 \left|K_{ez}^{\mathrm{TM}}(\varphi, \varphi_{\mathrm{in}} = 0°, \omega)\right|$$

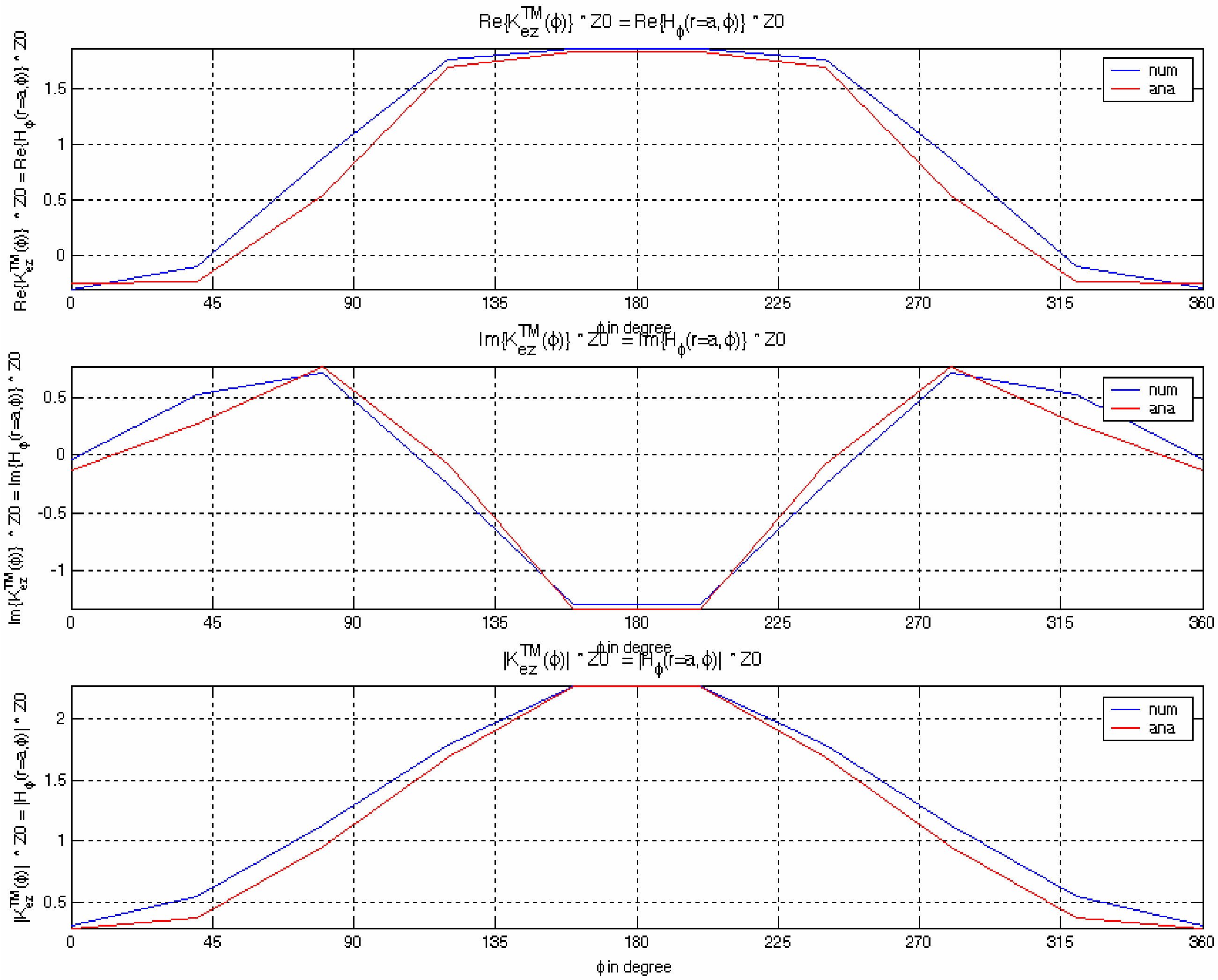


Induced Electric Surface Current for Different $ka = \{1, 5, 10\}$ and $N = 128$/ Induzierter elektrischer Flächenstrom für verschiedene $ka = \{1, 5, 10\}$ und $N = 128$

$$Z_0 \operatorname{Re}\left\{K_{ez}^{\mathrm{TM}}(\varphi, \varphi_{\mathrm{in}} = 0°, \omega)\right\}$$

