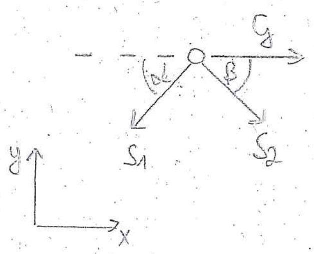
Aufgabe 4.1



GGW: $\begin{pmatrix} G \\ 0 \end{pmatrix} + S_1 \begin{pmatrix} -\cos\alpha \\ -\sin\alpha \end{pmatrix} + S_2 \begin{pmatrix} \cos\beta \\ -\sin\beta \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$$\Rightarrow \quad G - S_1 \cdot \cos\alpha + S_2 \cos\beta = 0 \qquad (1)$$

$$- S_1 \cdot \sin\alpha - S_2 \cdot \sin\beta = 0 \qquad (2)$$

$(1) \cdot \sin\beta + (2) \cos\beta$:

$$G \cdot \sin\beta - S_1 \left(\cos\alpha \cdot \sin\beta + \sin\alpha \cdot \cos\beta \right) = 0$$

$$S_1 = \frac{G \cdot \sin\beta}{(\cos\alpha \cdot \sin\beta + \sin\alpha \cdot \cos\beta)} = 0{,}896 \text{ kN} = 896 \text{ N}$$

aus (2)

$$S_2 = \frac{- S_1 \cdot \sin\alpha}{\sin\beta} = -0{,}731 \text{ kN} = -731 \text{ N}$$

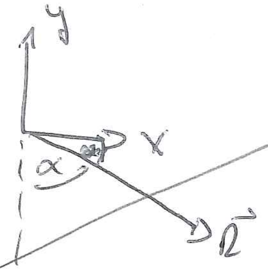
3.6:

a) $\vec{R} = F_1 \begin{pmatrix} \cos\alpha_1 \\ \sin\alpha_1 \end{pmatrix} + F_2 \begin{pmatrix} 1 \\ 0 \end{pmatrix} + F_3 \begin{pmatrix} \cos\alpha_3 \\ -\sin\alpha_3 \end{pmatrix} + F_4 \begin{pmatrix} \cos\alpha_4 \\ -\sin\alpha_4 \end{pmatrix} + F_5 \begin{pmatrix} \cos\alpha_5 \\ -\sin\alpha_5 \end{pmatrix}$

$\Rightarrow \vec{R} = \begin{pmatrix} 16{,}5896 \\ -5{,}40707 \end{pmatrix}$ kN

$|\vec{R}| = 17{,}4486$ kN

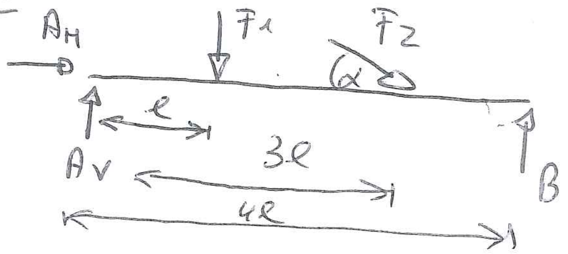
b)



$\alpha = \arctan\left(\frac{R_x}{-R_y}\right) = 71{,}95°$

4.2

~~3.7.~~



Gesucht: A_H, A_V, B

$\overset{\rightarrow}{\sum} F_{iH} = 0 = A_H + \underbrace{F_2 \cos\alpha}_{17{,}32N} \Rightarrow A_H = -F_2 \cos\alpha = \boxed{-17{,}32N = A_H}$

Momentengg.: Punkt suchen, durch den viele Wirkungslinien unbekannter Kräfte gehen!

