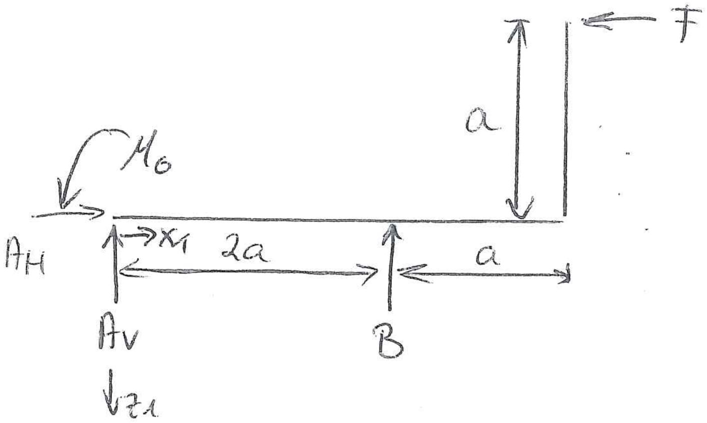
Gruppenübung 7:

Aufgabe 7.1:



Auflager:

$$\curvearrowleft \sum M_{iA} = 0 = F \cdot a + B \cdot 2a + M_0 \quad ; M_0 = Fa$$

$$0 = \cancel{2}F\cancel{a} + B \cdot \cancel{2a}$$

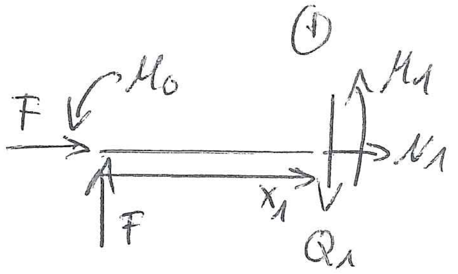
$$\rightarrow \boxed{B = -F}$$

$$\uparrow \sum F_{iV} = 0 = A_V + B \rightarrow \boxed{A_V = F}$$

$$\vec{\sum} F_{iH} = 0 = A_H - F \rightarrow \boxed{A_H = F}$$

Schnittgrößen:

Bereich (I):



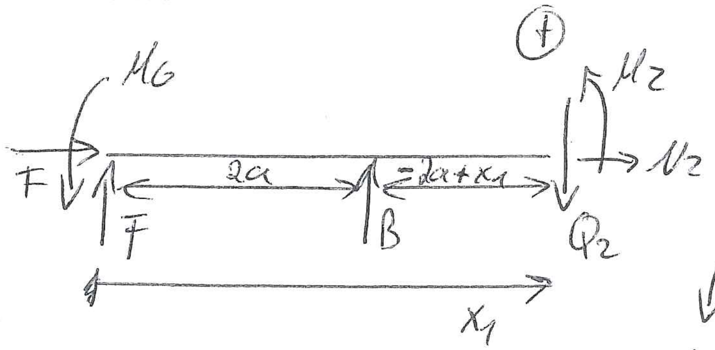
$$\vec{\sum} F_{iH} = 0 = F + N_1 \rightarrow \boxed{N_1 = -F}$$

$$\uparrow \sum F_{iV} = 0 = F - Q_1 \rightarrow \boxed{Q_1 = F}$$

$$\curvearrowleft M_{ix} = 0 = -Fx_1 + M_0 \overset{+M_1(x_1)}{=} -Fx_1 + Fa + M_1(x_1)$$

$$\rightarrow \boxed{M_1(x_1) = F(x_1 - a) = Fa\left(\frac{x_1}{a} - 1\right)}$$

Bereich (II):



$$\vec{\sum} F_{iH} = 0 = F + N_2 \rightarrow \boxed{N_2 = -F}$$

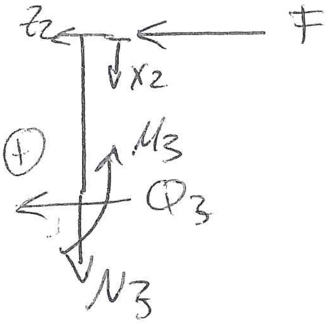
$$\uparrow \sum F_{iV} = 0 = F + B - Q_2$$

$$\rightarrow \boxed{Q_2 = 0}$$

$$\curvearrowleft \sum M_{ix} = 0 = -Fx_1 + M_0 - B(2a + x_1) + M_2$$

$$\rightarrow M_2(x_1) = \cancel{Fx_1} - Fa + F2a \cancel{- Fx_1} = \boxed{Fa = M_2(x_1)}$$

