Presented in May 2012 at Thessaloniki to the International Symposium *The Issue of Time in Aristotle,*

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Print version, submitted for publication with the Proceedings of the Symposium

Time as "measure".

Is Aristotle's account of time in *Physics* IV a theory of time-measurement?

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1. There is no straightforward answer to the question in the subtitle of my paper. Aristotle's account of time in *Physics* IV contains some preliminaries to, or fragments of, a theory of time-measurement. Hence, there are two question: (i) whether those fragments are essential to Aristotle's account of time in *Physics* IV (which, of course, is hard to deny) and (ii) whether the deficiencies of this account as a theory of time-measurement are detrimental to the theory.

With respect to the latter question I will argue they aren't. Since measurement assumes a standard unit, a fundamental question to be answered by any theory of measurement is this: What is it to be a standard unit of measurement? Taken as a theory of time measurement, Aristotle's account of time in *Physics* IV is deficient since that question is not even asked. But I will argue that the success of this account is not at all impaired by this deficiency.

Given a standard unit (such as the length of a rod, the time taken by a sand-glass to empty, the value of a coin, etc.), measurement is a matter of counting. Of course, Aristotle's account of time in *Physics* IV is a theory of marking off, and counting, temporal units – but just not standard units, as I said. I will argue that this, not time-measurement in any more technical sense, is sufficient both

- to account for the temporal unification described in *Phys*. IV 14, which of course is an important feature in Aristotle's cosmology,¹ and
- to make sense of Aristotle's analysis of linear continua, as presupposed in his discussion of Zeno's paradoxes of motion in *Phys*. VI.²

¹ Cf. Coope (2005, 5; cf. ibid. 113 ff.) who, however, follows Aristotle in confusing measurement with counting. Coope's remark that Aristotle's account of time in terms of counting and change is essentially incomplete (ibid. 80) should not be too rashly transferred from time-direction (see below 3.3.) to time-measurement since in the latter case, Aristotle would be easily able to fill the gap mentioned (see below 1.1.).

² It should be mentioned that in *Phys*. VI (c. 2. and 7), both metrical structures and uniform motion are presupposed by Aristotle's (attempted) proofs of the compactness theorems 233a31-4 and 237b24-5. In addition, the claim at *Phys*. IV 10, 218a6-7 that "the part measures (*metrei*)" the whole clearly involves time-measurement. But Aristotle's argument in that passage boils down to the observation that, given

1.1. The deficiencies mentioned are evident in Aristotle's claim that

T 1: *Phys.* IV 12, 220b32 - 221a4. ὁ χρόνος ... μετρεῖ δ' οὖτος τὴν κίνησιν τῷ ὁρίσαι τινὰ κίνησιν ἣ καταμετρήσει τὴν ὅλην (ὥσπερ καὶ τὸ μῆκος ὁ πῆχυς τῷ ὁρίσαι τι μέγεθος ὁ ἀναμετρήσει τὸ ὅλον), ...

Hussey translates "time ... measures change by defining some change which will measure out the whole change (just as the cubit measures length by defining some magnitude which will measure off the whole magnitude)"

A theory of time-measurement should be able to explain what sense can be made of that comparision of a change with a rod. Taken as it stands, it makes no sense.

I can use my ruler to measure the width of my desk – and find that it is less than four times the length of the ruler. The procedure is this. I have to mark successively the positions of the upper edge of the ruler as I move it step by step along the desk. It isn't just a truism that this amounts to making a measurement. Rather, measurement assumes that the length of my ruler is unchanged by the procedure described. In Aristotle's terms, *phora* and *auxêsis* / *phthisis* must be diverse kinds of change which regularly take place independently of each other. This, and hence the possibility to measure extension in space in the way described, was taken for granted in all cosmologies until Relativity Theory derived the relevant regularities from more refined principles.

How, then, to measure a change K? The analogy proposed in the passage quoted would have me make another change K_0 shift step by step along with K. This, of course, is sheer nonsense. What I can, and what it makes sense to claim I should, do is this: to repeat K_0 again and again as K takes place, thus producing a sequence of changes K_0 , K_1 , ... K_{n-1} , K_n such that K_{n-1} , but not K_n , finished while K is still taking place.³ This is the way in which Galileo's pendulum, or a mechanical clock is used which supplies the pendulum (or escapement, etc.) with a motive power, a counting device, and a display. There are cases in which a sand-glass may be employed similarly; in other cases the same purpose is served by the observation of astronomical periodicies (e.g., by counting days). Each of these instruments has a certain degree of resolution: there is nothing to count if the periodicity employed exceeds the change

an appropriate metrical framework, the assumption that the now is a part of time is inconsistent with the version of the Archimedean axiom thus quoted. As a heuristic preliminary, this is innocious.

³ This is also Aristotle's proposal: to refer to the repetition of a change, e.g. of the sun's moving around the ecliptic, or a certain part of that. Aristotle claims that "as" the change thus repeated is "one and the same change again and again, so too" is "time, e.g. a year or spring or autumn" (*Phys.* IV 12, 220b12-4: ώς ἐνδέχεται κίνησιν εἶναι τὴν αὐτὴν καὶ μίαν πάλιν καὶ πάλιν, οὕτω καὶ χοόνον, οἷον ἐνιαυτὸν ἢ ἔαο ἢ μετόπωοον). Evidently, the identity in question cannot be strict. At best, Aristotle is able to exhibit an isomorphism with respect to before and after (which entails identity of the respective numbers with which the corresponding times are equated). But Aristotle points to nothing to secure that the isomorphism extends to a metric.

to be measured in length. Don't try to determine how long my tea has to draw by counting days.

Instead, I may try measuring time through the act of breathing. Suppose I breathe seven times while my neighbour enters her car, and eleven times while a child crosses the street. Evidently, this counting does not amount to time-measurement since I may be relaxed and rested on the one occasion and out of breath from riding by bike on the other. My breathing seven times on the former occasion may even take more time than my breathing eleven times on the other. But the question is still, what does that mean? Why is counting days an appropriate method of measurement whereas counting the number of breathes is not?

This leads me back to my starting point: The measurement in question requires some equivalent to the rigidity of a rod upon which the measurement of extension in space is dependent. Of course, uniformity is that equivalent. Uniform repetition is a measure, non-uniform repetition is not. Assuming that the celestial motions are uniform whereas my breathing is not, the conclusion is straightforward. Counting days is an appropriate method of time-measurement since it relies on uniform repetition. Counting breathes is not since breathing provides no equivalent to the rigidity of a rod.