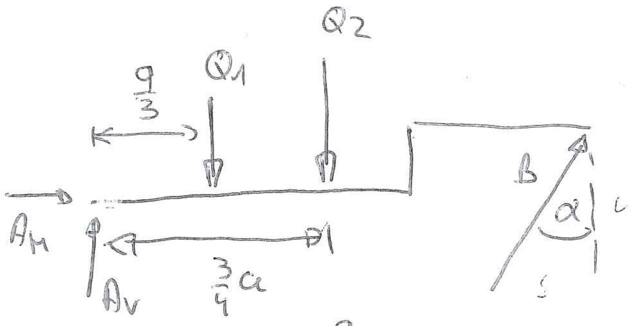
~~$\Rightarrow \sum M_{iA} \Rightarrow \sum M_{iA} = 0 = F_1 \cdot \frac{l}{4} + F_2 \cdot \frac{3l}{4} - B \cdot l =$~~

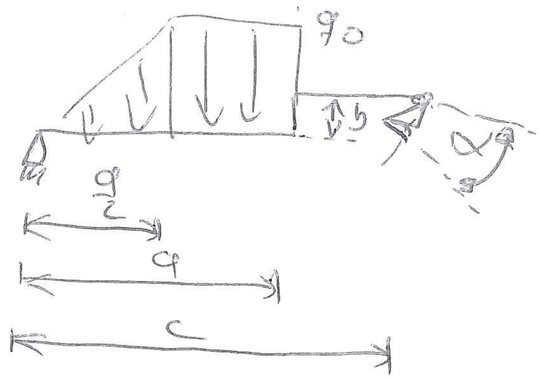
$\overset{\curvearrowright}{\sum} M_{iA} = 0 = F_1 \cdot \cancel{l} + \overset{\sin\alpha}{F_2} \cdot 3\cancel{l} - B \cdot 4\cancel{l}$

$\Rightarrow B = \frac{F_1}{4} + \frac{3F_2}{4} = \boxed{10N = B}$

$\uparrow \sum F_{iV} = 0 = A_V - F_1 - \underbrace{F_2 \cdot \sin\alpha}_{=10N} + B = 0 \Rightarrow \boxed{A_V = 10N}$

~~3.20~~ 4.5





$\cancel{Q_1 = \int_0^{\frac{a}{2}} q(x)\,dx = \int_0^{\frac{a}{2}}}$

$Q_1 = \frac{1}{2}\cdot\frac{a}{2}\cdot q_0 = \frac{1}{4} q_0 a = 1{,}5\,kN$

$Q_2 = \frac{1}{2} q_0 a = 3\,kN$

GGW - Bed.:

$\overset{\curvearrowleft}{\sum} M_{iA} = 0 = Q_1 \cdot \frac{1}{3}a + Q_2 \cdot \frac{3}{4}a - c\cdot B\cdot\cos\alpha + bB\sin\alpha$

$0 = \frac{a}{3}Q_1 + \frac{3a}{4}Q_2 - B(c\cdot\cos\alpha - b\cdot\sin\alpha)$

$\Rightarrow B = (\frac{1}{3}Q_1 + \frac{3}{4}Q_2)a \cdot \frac{1}{c\cdot\cos\alpha - b\sin\alpha} = \boxed{2{,}642\,kN = B}$

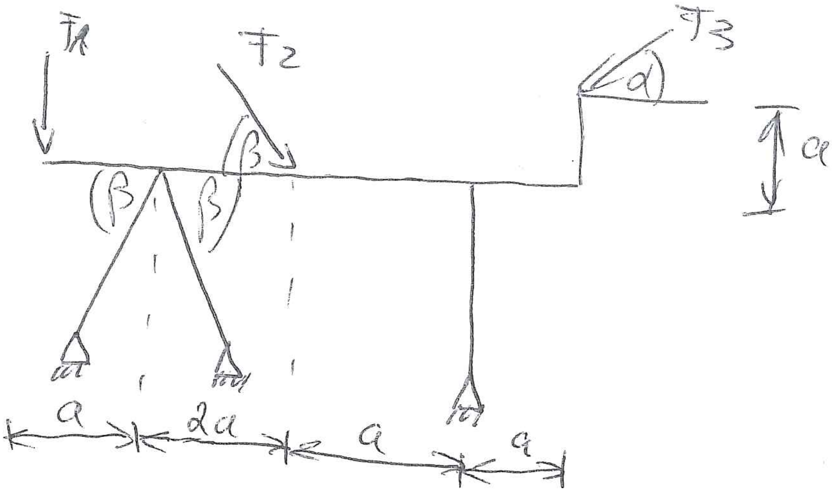
$\sum \vec{F}_{iH} = 0 = A_H + B\sin\alpha \Rightarrow A_H = -\frac{1}{2}B = \boxed{-1{,}321\,kN = A_H}$

