On nonlinear damage accumulation and creep-fatigue interaction of a damage model for the lifetime prediction of adhesively bonded joints

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Damage accumulation and creep-fatigue interaction are investigated and depicted for a continuum damage model proposed for the lifetime prediction of bonded steel joints with the structural adhesive BETAMATE 1496V™. The nonlinearity of damage accumulation is caused by the nonlinearity of damage interaction as a result of the identified parameters. In consequence of the nonlinear accumulation, the model allows for the loading sequence effect, which is the influence of the chronological order of the load values on the lifetime. Although the nonlinearity of the damage accumulation is not very pronounced, the model prediction is in good agreement with lifetimes from tests with service loading of the adhesive at hand.

1 Model for lifetime prediction and parameter identification

The following nonlinear first order ordinary differential equation (1) is proposed in [1] for the evolution of isotropic damage D in the toughened adhesive polymer BETAMATE 1496V™ ($c_0 = 1.0$ s, $\langle x \rangle = (x + |x|)/2$).

$$\frac{\mathrm{d}D}{\mathrm{d}t} = \dot{D} = \dot{D}_\mathrm{c} + \dot{D}_\mathrm{f} \quad , \quad \dot{D}_\mathrm{c} = \frac{1}{c_0}\left(\frac{\langle \sigma_\mathrm{eq} - \sigma_\mathrm{dc} \rangle}{\sigma_\mathrm{ref}(1-D)}\right)^n \quad , \quad \dot{D}_\mathrm{f} = \left(\frac{\langle \sigma_\mathrm{eq} - \sigma_\mathrm{df} \rangle}{(\sigma_\mathrm{u} - \sigma_\mathrm{df})(1-D)}\right)^k \frac{\langle \dot{\sigma}_\mathrm{eq} \rangle}{\sigma_\mathrm{u} - \sigma_\mathrm{df}} \tag{1}$$

Both the creep $\dot{D}_\mathrm{c}$ and fatigue damage part $\dot{D}_\mathrm{f}$ depend on the equivalent stress $\sigma_\mathrm{eq} = \sqrt{b_1 t_\mathrm{n}^2 + b_2 t_\mathrm{n} + t_\mathrm{t}^2 + t_\mathrm{b}^2}$, which is computed from the interface tractions in normal t_n, tangential t_t and binormal direction t_b. The virgin material is monotonically damaged by positive stresses σ_eq over time t from $D = 0$ at $t = 0$ until rupture at $D = 1$, where the lifetime $t = t_\mathrm{R}$ is attained. In the following, pure shear is considered, so the parameters b_1 and b_2 for shear-tension interaction are irrelevant, as $t_\mathrm{n} = t_\mathrm{b} = 0$, $\sigma_\mathrm{eq} = t_\mathrm{t} = \tau$. The model parameters in Tab. 1 are identified by means of lifetimes from creep and constant amplitude fatigue tests displayed in Fig. 1. The Backward Differentiation Formula of order two and the time step size $\Delta t = 0.00625$ s are applied for the numerical solution of Eq. (1) in consequence of the results in [2]. The predictions illustrated in Fig. 1 are in good agreement with the test data used for the identification and, thus, verify the implementation.

Table 1: Model parameters as identified in [2]

σ_ref [MPa]	σ_dc [MPa]	n [-]	σ_u [MPa]	σ_df [MPa]	k [-]
42.35	0	25.04	50.40	0	15.64

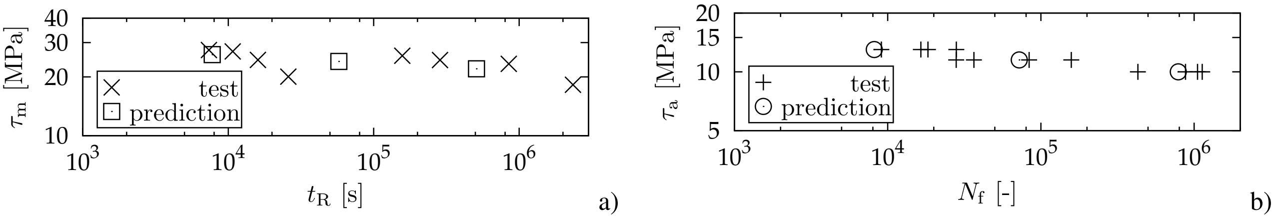

Fig. 1: Rupture times t_R and numbers of cycles to failure $N_\mathrm{f} = t_\mathrm{R} f$ of tests from [1] and numerical predictions for pure shear loading: (**a**) creep loading with stress τ_m; (**b**) fatigue loading $\tau = \tau_\mathrm{m} + \tau_\mathrm{a}\sin(2\pi f t)$ with frequency $f = 10$ Hz and load ratio $R = \min\tau / \max\tau = 0.1$