Bereich III:



$\overset{\rightarrow}{\sum} F_{iH} = 0 = -Q_3 - F$

$\rightarrow \boxed{Q_3 = -F}$

$\uparrow \sum F_{iV} = 0 = -N_3 \rightarrow \boxed{N_3 = 0}$

$\curvearrowleft \sum M_{ix} = 0 = M_3 + F \cdot x_2$

$\rightarrow \boxed{M_3 = -F x_2}$

graphische Darstellung:

max. Biegemoment:

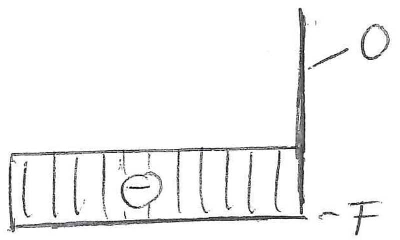
Dort wo $Q = 0$ oder am Bereichsrand:

① $x_1 = 0$; $x_1 = 2a$

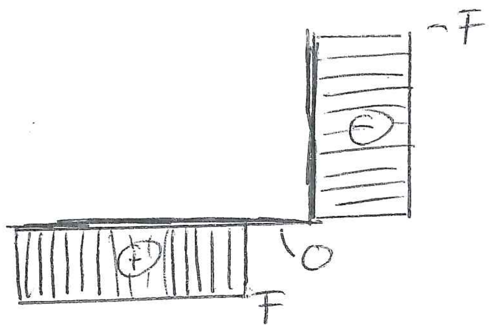
② $x_1 = [2a ; 3a]$

③ $x_2 = a$

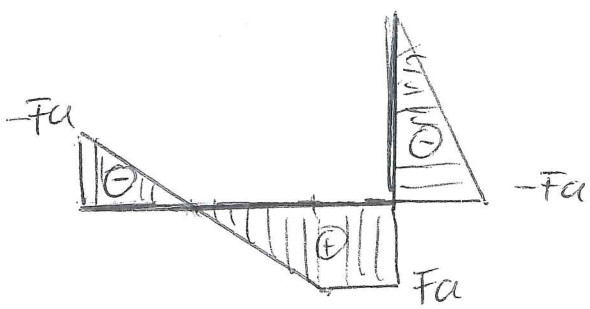
Normalkraft:



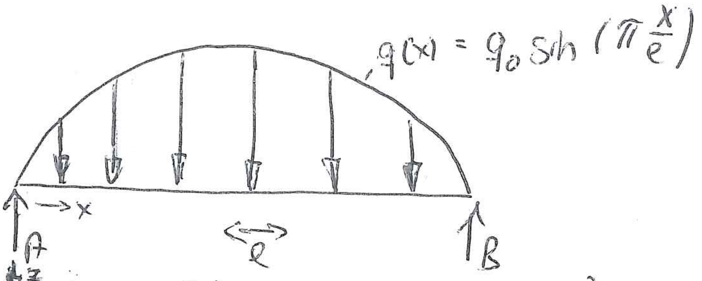
Querkraft:



Biegemoment:



Aufgabe 5.8:



Es gilt:

$$\left.\begin{aligned} N'(x) &= -n(x) \\ Q'(x) &= -q(x) \\ M'(x) &= Q(x) \; ; \; M''(x) = -q(x) \end{aligned}\right\} \text{DGL}$$

$$Q'(x) = -q(x) = -q_0 \sin(\pi \frac{x}{l})$$

$$Q(x) = q_0 \frac{l}{\pi} \cos(\pi \frac{x}{l}) + C_1$$

$$M(x) = q_0 \left(\frac{l}{\pi}\right)^2 \sin(\pi \frac{x}{l}) + C_1 x + C_2$$

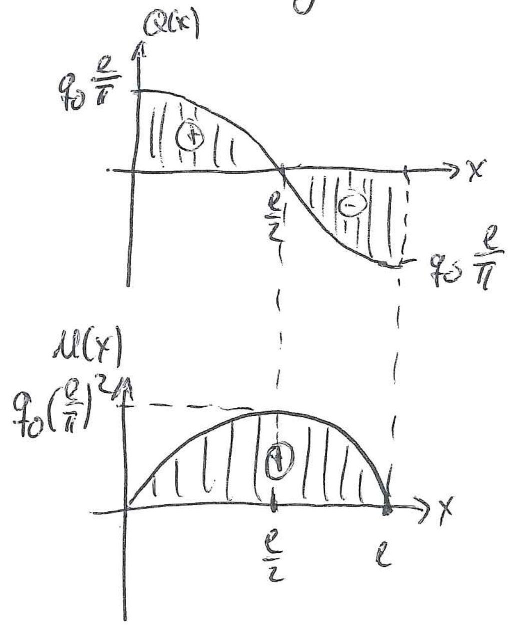
RB: $M(x=0) = \boxed{0 = C_2}$

$$M(x=l) = 0 = q_0 \left(\frac{l}{\pi}\right)^2 \underbrace{\cancel{\sin(\pi)}}_{=0} + C_1 l \rightarrow \boxed{C_1 = 0}$$

$$\Rightarrow Q(x) = q_0 \frac{l}{\pi} \cos(\pi \frac{x}{l})$$

$$M(x) = q_0 \left(\frac{l}{\pi}\right)^2 \sin(\pi \frac{x}{l})$$

Graphische Darstellung:



$Q(x=0) = q_0 \frac{l}{\pi}$; $Q(x=l) = -q_0 \frac{l}{\pi}$

$Q' = -q \rightarrow$ Da wo q maximal wird, hat Q den Nulldurchgang

M max, wo Q null ist

$$\rightarrow M_{max} = M(x=\frac{l}{2}) = q_0 \left(\frac{l}{\pi}\right)^2$$

Aufgabe 7.2

Möglichkeit 1)

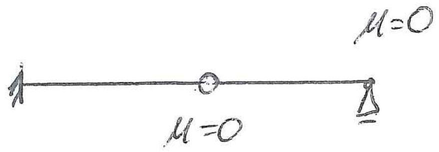
Lösung mit DGL:

$$\left.\begin{aligned} Q'(x) &= -q(x) \\ M'(x) &= Q(x) \end{aligned}\right\} \quad M''(x) = \underbrace{-q(x)}_{Q'(x)=} \quad ; \quad q(x) = \frac{2q_0}{2\ell}x = \frac{q_0}{\ell}x$$

$$N'(x) = -n(x)$$

$$M''(x) = Q'(x) = -q(x) = -\frac{q_0}{\ell}x$$

$$M'(x) = Q(x) = -\frac{q_0}{2\ell}x^2 + C_1$$



$$M(x) = -\frac{q_0 x^3}{6\ell} + C_1 x + C_2$$

RB: $M(\ell) = 0 = -\frac{q_0\ell^2}{6} + C_1\ell + C_2$ (1)

$M(2\ell) = 0 = -\frac{8q_0\ell^2}{6} + C_1 2\ell + C_2$ (2)

$\ominus$

$$\frac{7}{6}q_0\ell^{\cancel{2}} - C_1\cancel{\ell} = 0$$

$$\rightarrow \boxed{C_1 = \frac{7}{6}q_0\ell}$$

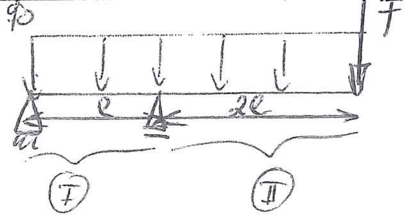
in (1): $0 = -\frac{q_0\ell^2}{6} + \frac{7}{6}q_0\ell^2 + C_2$

$$\rightarrow \boxed{C_2 = -q_0\ell^2}$$

$$\Rightarrow Q(x) = -\frac{q_0}{2\ell}x^2 + \frac{7}{6}q_0\ell = \frac{q_0\ell}{6}\left[-3\left(\frac{x}{\ell}\right)^2 + 7\right]$$

$$\Rightarrow M(x) = -\frac{q_0x^3}{6\ell} + \frac{7}{6}q_0\ell x - q_0\ell^2$$

$$= \frac{q_0\ell^2}{6}\left[-\left(\frac{x}{\ell}\right)^3 + 7\left(\frac{x}{\ell}\right) - 6\right]$$