Yet the question remains what it is for a change or motion to be uniform. As far as I can see, there is no answer to this question in *Phys*. IV. In particular, there is nothing of the sort in the two relevant passages in ch. 14 where uniformity is explicity mentioned.⁴ The first passage (222b33-223a4) describes the obvious way in which velocities of uniform motions are compared. In the second, it is claimed that

T 2: *Phys.* **IV 14, 223b18-20.** ... εἰ οὖν τὸ ποῶτον μέτρον πάντων τῶν συγγενῶν, ἡ κυκλοφορία ἡ ὁμαλὴς μέτρον μάλιστα, ὅτι ὁ ἀριθμὸς ὁ ταύτης γνωριμώτατος. Hussey translates: "... if, then, that which is first is the measure of all things of the same sort, then uniform circular motion is most of all a measure, because the number of this is most easily known."

It is hard to understand what it is for a number of change to be more or less easily known. One might assume that Aristotle refers to counting days or marks on astronomical instruments (t_1 = when the sun's shadow met this mark, t_2 = when the sun's shadow met the 5th mark to the right). But $gn\hat{o}rim\hat{o}tatos$ is claimed to reflect some priority which seems to have nothing to do with that. Aristotle does not explain in the passage quoted what priority is meant by $to pr\hat{o}ton$ (b18) but it is quite a safe guess that the First Movement of his cosmology is being referred to.⁵ That's why I propose to understand $gn\hat{o}rim\hat{o}tatos$ (b20) in the light of

⁴ Aristotle refers to uniformity of motion by both *homalos* (223a1 and passim) and *homalos* (223b19 ff. and passim). I agree with Wagner (1979, 586 – on 223a1/2) that the terms are "ziemlich synonym".

⁵ Cf. *Met.* XIII 3, 1078a13, with *to akribes* (a10) corresponding to *gnôrimôtatos* in T 2; see also *Phys.* VIII 6, 260b5-7 and passim.

Aristotle's distinction between *gnôrimôteron hêmin* and *haplôs gnôrimôteron* (*Phys.* I 1, 184a16 ff. and elsewhere). Measurement procedures exhibit something which is more easily known to us (*gnôrimôteron hêmin*). But as the celestial rotation is primary in the cosmic order, its "number" is "known best" (*gnôrimôtatos*) in a sense corresponding to that primacy. It serves as a principle upon which all other relevant knowledge depends. Hussey's translation is misleading in this respect.

Later in the *Physics* (VIII 6, 260b17-19 and passim) and in *De Caelo* (II 6), Aristotle will derive the uniformity of the celestial rotation from the uniformity of both its trajectory and its cause. It is pointless to ask whether this account of uniform motion was available to him when he wrote *Phys*. IV: what is missing here is not the answer but the question thus answered.

1.2. My second point is just a footnote to the first. It may appear less sophisticated, and maybe it is. Time measurement involves a variety of instruments only some of which have something to do with astronomical observation. Other tools I mentioned above are, e.g. waterclocks, sand-glasses, and the pendulum. Again, they function through uniform repetition. And again, therefore, one may ask how this is to be secured. As in the case of breathing, the repetitive character of the operation won't do. The question is, rather, why the motion of the pendulum is regular and how disturbances in the flow of sand or water may be avoided. Galileo was able to derive the regularity of the motion of the pendulum from the regularities of uniform and of "naturally accelerated motion". This was difficult enough – and is by far the easier case. As far as I can see, Aristotle was prudent enough just to avoid asking any such questions (and even to avoid mentioning any such time-keeping tools).

2. So far, I have said something about what Aristotle's account of time in *Physics* IV is not. It does, however, have merit. I will mention just three points. Aristotle's account of time is a model of conceptual analysis, in particular with respect to the interdependence of time with both change and consciousness. It is also a model of cosmological construction, illustrating the unity of one world-order in a pluralist cosmology in which no global regularity prevails. In either respect, there may be no point in asking whether Aristotle's account is true. But its use as a model may help us to ask the right questions about conceptual analysis and cosmological construction. Similarly, since Aristotle's account of time is given in terms of change and cognition, it may help us ask whether that is adequate to our concerns. Is there anything else to say about time? – I mean, beyond physics and cognitive science, Aristotelian or contemporary?

I think there is. But this is also beyond the scope of my paper. It may suffice just to mention two Aristotelian topics to which the framework described in *Phys*. IV does not seem to be

⁶ Note that $hapl \hat{o}s$ (184a18) = $t \hat{e}_i physei$ (184a17 and passim).

appropriate: (i) temporal features exhibited by action (see the papers by Couloubaritsis and Moutsopoulos in this volume); (ii) the "tripartite" (and more specifically, tensed) "structure of activity" which the joint paper by Harry and Polansky describes as a characteristic of "mortal beings".⁷

2.1. The questions asked at the beginning of Aristotle's account of time in *Physics* IV, and resumed at the end of chapter 13 (!), are routine:

T 3a: *Phys.* **IV 10, 217b29 / b31-2.** π ερὶ χρόνου ... π ότερον τῶν ὄντων ἐστὶν ἢ τῶν μὴ ὄντων, εἶτα τίς ἡ φύσις αὐτοῦ.

"About time: ... whether it is among things that are or among things that are not, and then [sc. after the former has been established, GH] what its nature is." (tr. Hussey, with modification)

T 3b: Phys. IV 13, 222b27-8. ὅτι μὲν οὖν ἔστιν ὁ χρόνος καὶ τί, καὶ ποσαχῶς λέγεται τὸ νῦν, καὶ τί τὸ ποτὲ καὶ τὸ ἄρτι καὶ τὸ ἤδη καὶ τὸ πάλαι καὶ τὸ ἐξαίφνης, εἴρηται.

Hussey tr.: "It has now been stated that time is, and what it is, and in how many ways 'now' is said, and what 'at some time' and 'recently' and 'just' and 'long ago' and 'suddenly' are."

Aristotle has asked similar questions concerning infinity,⁸ space,⁹ and the void.¹⁰ The Whatis-*F*-question involved owes its popularity as a mark of philosophy to a certain reading (or misreading) of Plato. Its place, and the place of the preliminary Is-*F*-question in Aristotle's methodology is described in the *Posterior Analytics* as follows.