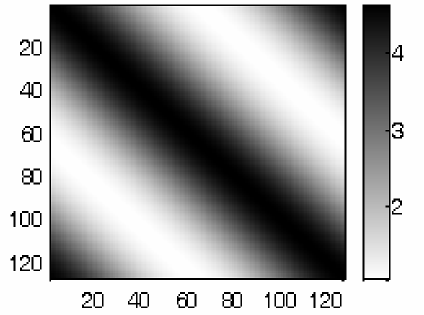
$$Z_0 \operatorname{Im}\left\{K_{ez}^{\mathrm{TM}}(\varphi, \varphi_{\mathrm{in}} = 0°, \omega)\right\}$$

$$Z_0 \left|K_{ez}^{\mathrm{TM}}(\varphi, \varphi_{\mathrm{in}} = 0°, \omega)\right|$$



EM Scattering by a Perfectly Electrically Conducting Cylinder: EFIE Discretized in the 2-D TM Case with Pulse Basis and Delta Testing Functions / EM-Streuung an einem ideal elektrisch leitendem Zylinder: EFIE diskretisiert im 2D-TM-Fall mit Impuls-Basisfunktionen und Delta-Testfunktionen

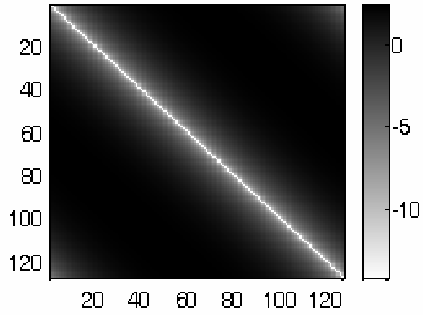
$\operatorname{Re}\{[\mathbf{Z}](\omega)\}$



Elements of the Impedance Matrix /
Elemente der Impedanzmatrix

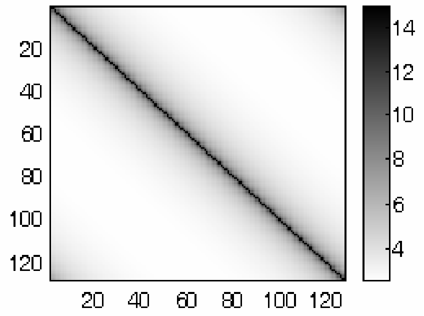
$$Z_{mn}(\omega) = \frac{\omega\mu_0}{4}\Delta^{(n)} \begin{cases} 1 + \mathrm{j}\frac{2}{\pi}\left[\ln\left(\frac{k}{4}\Delta^{(n)}\right) + \gamma - 1\right] & m = n \\ \mathrm{H}_0^{(1)}(k\, r_{mn}) & m \neq n \end{cases}$$