Aufgabe 7.3

Möglichkeit 1: Lösung mit DGL

$$\left.\begin{array}{l} Q'(x) = -q(x) \\ M'(x) = Q(x) \end{array}\right\} M''(x) = Q'(x) = -q(x) \quad ; \quad q(x) = q_0 \text{ in beiden Bereichen}$$

Bereich I:

$$M_1''(x_1) = -q_1(x_1) = -q_0$$

$$M_1'(x) = -q_0 x + C_1$$

$$M_1(x) = -\frac{1}{2} q_0 x^2 + C_1 x + C_2$$

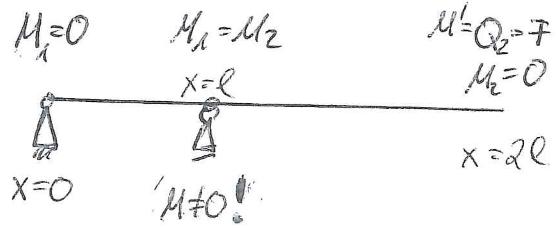
Bereich II:

$$M_2''(x) = -q_0$$

$$M_2'(x) = -q_0 x + D_1$$

$$M_2(x) = -\frac{1}{2} q_0 x^2 + D_1 x + D_2$$

RB:



$\rightarrow Q(3l) = F$

$$M_1(0) = \boxed{0 = C_2}$$

$$M_1(x=l) = -\frac{1}{2} q_0 l^2 + C_1 l$$

$$M_2(3l) = 0 = -\frac{1}{2} q_0\, 9l^2 + D_1 3l + D_2 \quad (1)$$

$$Q_2(3l) = F = -q_0 3l + D_1 \rightarrow D_1 = F + 3\underbrace{q_0 l}_{2F} = F + 6F \rightarrow \boxed{D_1 = 7F}$$

(1)

$$\rightarrow 0 = -\frac{1}{2} q_0 9 l^2 + 7F \cdot 3l + D_2$$

$$\left(q_0 l = 2F \rightarrow q_0 = \frac{2F}{l} \right)$$

$$0 = -\frac{9}{\cancel{2}} l \cancel{2} F + 21 Fl + D_2$$

$$0 = 12 Fl + D_2 \rightarrow \boxed{D_2 = -12 Fl}$$

$$M_1(x=l) = M_2(x=l)$$

$$\cancel{-\frac{1}{2} q_0 l^2} + C_1 \cancel{l} = \cancel{-\frac{1}{2} q_0 l^2} + 7F\cancel{l} - 12 F\cancel{l}$$

$$\rightarrow \boxed{C_1 = -5F}$$

$$\rightarrow Q_1(x) = -q_0 x - 5F = -\frac{2Fx}{l} - 5F = F\left(-2\frac{x}{l} - 5\right)$$

$$M_1(x) = -\frac{1}{2}q_0x^2 - 5Fx = -\frac{1}{\cancel{2}}\frac{\cancel{2}F}{l}x^2 - 5Fx = -\frac{Fx^2}{l} - 5Fx$$

$$\rightarrow M_1(x) = Fl\left[-\left(\frac{x}{l}\right)^2 - 5\frac{x}{l}\right]$$

$$Q_2(x) = -q_0x + 7F = -\frac{2F}{l}x + 7F = F\left[-2\left(\frac{x}{l}\right) + 7\right]$$