⁷ Harry and Polansky argue that the so-called tense test is not essential to Aristotle's *energeia-kinesis* distinction at *Met.* IX 6, 1048b18-35. I agree with much of their argument which, however, seems to boil down the observation that the tense test does not presuppose the account of time in *Phys* IV.

⁸ Phys. III 4, 202b35-6: περὶ ἀπείρου, εἰ ἔστιν ἢ μή, καὶ εἰ ἔστιν, τί ἐστιν – resumed at the end of the section by περὶ μὲν ἀπείρου, πῶς ἔστι καὶ πῶς οὐκ ἔστι καὶ τί ἐστιν, εἴρηται (ibid. 8, 208a22-3).

 $^{^9}$ Phys. IV 1, 208a27-9: περὶ τόπου ..., εἰ ἔστιν ἢ μή, καὶ πῶς ἔστι, καὶτί ἐστιν – resumed at the end of the section by περὶ μὲν τόπου, καὶ ὅτι ἔστι καὶ τί ἐστιν, εἴρηται (ibid. 5, 213a10-1).

¹⁰ Phys. IV 6, 213a13-4: περὶ κενοῦ, εἰ ἔστιν ἢ μή, καὶ πῶς ἔστι, καὶ τί ἐστιν, ὥσπερ καὶ περὶ τόπου – resumed at the end of the section by περὶ μὲν κενοῦ, πῶς ἔστι καὶ πῶς οὐκ ἔστι, διωρίσθω τὸν τρόπον τοῦτον (ibid. 9, 217b27-8).

T 4: *APo* **II 1, 89b24-5, 32, 34-5.** ζητοῦμεν δὲ τέτταρα, τὸ ὅτι, τὸ διότι, εἰ ἔστι, τί ἐστιν. ... οἷον εἰ ἔστιν ἢ μὴ ἔστι κένταυρος ἢ θεός ... γνόντες δὲ ὅτι ἔστι, τί ἐστι ζητοῦμεν, οἷον τί οὖν ἐστι θεός, ἢ τί ἐστιν ἄνθρωπος;

Barnes (*ROT*) tr.: "We seek four things: the fact, the reason why, if it is, what it is. ... e.g. if a centaur or god is or is not. ¹¹ ... And knowing that it is, we seek what it is (e.g. so what is a god? or what is a man?)."

As in the other sections of his treatise on infinity, place, the void, and time, ¹² Aristotle's point in asking the question "if it is" is just to secure that some subject matter of inquiry exists. Similarly with infinity, place, and the void, time turns out not to exist as such, but rather to depend on something else, i.e. on change (*kinêsis*). In a sense, there are no such things as infinity, place, the void, and time. Yet, in another, there are. The comparison with centaurs and gods suggested by my pairing of T 3 with T 4 is, therefore, misleading. If there are no centaurs, talk about centaurs is talk about nothing. ¹³ Talk about time is far from that. Rather, Aristotle suggests that talk about time is a certain way of talking about change. Our temporal vocabulary – including the tenses and such terms as 'now', 'recently', etc. – conveys meanings to be explained by reference to an appropriate analysis of chance. Accordingly, the relevant question as to "what it is" takes the form described as follows.

T 5: Phys. IV 11, 219a1-3. ὅτι μὲν οὖν οὖτε κίνησις οὖτ' ἄνευ κινήσεως ὁ χοόνος ἐστί, φανερόν ληπτέον δέ, ἐπεὶ ζητοῦμεν τί ἐστιν ὁ χοόνος, ἐντεῦθεν ἀρχομένοις, τί τῆς κινήσεως ἐστιν.

"It is manifest, then, that time neither is change nor is apart from change, and since we are looking for what time is, we must start from this result, and grasp what it is of change."¹⁴

Time is "something of change" (219a9-10: *tês kinêseôs ti*, tr. Coope). In a way, this is very vague. The phrase quoted does not specify how the relation between time and change is to be described. Time is just claimed to belong somehow to change. Yet in another way, this is

¹¹ Aristotle adds: τὸ δ' εἰ ἔστιν ἢ μὴ άπλῶς λέγω, ἀλλ' οὐκ εἰ λευκὸς ἢ μή ("I mean if it is or is not *simpliciter*, and not if it is white or not", 89b33, my tr.).

¹² That is, *Phys.* III 4 - IV 14. – *Phys.* III 1-3 is not as continuous with the rest as the introduction to books III and IV (*Phys.* III 1, 200b15-25) indicates. This is quite evident from my experience as a translator who tries to be transparent in terminology: There is no translation of *energeia* and *entelecheia* that works for both III 1-3 and the rest.

¹³ This is not to deny that dreams, tales, or myths with centaurs playing some rôle are about something – for the exhibition of which, however, semantic analysis of relevant texts will not do. The more sophisticated ways of interpretation required are beyond my present topic.

¹⁴ Hussey's tr., with corrections: "result" and "grasp" (for *lêpteon*) are mine; "what it is of change" is Coope's (2005, 37), replacing Hussey's "what aspect of change it is". I agree with Coope that it is important not to "suggest that time is a *property* of change" (ibid. 31n1, her italics).

very precise. Aristotle's *ti* is just a blank to be filled in later. Aristotle eventuelly fills in "number with respect to before and after", claiming that.

T 6a: *Phys.* **IV 11, 219b1-2.** τοῦτο ... ἐστιν ὁ χρόνος, ἀριθμὸς κινήσεως κατὰ τὸ πρότερον καὶ ὕστερον.

"... time is this: number of change with respect to before and after"

Aristotle adds that in this case, "number" (arithmos) is just something that is counted (arithmoumenon, b8).

T 6b: *Phys.* **IV 11, 219b5-8.** $\dot{\epsilon}\pi\epsilon$ ὶ δ' ἀριθμός ἐστι διχῶς (καὶ γὰρ τὸ ἀριθμούμενον καὶ τὸ ἀριθμητὸν ἀριθμὸν λέγομεν, καὶ ῷ ἀριθμοῦμεν), ὁ δὴ χρόνος ἐστὶν τὸ ἀριθμούμενον καὶ οὐχ ῷ ἀριθμοῦμεν.

Hussey tr.: "Number is [sc. spoken of, GH] in two ways: we call number both (a) that which is counted and countable and (b) that by which we count. Time is that which is counted and not that by which we count."

Accordingly when there is time, there must be both some counting and something to count (*arithmêton*, b25). The latter is equated with the "now" (*nyn*):

T 6c: *Phys.* **IV 11, 219b25.** $\tilde{\eta}$ δ' ἀριθμητὸν τὸ πρότερον καὶ ὕστερον, τὸ νῦν ἔστιν Hussey tr.: "... and the before and after, considered as countable, is the now."