$\operatorname{Im}\{[\mathbf{Z}](\omega)\}$



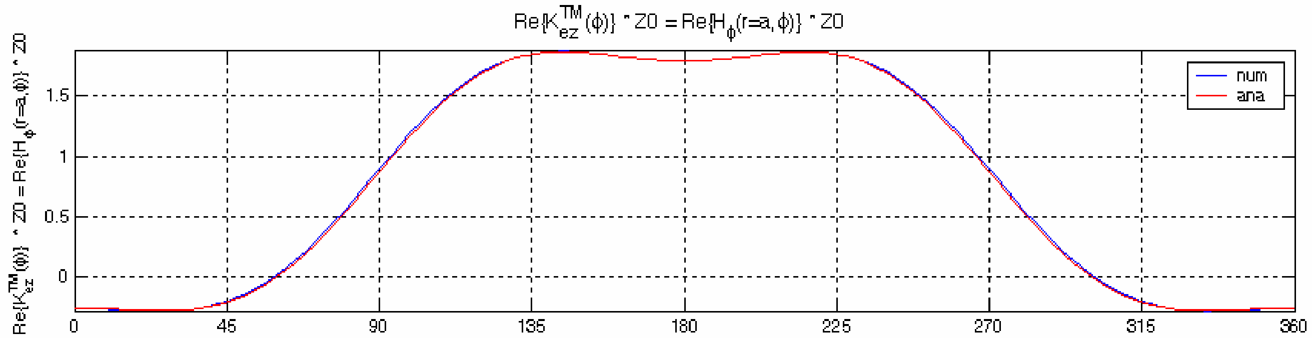
$ka = 1, \qquad N = 128$

$|[\mathbf{Z}](\omega)|$

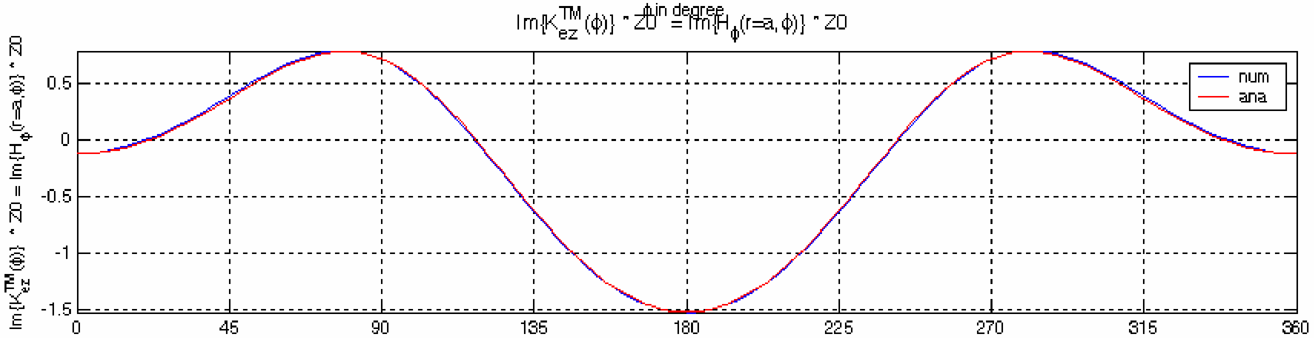


EM Scattering by a Circular PEC Cylinder – EFIE – 2-D TM Case – Results / EM-Streuung an einem kreisrunden IEL-Zylinder – EFIE – 2D-TM-Fall – Results

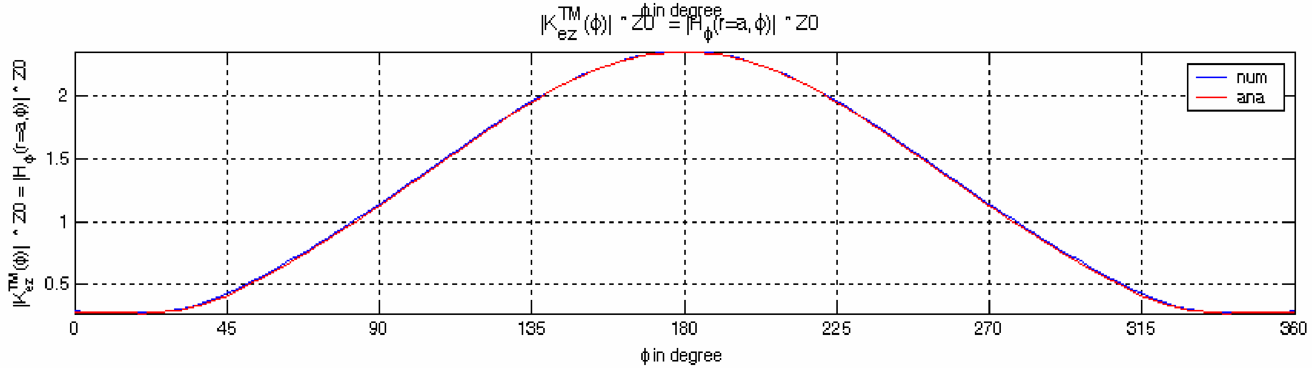
$$Z_0 \,\mathrm{Re}\left\{K_{\mathrm{ez}}^{\mathrm{TM}}(\varphi, \varphi_{\mathrm{in}} = 0°, \omega)\right\}$$