$$M_2(x) = -\frac{1}{2}q_0x^2 + 7Fx - 12Fl$$

$$= -\frac{1}{\cancel{2}}\frac{\cancel{2}F}{l}x^2 + 7Fx - 12Fl$$

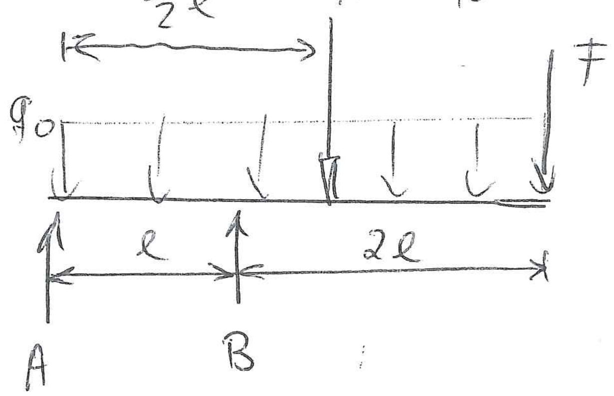
$$= -\frac{Fx^2}{l} + 7Fx - 12Fl$$

$$\Rightarrow M_2(x) = Fl\left[-\left(\frac{x}{l}\right)^2 + 7\left(\frac{x}{l}\right) - 12\right]$$

<u>Möglichkeit 2:</u> Schnittprinzip

<u>Auflager:</u>

$$R = 3q_0l = 3\cdot\frac{2F}{\cancel{l}}\cancel{l} = 6F$$



$$q_0 = \frac{2F}{l}$$

$$\circlearrowleft \sum M_{iA} = 0 = -R\cdot\frac{3}{2}l - F\cdot 3l + Bl$$

$$0 = -3q_0l\cdot\frac{3}{2}l - F\cdot 3l + Bl$$

$$0 = -3\frac{2F}{\cancel{l}}\cancel{l}\cdot\frac{3}{\cancel{2}}l - F\cdot 3l + Bl$$

$$0 = -9F\cancel{l} - 3F\cancel{l} + B\cancel{l} \rightarrow \boxed{B = 12F}$$

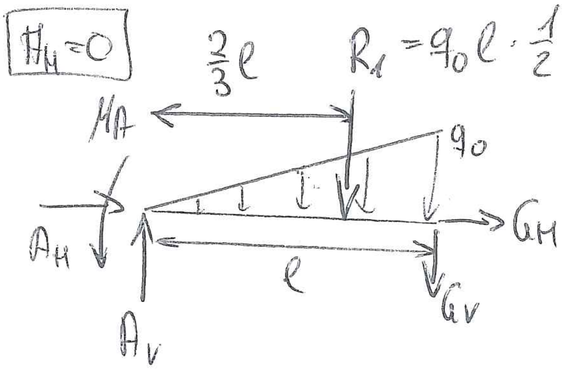
$$\uparrow \sum F_{iv} = 0 = A + B - R - F$$

$$0 = A + 12F - 6F - F \rightarrow \boxed{A = -5F}$$

Möglichkeit 2:

Schnittprinzip:

System (I):



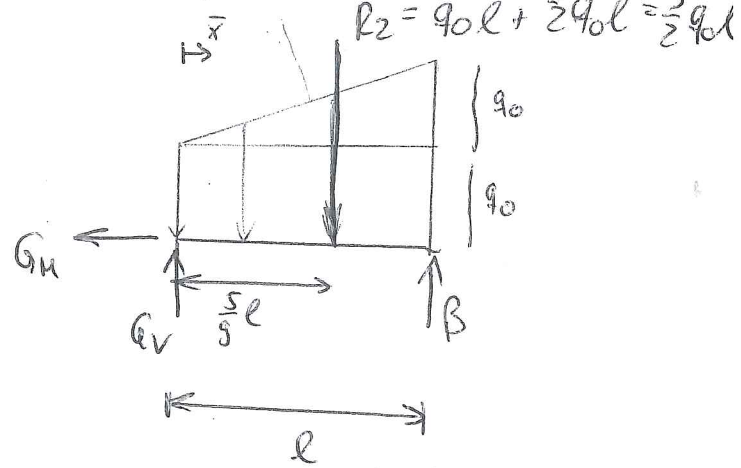
statisch bestimmt?