Both the counting and what is counted are exhibited in the paragraph between T 5 and T 6.

2.2. Analysis of *Phys.* **IV 11, 219a10-b1.** The text of each portion (from *TLG* CD-ROM #D) is accompanied by Hussey's translation¹⁵ and my analysis and remarks. For a systematic exposition see section 2.3. below.

T 7: *Phys.* **IV 11, 219a10-4.** ἐπεὶ δὲ τὸ κινούμενον κινεῖται ἔκ τινος εἴς τι καὶ πᾶν μέγεθος συνεχές, ἀκολουθεῖ τῷ μεγέθει ἡ κίνησις διὰ γὰο τὸ τὸ μέγεθος εἶναι συνεχὲς καὶ ἡ κίνησίς ἐστιν συνεχής, διὰ δὲ τὴν κίνησιν ὁ χρόνος ὅση γὰο ἡ κίνησις, τοσοῦτος καὶ ὁ χρόνος αἰεὶ δοκεῖ γεγονέναι.

Hussey tr.: "Now since what changes changes from something to something, and every magnitude [sc. thus traversed] is continuous, the change follows the magnitude: it is because the magnitude is continuous that the change is too. And it is because the change is that the time is. (For the time always seems [or: is believed (*dokei*)] to have been of the same amount as the change.)"

Analysis of 219a10-14. Let *K* (Aristotle's *kinêsis*) be a change. Accordingly,

(1) There is some object a changing from an initial state α to a final state ω .¹⁶

¹⁵ Insertions in brackets are mine; Hussey's brackets are replaced by braces.

- (2) There is a continuous trajectory M (Aristotle's megethos) from α to ω .¹⁷
- (3) *K* derives its structure from *M*.¹⁸
- (4) Hence, since *M* is continuous, *K* is continuous.¹⁹
- (5) The increase in the time that appears to have passed corresponds to the increase in the change that has come to pass.²⁰
- (6) Hence, since *K* is continuous, time is continuous.²¹

Remark. No account of continuity precedes this argument. So continuity may be taken in a non-technical sense, suggesting that there are no gaps and no jumps. The presupposition is just that in the trajectory there is a distinction of

- more or less with respect to the distance from the initial state which gives rise to a distinction of
- more or less with respect to the progress of the change and, in turn, to a distinction of
- more or less with respect to the time passed
 all of which correspond to each other. Again, no technical sense of "more or less" is required
 no more than in asking "How far have we reached?" and "How long have we been travelling now?"

Τ 8: Phys. IV 11, 219a14-9. τὸ δὴ πρότερον καὶ ὕστερον ἐν τόπῳ πρῶτόν ἐστιν. ἐνταῦθα μὲν δὴ τῆ θέσει ἐπεὶ δ' ἐν τῷ μεγέθει ἔστι τὸ πρότερον καὶ ὕστερον, ἀνάγκη καὶ ἐν κινήσει εἶναι τὸ πρότερον καὶ ὕστερον, ἀνάλογον τοῖς ἐκεῖ. ἀλλὰ μὴν καὶ ἐν χρόνῳ ἔστιν τὸ πρότερον καὶ ὕστερον διὰ τὸ ἀκολουθεῖν ἀεὶ θατέρῳ θάτερον αὐτῶν.

Hussey tr.: "Now the before and after is in place primarily; there it is by convention [or rather: by relative position²²] But since the before and after is in magnitude, it must also

¹⁶ κινεῖται ἔκ τινος εἴς τι (a10-1). – Note that any questions of time direction are settled by this.

 $^{^{17}}$ πᾶν μέγεθος συνεχές (a11), with μέγεθος tacitly referring to the trajectory. – I prefer 'trajectory' to the less technical 'path' since the latter term may suggest only locomotion to be at issue.

¹⁸ ἀκολουθεῖ τῷ μεγέθει ἡ κίνησις (a11-2).

¹⁹ διὰ γὰο τὸ τὸ μέγεθος εἶναι συνεχὲς καὶ ἡ κίνησίς ἐστιν συνεχής (a12-3).

 $^{^{20}}$ ὅση γὰο ἡ κίνησις [sc. γέγονε], τοσοῦτος καὶ ὁ χρόνος αἰεὶ δοκεῖ γεγονέναι. (a13-4). – Two remarks are in order. (i) The regularity, indicated by αἰεί, in the correspondence of the *quantities* mentioned is indicated by my having the *increase* of either corrrespond to the increase of the other. (ii) The meaning of δοκεῖ must be the same as in a6 and a8, indicating such experiences as are referred to earlier by ἄμα γὰο κινήσεως αἰσθανόμεθα καὶ χρόνου (a3-4).

 $^{^{21}}$ διὰ δὲ τὴν κίνησιν [sc. εἶναι συνεχή] ὁ χρόνος [sc. ἐστιν συνεχής] (a13).

be in change, by analogy with [or: correspondingly to] what there is there. But in time, too, the before and after is present, because the one always follows the other of them."

Analysis of 219a14-19. Aristotle turns now to the topological structure common to the relevant continua. There is a sense in which the "before and after" of a14 ff. corresponds to the "more or less" presupposed at a13-4. But there is a difference since a13-4 refers to initial sections only.²³ – Aristotle's starting point is commonplace.

- (7) There are positions relatively ordered by "before and after" in the tracectory of locomotion.²⁴
- (8) In general, there are intermediate states ϕ , ψ , etc. in M ordered by "before and after".²⁵
- (9) Intermediate positions or states are touched in passing by *K*.
- (10) Hence, there must be intermediate events a-in- ϕ , a-in- ψ , etc. in K correspondingly ordered by "before and after". ²⁶
- (11) As K derives its structure from M, so does time derive its structure from K.²⁷
- (12) Hence there is also some intermediate "before and after" in time.²⁸

Remark. Since the trajectory is continuous, there are as many intermediate positions (or states) as there are potential divisions of the trajectory. Note that this – just to correspond to potential divisions of a linear continuum – also holds of points in geometry. Some divisions may be actual in a sense, e.g. by corresponding to places in the more technical sense of Aristotle's account in *Phys*. IV 1-5. But these are exceptions. In general, the relevant divisions are potential.

(in the sequel, "if and only if will" be abbreviated, as usual, by iff).