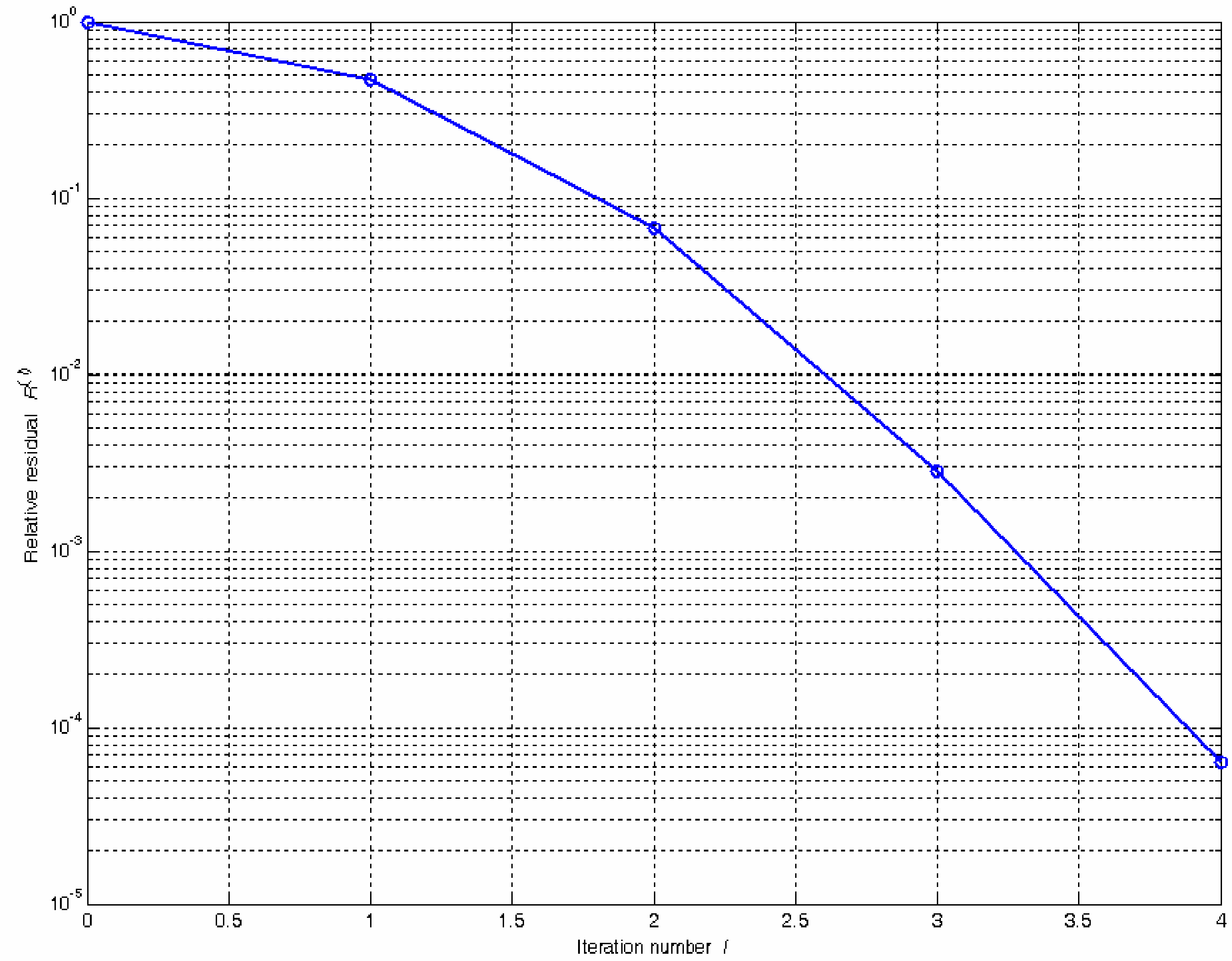
$$Z_0 \,\mathrm{Im}\left\{K_{\mathrm{ez}}^{\mathrm{TM}}(\varphi, \varphi_{\mathrm{in}} = 0°, \omega)\right\}$$