$$3n - a - z = 0$$

$$\Rightarrow 3 \cdot 2 - 4 - 2 = 0$$

$$0 = 0 \checkmark \text{ stat. bestimmt}$$

System (II):



Angriffspunkt Resultierender

$$\bar{x}_S = \frac{\sum \bar{x}_i A_i}{\sum A_i} = \frac{\frac{l}{2} \cdot \cancel{q_0 l} + \frac{2}{3} l \frac{1}{2} \cancel{q_0 l}}{\frac{3}{2} \cancel{q_0 l}}$$

$$\Rightarrow \bar{x}_S = \frac{\frac{1}{2} + \frac{1}{3}}{\frac{3}{2}} l = \frac{\frac{5}{6}}{\frac{3}{2}} l = \frac{5 \cdot 2}{6 \cdot 3} l = \frac{10}{18} l = \ldots$$

Auflager:

System (I):

$$\sum^{\rightarrow} F_{iH} = \boxed{0 = G_H}$$

$$\uparrow \sum F_{iV} = 0 = A_V + G_V - R_1$$

$$0 = A_V + G_V - \frac{1}{2} q_0 l \quad (1)$$

$$\curvearrowleft \sum M_{iA} = 0 = M_A - R_1 \cdot \frac{2}{3} l - G_V \cdot l$$

$$0 = M_A - \frac{q_0 l}{\cancel{2}} \cdot \frac{\cancel{2} l}{3} - G_V l$$

$$0 = M_A - \frac{q_0 l^2}{3} - G_V l \quad (2)$$

System (II)

$$\uparrow \sum F_{iV} = 0 = G_V + B - R_2$$

$$0 = G_V + B - \frac{3}{2} q_0 l \quad (3)$$

$$\curvearrowleft \sum M_{iG} = 0 = B\cancel{l} - R_2 \frac{5}{9} \cancel{l}$$

$$0 = B - \frac{3}{2} q_0 l \cdot \frac{5}{9_3}$$

$$\Rightarrow \boxed{B = \frac{5}{6} q_0 l}$$

in (3): $0 = G_V + \frac{5}{6} q_0 l - \frac{3}{2} q_0 l$

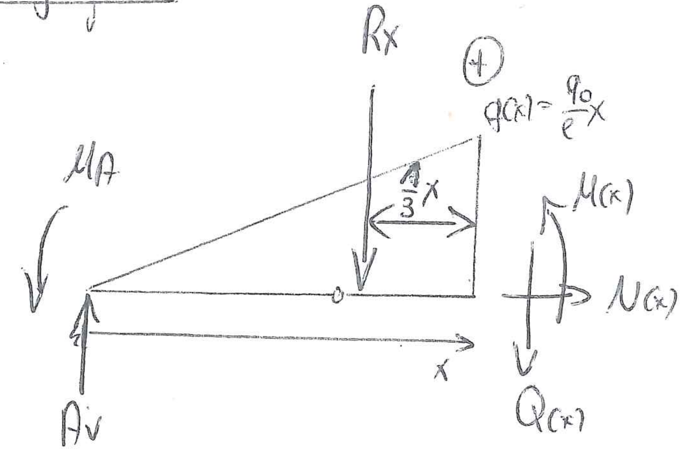
$$\Rightarrow \boxed{G_V = \frac{2}{3} q_0 l}$$

in (1): $0 = A_V + \frac{2}{3} q_0 l - \frac{1}{2} q_0 l$

$$\Rightarrow \boxed{A_V = \frac{7}{6} q_0 l}$$

in (2): $M_A = \frac{q_0 l^2}{3} + \frac{2}{3} q_0 l^2 = \boxed{q_0 l^2 = M_A}$ 3

Schnittgrößen:



$$R_x = \frac{q_0}{l} x \cdot x \cdot \frac{1}{2} = \frac{1}{2}\frac{q_0 x^2}{l}$$