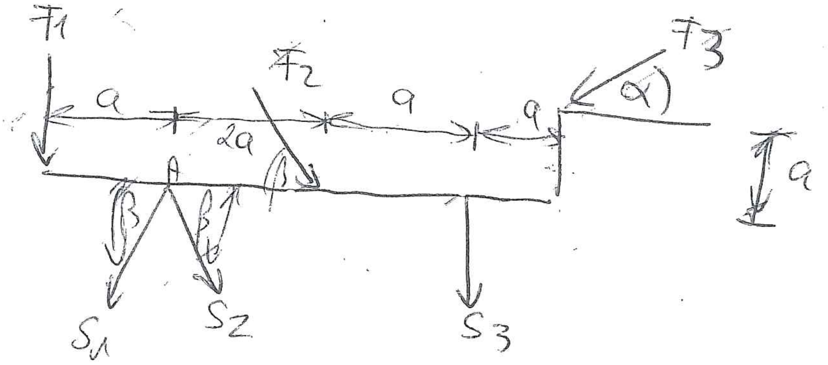
$\sum F_{iV} = 0 = A_V - Q_1 - Q_2 + B\cos\alpha$

$\Rightarrow \boxed{A_V = 2{,}212\,kN}$

~~4.4~~

4.3

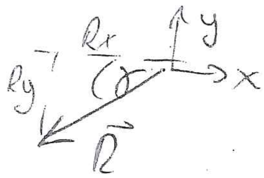




a) $\vec{R} = F_1\begin{pmatrix}0\\-1\end{pmatrix} + F_2\begin{pmatrix}+\cos\beta\\-\sin\beta\end{pmatrix} + F_3\begin{pmatrix}-\cos\alpha\\-\sin\alpha\end{pmatrix} = \begin{pmatrix}-8\\-36{,}2\end{pmatrix} N$

$|\vec{R}| = 37{,}03 N$

$\gamma = \arctan \frac{|R_y|}{|R_x|} = 77{,}5°$



b)

$\circlearrowright \sum M_{iA} = 0 = +F_1 a - F_2 \sin\beta\, 2a - S_3\, 3a + a F_3 \cos\alpha - F_3 \sin\alpha\, 4a$

$\Rightarrow S_3 = \frac{+F_1 - F_2 \sin\beta \cdot 2 + F_3 \cos\alpha - 4 F_3 \sin\alpha}{3} = \boxed{-4{,}77 N = S_3}$

$\uparrow \sum F_{iV} = 0 = -F_1 - F_2 \sin\beta - F_3 \sin\alpha - S_3 - S_2 \sin\beta - S_1 \sin\beta \quad (1)$

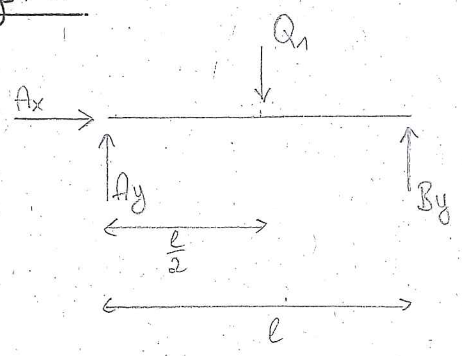
$\overset{\rightarrow}{\sum} F_{iH} = 0 = -S_1 \cos\beta + S_2 \cos\beta + F_2 \cos\beta - F_3 \cos\alpha \quad (2)$

aus (1) und (2): $\boxed{\begin{matrix} S_1 = -26{,}11 N \\ S_2 = -10{,}13 N \end{matrix}}$

Aufgabe 4.4 Geg: $l = 4m$, a, $q_1(x) = 10\frac{kN}{m}$, $F = 2N$, $q_2(x) = \frac{1N}{a}$

Linkes System:

Freischneiden:



$$Q_1 = q_1(x) \cdot l = 10\frac{kN}{m} \cdot 4m = 40kN$$

Gleichgewichtsbedingungen:

$$\sum F_{iy} = 0 \quad : \quad A_y + B_y - Q_1 = 0 \qquad (1)$$

$$\sum F_{ix} = 0 \quad : \quad A_x = 0$$

$$\sum M_{iA} = 0 \quad : \quad B_y \cdot l - Q_1 \cdot \frac{l}{2} = 0$$

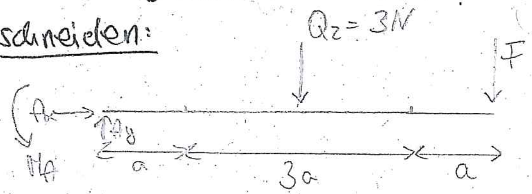
$$B_y = \frac{Q_1}{2} = 20kN$$

B_y in (1):

$$A_y = -B_y + Q_1 = -20kN + 40kN = 20kN$$

Rechtes System:

Freischneiden:



GGW:

$$\sum F_{iy} = 0 : A_y - Q_2 - F = 0 \quad \Rightarrow A_y = Q_2 + F = 3N + 2N = 5N$$

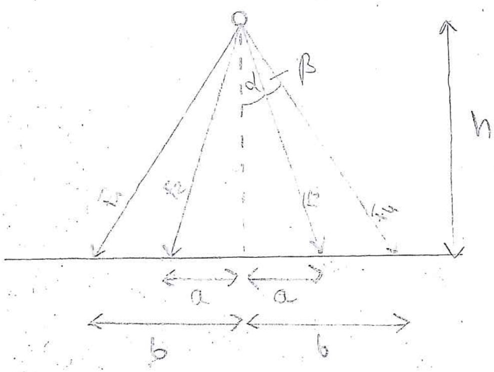
$$\sum F_{ix} = 0 : A_x = 0$$

$$\sum M_{iA} = 0 : M_A - Q_2 \cdot 2{,}5a - F \cdot 5a = 0$$

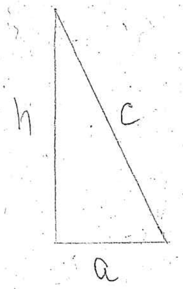
$$M_A = F \cdot 5a + Q_2 \cdot 2{,}5a = 2N \cdot 5a + 3N \cdot 2{,}5a$$

$$M_A = 17{,}5N \cdot a$$

Aufgabe 4.6



Winkelberechnung



$$a^2 + h^2 = c^2$$

$$c = \sqrt{10^2 + 25^2} = 26{,}9258\,m$$

$$\sin\alpha = \frac{a}{c} = \frac{10\,m}{26{,}9258\,m}$$

$$\alpha = \sin^{-1}\left(\frac{10\,m}{26{,}9258\,m}\right) = 0{,}38\,rad \mathrel{\hat{=}} 21{,}77^0$$

Analoges Vorgehen für β liefert:

$$\beta = 45^0$$

a)

$$R = \begin{pmatrix} R_x \\ R_y \end{pmatrix} = F_1 \begin{pmatrix} -\sin\beta \\ -\cos\beta \end{pmatrix} + F_2 \begin{pmatrix} -\sin\alpha \\ -\cos\alpha \end{pmatrix} + F_3 \begin{pmatrix} \sin\alpha \\ -\cos\alpha \end{pmatrix} + F_4 \begin{pmatrix} \sin\beta \\ -\cos\beta \end{pmatrix}$$

$$R_x = -140\,kN \cdot \sin\beta - 200\,kN \sin\alpha + 240\,kN \sin\alpha + 200\,kN \cdot \sin\beta$$

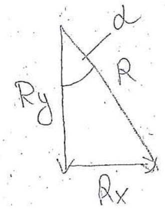
$$= 60\,kN \cdot \sin\beta + 40\,kN \cdot \sin\alpha = \underline{57{,}261\,kN}$$

$$R_y = -140\,kN \cos\beta - 200\,kN \cdot \cos\alpha - 240\,kN \cos\alpha - 200\,kN \cdot \cos\beta$$

$$= -340\,kN \cos\beta - 440\,kN \cos\alpha = \underline{-649{,}035\,kN}$$

$$|R| = \sqrt{R_x^2 + R_y^2} = 651{,}556\,kN$$

b)



$$\sin\alpha = \frac{R_x}{R}$$

$$\alpha \approx 5^{\circ}$$

c)

$\overleftarrow{F_{1x}}$ $\overleftarrow{F_{2x}}$ $\overrightarrow{F_{3x}}$ $\overrightarrow{F_{4x}}$ $\overleftarrow{A_x}$

$$A_x = F_{1x} + F_{2x} - F_{3x} - F_{4x} = -59{,}828\ kN$$