²² This is the second option mentioned by Bonitz (*Index* 327b17-8). See below section 3.2 (my footnote on *Phys.* III 5, 205b33-4).

²³ Both Hussey (1983, 148 ad loc) and Coope (2005, 72 ff.) suggest that Aristotle derives the "before and after" of intermediate events from the "more or less" of corresponding initial segments. This is unnecessarily complicated since given the distinction of initial from final states, betweenness on the linear continuum will do.

 $^{^{24}}$ τὸ δὴ πρότερον καὶ ὕστερον ἐν τόπω πρωτόν ἐστιν. ἐνταῦω μὲν δὴ τῆ θέσει (a14-6) – "relatively ordered": τῆ θέσει; "in the tracectory of locomotion": ἐν τόπω.

 $^{^{25}}$ ἐν τ $\tilde{\omega}$ μεγέθει ἔστι τὸ πρότερον καὶ ὕστερον (a16).

²⁶ ἀνάγκη καὶ ἐν κινήσει εἶναι τὸ πρότερον καὶ ὕστερον, ἀνάλογον τοῖς ἐκεῖ (a17-8) – "correspondingly": ἀνάλογον. Note that the correspondence is strict, amounting to isomorphism:

 $[\]phi$ occurs before ψ if and only if *a*-in- ϕ occurs before *a*-in- ψ

²⁷ ... ἀκολουθεῖν ἀεὶ θατέρω θάτερον αὐτῶν (a19).

²⁸ καὶ ἐν χρόνω ἔστιν τὸ πρότερον καὶ ὕστερον (a18-9).

Intermediate positions or states are touched in passing while the motion traverses its trajectory. In a sense, this is obviously so. As there is a place where I stood waiting for the street-lights yesterday and which I touch in passing today so there is another position three meters above which a pigeon touched in passing as I aproached; there is an intermediate state of equilibrium touched in passing when domination fades to its opposite, etc. But the question is, what is it to touch a position or state in passing by some motion? Lacking an answer to this question, the events mentioned in (10) and, hence, the "before and after" in both change and time are still ill-defined. – This is the message in the somewhat cryptical statement to follow.²⁹

T 9: Phys. IV 11, 219a19-25. ἔστι δὲ τὸ πρότερον καὶ ὕστερον ἐν τῆ κινήσει ὃ μέν ποτε ὂν κίνησις ἐστιν³⁰ τὸ μέντοι εἶναι αὐτῷ ἕτερον καὶ οὐ κίνησις. ἀλλὰ μὴν καὶ τὸν χρόνον γε γνωρίζομεν ὅταν ὁρίσωμεν τὴν κίνησιν, τῷ πρότερον καὶ ὕστερον ὁρίζοντες καὶ τότε φαμὲν γεγονέναι χρόνον, ὅταν τοῦ προτέρου καὶ ὑστέρου ἐν τῆ κινήσει αἴσθησιν λάβωμεν.

Hussey tr.: "The before and after in change is, in respect of what makes it what it is, change [but see below (13)]; but its being is different and is not change. But time, too, we become acquainted with when we mark off change, marking it off by the before and after, and we say that [sc. a lapse of] time has passed when we get a perception of the before and after in change."

Analysis of 219a19-25.

- (13) The "before and after" in change is that, by being which on occasion change is.³¹
- (14) Yet, what it is to be before and after in change is something else, and is not the same thing as change.³²
- (15) We become acquainted with time when we mark off the change, that is, when we mark it off by what is before and after.³³
- (16) We say that (a lapse of) time has passed when we perceive the before and after in change.³⁴

²⁹ "Cryptical": Coope 2005, 65.

³⁰ ἐστιν (a21): secl. Torstrik, Ross

 $^{^{31}}$ ἔστι δὲ τὸ πρότερον καὶ ὕστερον ἐν τῆ κινήσει ὃ μέν ποτε ὂν κίνησις ἐστιν (a19-21) – "on occasion": *pote* (a20) which corresponds to my "in passing" (9), left unexplained in my analysis of T 8.

 $^{^{32}}$ τὸ μέντοι εἶναι αὐτῷ ἕτερον καὶ οὐ κίνησις (a21).

 $^{^{33}}$ άλλὰ μὴν καὶ τὸν χρόνον γε γνωρίζομεν ὅταν ὁρίσωμεν τὴν κίνησιν, τῷ πρότερον καὶ ὕστερον ὁρίζοντες (a22-3) – "by": $t\hat{o}_i$ (a23).

Remarks. I will not engage now in a discussion of Aristotle's notoriously difficult *ho pote on*.³⁵ My "on occasion" – for *pote* is indepted to Bowin's interpretation of the passage as indicating "diacronic diversity" (Bowin 2008, 69 f.). But it is essential not to take *pote* as just referring to time (which would lead into a vicious circle). Rather, the explanition must keep within the non- (or pre-) temporal framework of the passage.

Aristotle's point in (14) is that the questions as to (i) what it is to be before and after in change and, in particular, (ii) what of change it is to which the distinction between before and after applies, have not yet been answered. The clause *kai ou kinêsis* may even be understood as indicating that the questions are not to be answered just in terms of change.

In my analysis, (ii) should be answered in terms of events a-in- φ , a-in- ψ , etc. So the canonical questions are (iii) whether events a-in- φ etc. exist at all, and if so, (iv) what an event such as a-in- φ is. The negative part of Aristotle's answer is evident from his discussion of Zeno's arrow in *Phys*. VI 9. Events such as a-in- φ may occur in a change K but must not be considered as parts of which K is composed. So, what is it for an event such as a-in- φ to exist? The positive part of Aristotle's answer is adumbrated in the sequel. For events such as a-in- φ to exist is just to be grasped separately, and to be exhibited in experience as the present event. Derivatively, events thus grasped may be seen as diverse and, hence, may be used in marking off change.

Τ 10: *Phys.* **IV 11, 219a25-30.** όρίζομεν δὲ τῷ ἄλλο καὶ ἄλλο ὑπολαβεῖν αὐτά, καὶ μεταξύ τι αὐτῶν ἕτερον· ὅταν γὰρ ἕτερα τὰ ἄκρα τοῦ μέσου νοήσωμεν, καὶ δύο εἴπῃ ἡ ψυχὴ τὰ νῦν, τὸ μὲν πρότερον τὸ δ' ὕστερον, τότε καὶ τοῦτό φαμεν εἶναι χρόνον· τὸ γὰρ ὁριζόμενον τῷ νῦν χρόνος εἶναι δοκεῖ· καὶ ὑποκείσθω.