$\boxed{N(x)=0}$

$$\uparrow \sum F_{iv} = 0 = A_v - R_x - Q(x)$$

$$0 = \frac{7}{6} q_0 l - \frac{q_0 x^2}{2l} - Q(x)$$

$$\Rightarrow Q(x) = \frac{7}{6} q_0 l - \frac{q_0 x^2}{2l} = \frac{q_0 l}{6}\left[-3\left(\frac{x}{l}\right)^2 + 7\right]$$

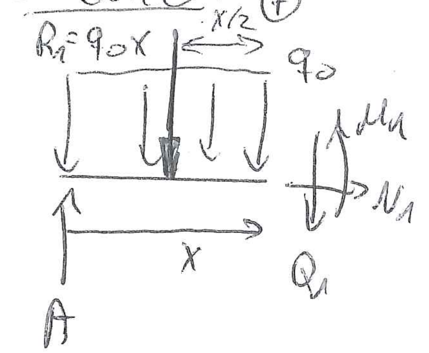
$$\circlearrowright \sum M_{ix} = 0 = -A_v x + R_x \frac{1}{3} x + M(x) + M_A$$

$$M(x) = \frac{7}{6} q_0 l x - \frac{q_0 x^2}{2l} \cdot \frac{x}{3} - q_0 l^2$$

$$M(x) = \frac{7}{6} q_0 l x - \frac{q_0 x^3}{6l} - q_0 l^2 = \frac{q_0 l^2}{6}\left[-\left(\frac{x}{l}\right)^3 + 7\left(\frac{x}{l}\right) - 6\right]$$

b) Schnittgrößen:

Bereich (I):



$$R = q_0 x = \frac{2F}{\ell} x$$

$$\boxed{N_1 = 0}$$

$$\uparrow \Sigma F_{iv} = 0 = A - R_1 - Q_1(x)$$

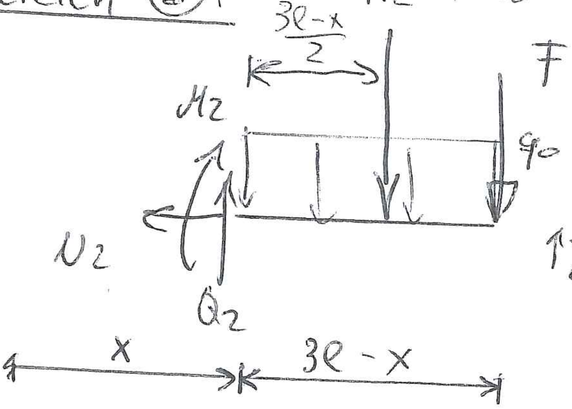
$$0 = -5F - \frac{2F}{\ell} x - Q_1(x) \rightarrow \boxed{Q_1(x) = -5F - \frac{2F}{\ell} x}$$

$$\Sigma M_{ix} = 0 = -Ax + R_1 \frac{x}{2} + M_1(x)$$

$$0 = 5Fx + \frac{2Fx}{\ell} \cdot \frac{x}{2} + M_1(x) \rightarrow \boxed{M_1(x) = -\frac{Fx^2}{\ell} - 5Fx}$$

Bereich (II):

$$R_2 = q_0(3\ell - x) = \frac{2F}{\ell}(3\ell - x)$$



$$\boxed{N_2 = 0}$$

$$\uparrow \Sigma F_{iv} = 0 = Q_2 - R_2 - F$$

$$0 = Q_2 - \frac{2F}{\ell}(3\ell - x) - F$$

$$0 = Q_2 - 6F + \frac{2Fx}{\ell} - F$$

$$\rightarrow \boxed{Q_2 = 7F - \frac{2Fx}{\ell}}$$

$$\Sigma M_{ix} = 0 = -F(3\ell - x) - R_2 \frac{3\ell - x}{2} - M_2(x)$$

$$0 = -3F\ell + Fx - \frac{2F}{\ell}(3\ell - x)^2 \cdot \frac{1}{2} - M_2(x)$$

$$M_2(x) = -3F\ell + Fx - \frac{F}{\ell}(9\ell^2 - 6\ell x + x^2)$$

$$M_2(x) = -3F\ell + Fx - 9F\ell + 6Fx - \frac{Fx^2}{\ell}$$

$$M_2(x) = -12F\ell + 7Fx - \frac{Fx^2}{\ell}$$

1) Maximum, wo $Q=0$ oder Bereichsgrenzen:

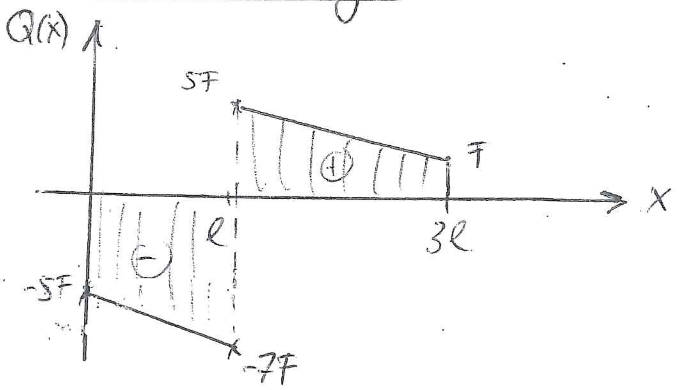
$$Q_1' = -q_0 \neq 0 \; ; \; Q_2' = -q_0 \neq 0$$

Bereichsgrenzen:

$$M_1(x=0)=0$$
$$M_1(x=l) = Fl[-1-5] = -6Fl$$
$$M_2(x=l) = Fl[-1+7-12] = -6Fl$$
$$M_2(x=3l] = Fl[-9+21-12] = 0$$

$$\Rightarrow |M(x=l)| = 6Fl = |M_{max}|$$

graphische Darstellung:

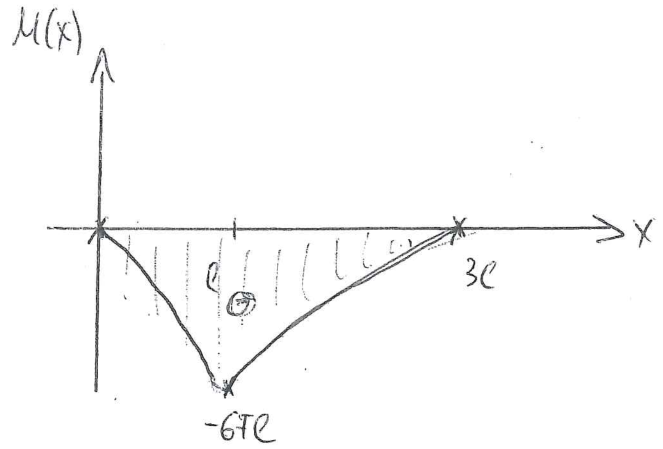


$Q_1(0) = -5F$

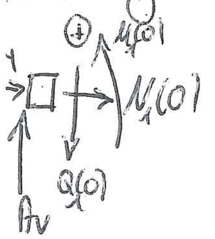
$Q_1(l) = -7F$

$Q_2(l) = 5F$

$Q_2(3l) = F$

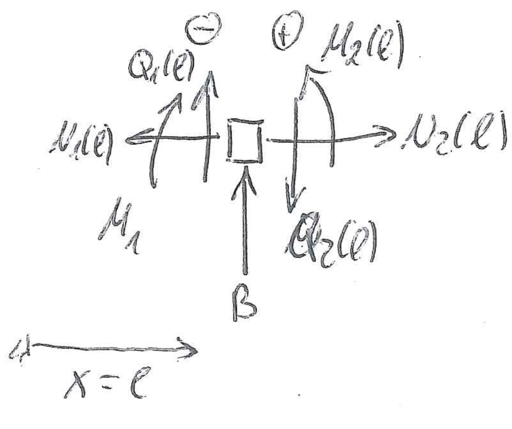


Auflager aus DGL



$A_v = Q_1(0) = -5F$

$A_H = N_1(0) = 0$



$\uparrow \sum F_{iv} = 0 = Q_1(l) - Q_2(l) + B$

$\Rightarrow B = Q_2(l) - Q_1(l)$

$\Rightarrow B = 5F + 7F = 12F$