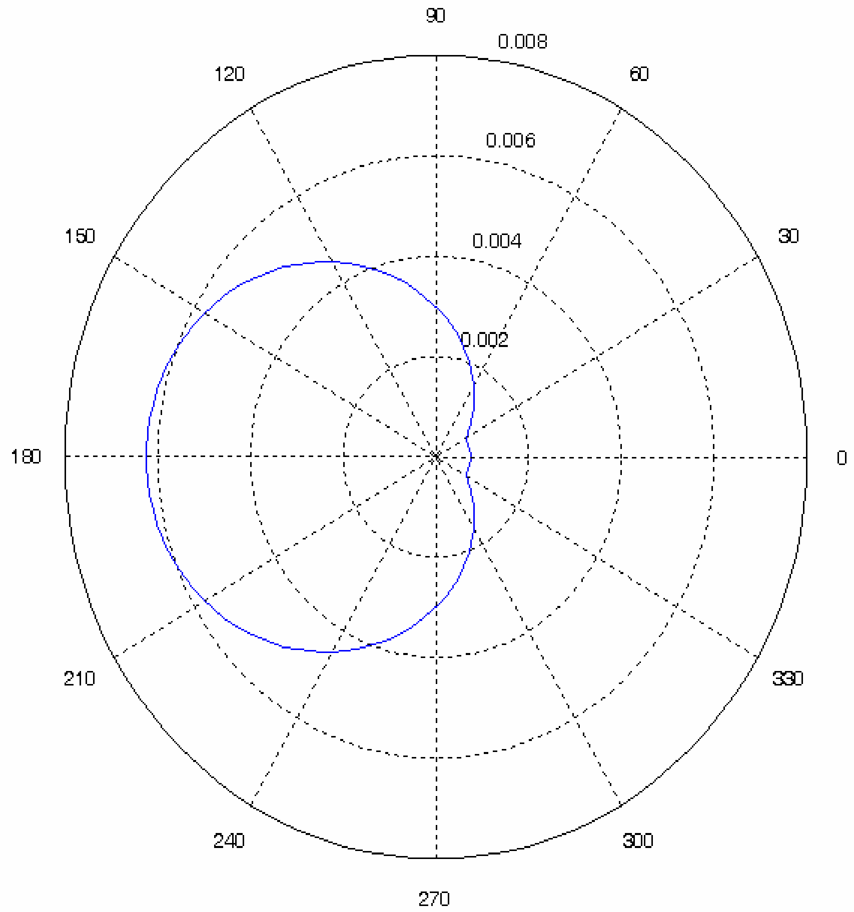
$$Z_0 \left|K_{\mathrm{ez}}^{\mathrm{TM}}(\varphi, \varphi_{\mathrm{in}} = 0°, \omega)\right|$$



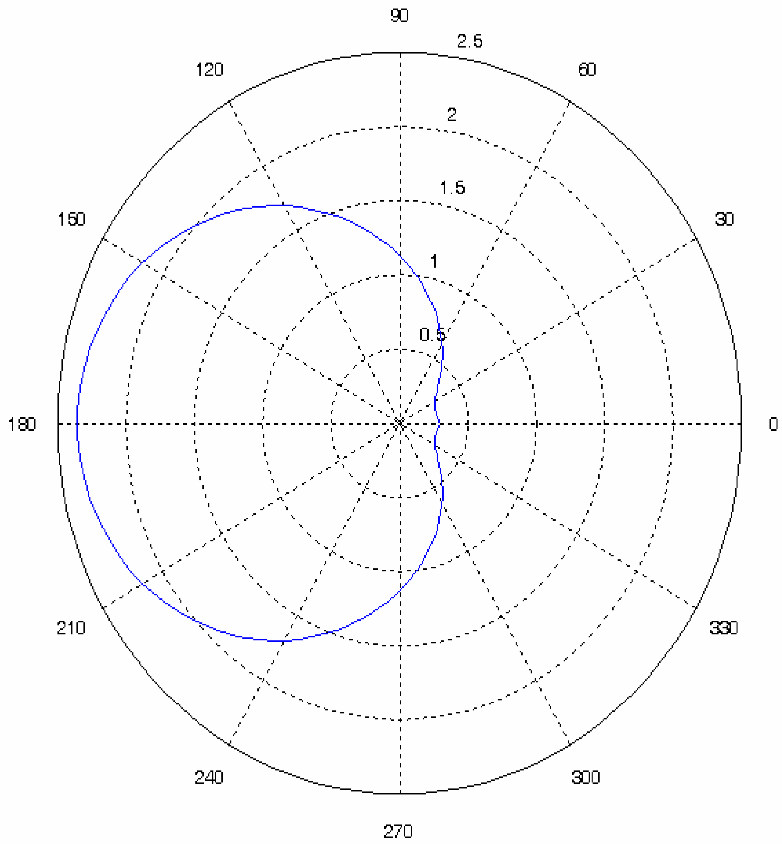
EM Scattering by a Circular PEC Cylinder – EFIE – 2-D TM Case – Results / EM-Streuung an einem kreisrunden IEL-Zylinder – EFIE – 2D-TM-Fall – Results



EM Scattering by a Circular PEC Cylinder – EFIE – 2-D TM Case – Results / EM-Streuung an einem kreisrunden IEL-Zylinder – EFIE – 2D-TM-Fall – Results



EM Scattering by a Circular PEC Cylinder – EFIE – 2-D TM Case – Results / EM-Streuung an einem kreisrunden IEL-Zylinder – EFIE – 2D-TM-Fall – Results



End of 5th Lecture / Ende der 5. Vorlesung