Hussey tr.: "We mark off change by taking them [i.e. the before and after in change (a24-5)] to be different things, and some other thing between them; for whenever we conceive of the limits as other than the middle, and the soul says that the nows are two, one before and one after, then it is and this it is that we say time is [but see below (20)]. (What is marked off by the now is thought to be time: let this be taken as true [or rather: be assumed].)"

Analysis of 219a25-30. Notation: K is a change whereby a changes from an initial state α to a final state ω along the trajectory M; ϕ_1 and ϕ_2 are intermediate states in M such that ϕ_1 occurs before ϕ_2 ; e_1 and e_2 are the events a-in- ϕ_1 and a-in- ϕ_2 , respectively.

 $^{^{34}}$ καὶ τότε φαμὲν γεγονέναι χρόνον, ὅταν τοῦ προτέρου καὶ ὑστέρου ἐν τῆ κινήσει αἴσθησιν λάβωμεν (a23-5).

³⁵ But see my notes on T 12 in section 2.3.

- (17) We mark off the events e_1 and e_2 by grasping them separately, and something else between.³⁷
- (18) That is, if (i) we conceive of the limits now-*e*¹ and now-*e*² thus grasped as "other than the middle", 38
- (19) and (ii) our soul says the nows involved are two,³⁹ i.e.

 $t_1 =_{\text{def}}$ the now involved in now- e_1

and

 t_2 =_{def} the now involved in now- e_2 , such that t_1 occurs before t_2 ,⁴⁰

- (20) then we call the middle the time interval $[t_1, t_2]$.⁴¹
- (21) What is marked off by the now is time. 42

Remarks. Note that in (21), the phrase "the now" may refer both

- (a) to one now, t, and
- (b) to the now qua being before and after,⁴³ i.e. qua being two "nows", t_1 and t_2 .

In the former case, two times, the time before t and the time after t, are marked off. Only in the latter case is the time marked off a finite time interval, $[t_1, t_2]$. Aristotle's point in (21) is that in either case what is marked off is time but not change (or anything else).

It is important to see that no change but time is primarily marked off by the construction described. Aristotle does not proceed from given events e_1 and e_2 , directly to the corresponding section [e_1 , e_2] in K.⁴⁴ Rather, events e_1 and e_2 exist just by virtue of being grasped as fea-

 $^{^{36}}$ Phys. VI 9, 239b8-9: οὐ γὰο σύγκειται ὁ χρόνος ἐκ τῶν νῦν τῶν ἀδιαιρέτων, ὥσπερ οὐδ' ἄλλο μέγεθος οὐδέν ("time is not composed of nows, i.e. of indivisibles, nor is any other magnitude"); see section 3.5. below.

³⁷ ὁρίζομεν (sc. πρότερον καὶ ὕστερον a23) δὲ τῷ ἄλλο καὶ ἄλλο ὑπολαβεῖν αὐτά, καὶ μεταξύ τι αὐτῶν ἕτερον (a25-6).

³⁸ ὅταν γὰς ἕτεςα τὰ ἄκςα τοῦ μέσου νοήσωμεν (a26-7).

³⁹ Note that my "the nows" is just as ungrammatical as Aristotle's *ta nyn*.

 $^{^{40}}$ καὶ δύο εἴπη ή ψυχὴ τὰ νῦν, τὸ μὲν πρότερον τὸ δ' ὕστερον (a27-8).

 $^{^{41}}$ τότε καὶ τοῦτό (sc. τό μέσον, a27) φαμεν εἶναι χρόνον (a28-9). – As often, *chronos* means "lapse of time" rather than "time" here.

 $^{^{42}}$ τὸ γὰρ ὁριζόμενον τῷ νῦν χρόνος εἶναι δοκεῖ· καὶ ὑποκείσ 9ω (a29-30).

 $^{^{43}}$ $\tilde{\eta}$ πρότερον καὶ ὕστερον (219b12). – Hence, metrei may be retained in the context (Ross: horizei).

⁴⁴ Differently Loughlin (2011, 310) who is hence, in view of (21), committed to claim that Aristotle somehow equates the time [t_1 , t_2] with the change $K \setminus [t_1, t_2]$.

tures in the cognitions described, i.e. in now- e_1 and now- e_2 respectively.⁴⁵ Hence, the time interval [t_1 , t_2] is exhibited first. As a second step, then, a corresponding section in K may be defined in terms of time as follows. $K \setminus [t_1, t_2]$ is the part of K that takes place within [t_1 , t_2]. That is to say, the relevant section in K exists just by virtue of being measured by time.

Aristotle adds:

T 11: *Phys.* **IV 11, 219a30-b1.** ὅταν μὲν οὖν ὡς εν τὸ νῦν αἰσθανώμεθα, καὶ μὴ ἤτοι ὡς πρότερον καὶ ὕστερον ἐν τῆ κινήσει ἢ ὡς τὸ αὐτὸ μὲν προτέρου δὲ καὶ ὑστέρου τινός, οὐ δοκεῖ χρόνος γεγονέναι οὐδείς, ὅτι οὐδὲ κίνησις. ὅταν δὲ τὸ πρότερον καὶ ὕστερον, τότε λέγομεν χρόνον·

Hussey tr.: "So whenever we perceive the now as one, and not either as before and after in the change, or as the same but pertaining [or: belonging] to something which is before and after, no time seems to have passed, because no change {seems to have occurred} either. But whenever {we do perceive} the before and after, then we speak of time."

Analysis of 219a30-b1.

- (22) No change and, hence, no time appears to have passed,⁴⁶ when we perceive the now as one,⁴⁷ rather than either
 - as one before, one after in a change,⁴⁸ or
 - as the same but belonging to certain sections of a change before and after it.⁴⁹
- (23) But whenever (sc. we perceive) the before and after, we speak of time.⁵⁰

Remark. Aristotle's point is that time involves both the now and the before-after. The former is a feature in cognition, the latter in change. Hence, for there to be time, both cognition and change are required, respectively contributing either component.

2.3. As it turns out, the operation referred to in T 6bc as counting is this.⁵¹ Consider some change K whereby an object a changes from an initial state α to a final state ω along the tra-

 $^{^{45}}$ My notation is designed to exhibit two features in a cognition: the event represented and the immediacy of being aware of it. Accordingly, now-e is the cognition that exhibits e in experience as the present event.