2 Damage accumulation and creep-fatigue interaction

A K-level loading consists of K functions $\tau_i = \tau_{\mathrm{m}i} + \tau_{\mathrm{a}i}\sin(2\pi f_i t)$ defined one after the other in time. The load level is denoted with $i = 1, ..., K$. The hypothesis of linear damage accumulation $(2)_a$ states that failure occurs due to K-level loading, when the sum of life fractions $t_i/t_{\mathrm{R}i}$ equals one. In Eq. $(2)_a$, t_i is the time spent on load level i and $t_{\mathrm{R}i}$ is the rupture time for the case, where one-level loading with τ_i is applied. The Eq. $(2)_a$ with $K = 2$ yields Eq. $(2)_b$ for a two-level loading.

$$\sum_{i=1}^{K} \frac{t_i}{t_{\mathrm{R}i}} = 1 \quad , \quad \sum_{i=1}^{2} \frac{t_i}{t_{\mathrm{R}i}} = \frac{t_1}{t_{\mathrm{R}1}} + \frac{t_2}{t_{\mathrm{R}2}} = 1 \tag{2}$$

The summands in Eqs. (2) are commutative and independent of the chronological order of the life fractions $t_i/t_{\mathrm{R}i}$. So the loading sequence is not taken into account. The hypothesis of linear creep and fatigue damage interaction $(3)_a$ states that the life fraction $t_i/t_{\mathrm{R}i}$ is the sum of the creep $t_i/t_{\mathrm{Rc}i}$ and the fatigue contribution $t_i/t_{\mathrm{Rf}i}$. The times $t_{\mathrm{Rc}i}$ and $t_{\mathrm{Rf}i}$ denote the failure times for pure creep damage, i. e. $\dot{D}_\mathrm{f} = 0, \dot{D} = \dot{D}_\mathrm{c}$ in Eq. (1), or pure fatigue damage, i. e. $\dot{D}_\mathrm{c} = 0, \dot{D} = \dot{D}_\mathrm{f}$ in Eq. (1). The substitution of Eq. $(3)_a$ into Eq. $(2)_a$ yields the hypothesis of linear damage accumulation and creep-fatigue

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damage interaction $(3)_b$, which becomes Eq. $(3)_c$ for one-level loading until rupture, as $K = 1$ and $t_1 = t_{\mathrm{R}1}$, see Eq. $(2)_a$.

$$\frac{t_i}{t_{\mathrm{R}i}} = \frac{t_i}{t_{\mathrm{Rc}i}} + \frac{t_i}{t_{\mathrm{Rf}i}} \quad , \quad \sum_{i=1}^{K} \left(\frac{t_i}{t_{\mathrm{Rc}i}} + \frac{t_i}{t_{\mathrm{Rf}i}} \right) = 1 \quad \overset{K=1}{\Rightarrow} \quad \frac{t_{\mathrm{R}1}}{t_{\mathrm{Rc}1}} + \frac{t_{\mathrm{R}1}}{t_{\mathrm{Rf}1}} = 1 \tag{3}$$

Data from tests with K-level loadings usually show a significant loading sequence effect and, thus, disprove Eq. $(2)_a$, but encourage the need for nonlinear damage accumulation models. The following Figs. 2a) through c) demonstrate the nonlinear accumulation of model (1) through the nonlinear interaction of the creep $\dot{D}_{\mathrm{c}}$ and fatigue damage part $\dot{D}_{\mathrm{f}}$.

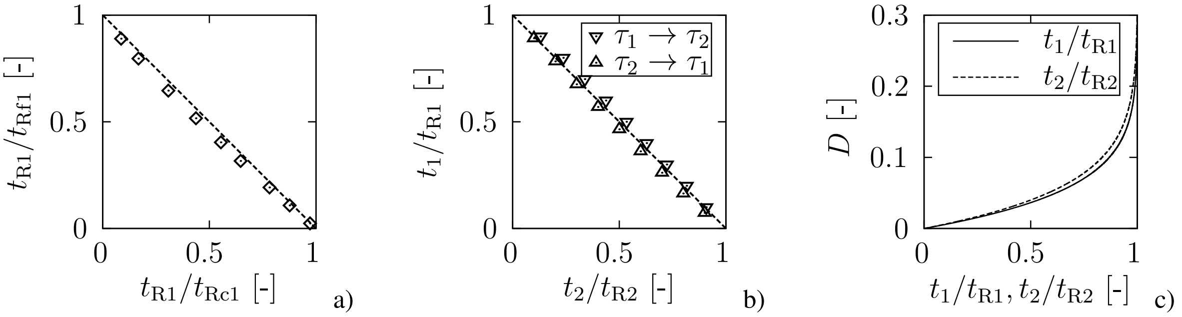

Fig. 2: (**a**) Creep-fatigue interaction for different one-level loadings; (**b**) life fraction for two-level loadings with $\tau_1 = \tau_{1\mathrm{m}} + \tau_{1\mathrm{a}} \sin(2\pi f t)$ ("high" loading) and $\tau_2 = \tau_{2\mathrm{m}} + \tau_{2\mathrm{a}} \sin(2\pi f t)$ ("low" loading), $f = 10$ Hz, $R = \min \tau_1 / \max \tau_1 = \min \tau_2 / \max \tau_2 = 0.1$, $\tau_{1\mathrm{a}} = 13$ MPa, $\tau_{2\mathrm{a}} = 10$ MPa, $t_{\mathrm{R}1} = 818$ s, $t_{\mathrm{R}2} = 78898$ s; (**c**) damage over life fraction for the loadings τ_1 and τ_2 in b)