⁴⁶ οὐ δοκεῖ χρόνος γεγονέναι οὐδείς, ὅτι οὐδὲ κίνησις (a32-3).

 $^{^{47}}$ ώς 20 ν τὸ νῦν αἰσθανώμεθα (a30-1).

 $^{^{48}}$ ώς πρότερον καὶ ὕστερον ἐν τῆ κινήσει (a31).

 $^{^{49}}$ ώς τὸ αὐτὸ μὲν προτέρου δὲ καὶ ὑστέρου τινός (a32). – In this case, the times marked off, say, T_1 = (...t] and T_2 = [t ...) or, in modern notation, T_1 = (-∞,t] and T_2 = [t ,∞), are unlimited in opposite directions.

⁵⁰ ὅταν δὲ τὸ πρότερον καὶ ὕστερον (sc. αἰσθανώμεθα, a30-1), τότε λέγομεν χρόνον (a33-b1).

jectory M.⁵² M is a linear continuum, with the direction indicated by "before" and "after" derived from α occurring before ω . Let φ_1 and φ_2 be intermediate states on M touched in passing by K such that φ_1 occurs before φ_2 . Events a-in- φ_1 and a-in- φ_2 are marked off by cognitions now-(a-in- φ_1) and now-(a-in- φ_2). Time is marked off by recognizing that "the nows involved are two, one before, one after" (219a27-8), forming the "limits" (akra, a27) of something else "between" (meson, ibid.).

A misleadingly straightforward definition of the relevant order would run as follows.⁵³

- As ϕ_1 occurs before ϕ_2 , so of the two events a-in- ϕ_1 and a-in- ϕ_2 the former occurs before the latter.
- As a-in- ϕ_1 occurs before a-in- ϕ_2 , so of the two "nows" involved in now-(a-in- ϕ_1) and now-(a-in- ϕ_2), respectively, the former occurs before the latter.

The trouble with that construction is this. The relevant states and events correspond to potential divisions of M and K, respectively, and hence do not exist, or are given, as such.⁵⁴ Rather, events a-in- φ_1 and a-in- φ_2 exist, and are given, by virtue of being marked off by cognitions now-(a-in- φ_1) and now-(a-in- φ_2). States φ_1 and φ_2 are marked off by the relevant events and therefore exist, and are given, derivatively of the latter.

The ordering of the trajectory M is described above in terms of potential divisions and hence, in a sense, as existing potentially. As the ordering is transferred from M to the change K and from K to time, the orderings derived inherit that character. Actual divisions are only effected by the relevant cognitions, and transfer themselves via time backwards to K and M so as to exhibit events and states actually ordered by the same ordering.⁵⁵ For states ϕ_1 and ϕ_2 , events a-in- ϕ_1 and a-in- ϕ_2 , and for the relevant "nows", what it is to exhibit actual order-

⁵¹ Of course, the operation can be repeated indefinitely, whereby increase in number (i.e. in time) and increase in the amount of change passed correspond to each other (cf. 219b3-5). Evidently, this works only with a series of sections in the underlying change K, where each section is contained in its successor. No metric is involved (or can be thus defined).

⁵² Initial and final states are referred to at 219a10-1 by *ek tinos* and *eis ti*, respectively. The distinction between initial and final states is entailed by the definition of change (*Phys*. II 1, 201a10-1, see below 3.1.). Celestial rotations are the anomalous case in which that definition does not apply and, therefore, the requirement stated above is only met by non-cyclic sections of the change. Loughlin (2011) rightly points to that anomaly (though many details of his argument are dubious). But it should be also noted that non-cyclical sections of the whole system of celestial rotations may be indefinitely long – or at least as long as a Great Year (if the celestial rotations are commensurable, about which Aristotle, differently from Plato *Tim*. 39b2 ff., is silent).

⁵³ This seems to be the way in which Loughlin (2011, 310) reads 219a22-30. – See also my remarks in section 2.2. on T 10 above.

⁵⁴ Otherwise, there is nothing to distinguish change from a mere collection of events, and "Zeno gets at you" – as Whitehead seems to have put it in his Harvard Lectures for 1924-5 (Ford 1984, 283).

⁵⁵ Why the same ordering? Because (with respect to the same states and events) no different potential to be thus activated exists.

ing must be therefore explained in terms of what is to be marked off in cognition and, hence, by reference to the relevant cognitions now- $(a-in-\varphi_1)$ and now- $(a-in-\varphi_2)$.

The construction described above should be revised as follows.

- (a) As ϕ_1 occurs before ϕ_2 , so in view of a change touching those states in passing, of the two cognitions now-(a-in- ϕ_1) and now-(a-in- ϕ_2) the former occurs before the latter.
- (b) As now- $(a-in-\phi_1)$ occurs before now- $(a-in-\phi_2)$, so of the two "nows" involved, viz.

 $t_1 =_{\text{def}} \text{ the now involved in now-}(a-\text{in-}\phi_1)$

 $t_2 =_{\text{def}}$ the now involved in now-(a-in- ϕ_2),⁵⁶

the former occurs before the latter, thus forming the limits of the time-interval $[t_1, t_2]$.⁵⁷

If ϕ is a state in the trajectory M, touched in passing by the change K, the now t involved in the cognition now-(a-in- ϕ) is an indivisible position, or instant, in time. t is the instant at which a is in ϕ – that formula being defined by

a is in ϕ at t iff_{def} t is the now involved in the cognition now-(a-in- ϕ).⁵⁸

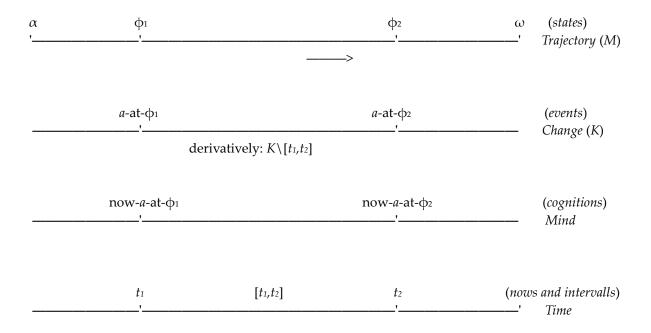


Fig. 1

⁵⁶ I will comment on this presently, with a view on 219b9-28.

⁵⁷ It is important to see that *chronos* at 219a24, a29 (both occurrences), a32-3, and at the first occurrence in b1 is not just time, but a time-interval. Only at the second occurrence in b1, *chronos* refers to the overall structure exhibited by marking off time-intervalls. On a32-3 see my analysis of T 11 above, footnote to the second case in (22).