Fig. 2a) depicts the predicted creep and fatigue damage contributions by Eq. (1) for different one-level loadings until failure. The dashed line represents linear accumulation and interaction according to Eq. $(3)_c$. The predictions show rather weak nonlinear interaction and accumulation as they do not match with the dashed line. The nonlinear accumulation is also illustrated by the failure predictions with Eq. (1) in Fig. 2b) for the indicated High-Low ($\tau_1 \to \tau_2$) and Low-High ($\tau_2 \to \tau_1$) two-level loadings. The dashed line represents linear accumulation according to Eq. $(2)_b$. The predictions show nonlinear accumulation, because they do not coincide with the dashed line. In Fig. 2c), the nonlinear accumulation is confirmed by the comparison of damage D over the life fractions $t_1/t_{\mathrm{R}1}$ and $t_2/t_{\mathrm{R}2}$ due to one-level loading with τ_1 or τ_2 until failure, since in the case of linear accumulation, the curves would match in order to comply with Eq. $(2)_b$.

The nonseparability of differential equation (1) for $n \neq k$, see Tab. 1, is in accordance with the evidence of nonlinear accumulation, because separability is the necessary and sufficient condition for linear accumulation [3], [4]. In Eq. (1), pure creep ($\dot{D}_{\mathrm{f}} = 0, \dot{D} = \dot{D}_{\mathrm{c}}$) and pure fatigue damage ($\dot{D}_{\mathrm{c}} = 0, \dot{D} = \dot{D}_{\mathrm{f}}$) lead to linear accumulation, because every damage part $\dot{D}_{\mathrm{c}}$ and $\dot{D}_{\mathrm{f}}$ is separable on its own. As a conclusion, the nonlinearity of damage accumulation according to Eq. (1) is determined through the nonlinearity of damage interaction as shown in Fig. 2a) due to the identified parameters n and k in Tab. 1.

3 Model and parameter validation by service loading

As illustrated in Figs. 2a) through c), the nonlinear damage accumulation of Eq. (1) due to the nonlinear damage interaction is not very pronounced. Nevertheless, the damage model is successfully validated by means of pure shear tests with service loading, whereby the multiple $\tau = \alpha\tau^*$ of the loading sequence τ^* shown in Fig. 3a) is applied to the adhesive layer repeatedly until failure. Fig. 3b) shows the good agreement between the numerical predictions by Eq. (1) and the numbers of cycles to failure N_{f} from the tests. So the model with its parameters as identified is validated for the service loading at hand.

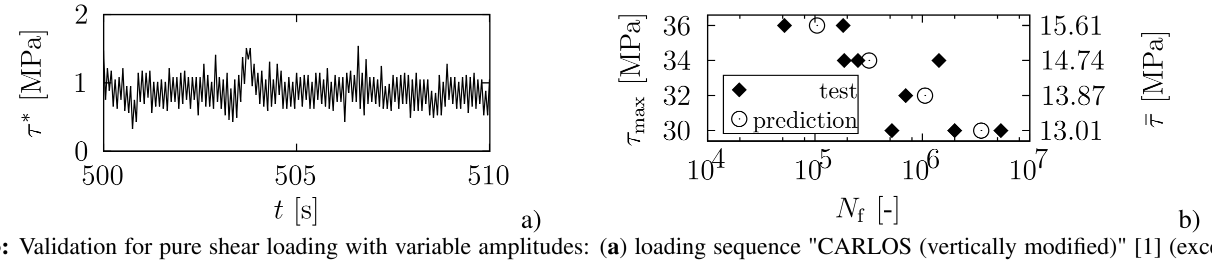

Fig. 3: Validation for pure shear loading with variable amplitudes: (**a**) loading sequence "CARLOS (vertically modified)" [1] (excerpt of 200 peaks, total time of sequence: 3.8 h, $\min \tau^* = 0$ MPa, $\max \tau^* = 2$ MPa, arithmetic mean of all peak values $\bar{\tau}^* = 0.866$ MPa); (**b**) tested lifetime and prediction due to shear stress $\tau = \alpha\tau^*$ with maximum $\tau_{\max} = \max \tau$ and arithmetic mean $\bar{\tau}$ in the bonding layer

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