⁵⁸ As usual, "if and only if" is here and in the sequel abbreviated by iff.

Accordingly, as t_1 occurs before t_2 , so of the two events a-in- ϕ_1 , occurring at t_1 , and a-in- ϕ_2 , occurring at t_2 , the former occurs before the latter. Sections in K are marked off by the relevant time intervals. If x is in ϕ_1 at t_1 and in ϕ_2 at t_2 , then $K \setminus [t_1, t_2]$ is the section in K that takes place within $[t_1, t_2]$ and, hence, corresponds to the section in the trajectory between ϕ_1 and ϕ_2 . In a sense, this is a change of which ϕ_1 is the initial, and ϕ_2 is the final state. But Aristotle would insist that K is not composed of sections. For otherwise, his account of time would be exposed to the Zenonian trouble which Aristotle is so careful to avoid.

So far, the crucial step is the introduction of instants of time. The idea behind my formula $t =_{def}$ the now involved in now-(a-in- φ)

may be most easily explained in terms of of the distinction between (linguistic) meaning and reference (or content).⁶⁰ On the one hand, the meaning of 'now' is always the same – since the rule to follow using that term is the same. On the other hand, since by virtue of that very rule 'now' is a token-reflexive term, its reference is "other and other". Hence on the one hand, the meaning of 'now' is the same with respect to all cognitions now-(a-in- φ). In particular, the meaning of 'now' with respect to the cognition now-(a-in- φ 1) is the same as the meaning of 'now' with respect to the cognition now-(a-in- φ 2). This is exactly what makes it possible to distinguish between the cognitions now-(a-in- φ 1) and now-(a-in- φ 2) in terms of the relevant states φ 1 and φ 2. On the other hand, since the states φ 1 and φ 2 are distinct and exclude each other,⁶¹ the principle of non-contradiction requires that events a-in- φ 1 and a-in- φ 2 cannot jointly occur.⁶² The distinction of aspects that separates them⁶³ is just a distinction in the reference of 'now' with respect to the relevant cognitions now-(a-in- φ 1) and now-(a-in- φ 2). Hence with my t indicating that reference, it follows that just as φ 1 and φ 2 are distinct, so t1 and t2 are also distinct.

This, however, is still too easy – and too far from Aristotle who avoids the second order discourse employed. Aristotle would not describe the distinction between the nows involved in now- $(a-in-\varphi_1)$ and in now- $(a-in-\varphi_2)$ just as a distinction in reference. He would rather de-

⁵⁹ Note that this is the paradigm case in which a change, i.e. $K \setminus [t_1, t_2]$, is "in time" (*en chronôi, Phys.* IV 12, 221a9) – viz. in the time-intervall $[t_1, t_2]$: it takes place just "when that time is" (*hote ho chronos estin*, ibid. a10).

⁶⁰ See D. Brown's *SEP* article on "Indexicals": http://plato.stanford.edu/entries/indexicals (2001, rev. Oct. 15, 2007, viewed Feb. 20, 2013). It shoubt be noted, that Frege's distinction of meaning ("Sinn") and reference ("Bedeutung") does nor coincide with the above-mentioned distinction since the meaning of indexicals is incomplete according to Frege (see his posthumously published *Logik*, p. 48 in Gabriel's edition).

⁶¹ That is to say, the trajectory is a realm of incompatibles ("Inkompatibilitätsbereich" in Tugendhat and Wolf 1983, 60 f.).

^{62 &}quot;jointly": *hama* (*Met.* IV 3, 1005b19 – see below section 3).

^{63 &}quot;distinction of aspects": (ou) kata auto (ibid. b20).

scribe it as a distinction in "being" or "in definition",⁶⁴ which is derivative of a distinction in reference. For any ϕ_1 and ϕ_2 , what it is to be the now involved in now-(a-in- ϕ_1) is distinct from what it is to be the now involved in now-(a-in- ϕ_2). But what it is, for any ϕ , to be the now involved in now-(a-in- ϕ) is always the same. As I understand it, this is just Aristotle's claim in the following passage. ⁶⁵

T 12: Phys. IV 11, 219b12-15

τὸ δὲ νῦν ἔστι μὲν ώς τὸ αὐτό, ἔστι δ' ώς οὐ τὸ αὐτό ή μὲν γὰο ἐν ἄλλ ω καὶ ἄλλ ω , ἕτερον (τοῦτο δ' ἦν αὐτ $\tilde{\omega}$ τὸ νῦν <εἶναι>), ὃ δέ ποτε ὄν ἐστι τὸ νῦν,⁶⁶ τὸ αὐτό.

"The now is in a way the same, in a way not the same. For as it is in other and other occasions it is diverse (that's what being now is for it – i.e., being in other and other occasions). But to the question

By virtue of being what, on any occasion, is the now what it is? the answer is always the same."

The answer – viz. by virtue of being "the before and after in change" – leads directly back to the construction described at a14-30.

My "being what, on any occasion" represents Aristotle's *ho pote on* (b14-5). – As I understand it, this phrase indicates that in the definition of the thing in question a free variable is involved. In the present case, the free variable is φ , the thing in question is the now involved in now-(a-in- φ). With φ as a free variable, the term 'the now involved in now-(a-in- φ)' refers to a function (in the mathematical sense). It refers to the same function, whatever values φ

^{64 &}quot;being": *einai* (219b11, b14, b27); "in definition": *tôi logôi* (219b19-20).

⁶⁵ Here are the direct parallels to T 12 in the context.

b10-1: τὸ γὰο νῦν τὸ αὐτὸ ὅ ποτ' ἦν—τὸ δ' εἶναι αὐτῷ ἕτερον ...

[&]quot;For the now is the same as it was on any occasion. On the other hand, its being is different ..." b26-8: ωστε καὶ ἐν τούτοις ὁ μέν ποτε ὂν νῦν ἐστι, τὸ αὐτό (τὸ πρότερον γὰρ καὶ ὕστερόν ἐστι τὸ ἐν κινήσει), τὸ δ' εἶναι ἕτερον (ἡ ἀριθμητὸν γὰρ τὸ πρότερον καὶ ὕστερον, τὸ νῦν ἔστιν).

[&]quot;Again, to the question 'By virtue of being what, on any occasion, is the now what it is?' the answer is therefore always the same: It is (that is, it is what it is by virtue of being) the before and after in change. But its being is different. For the before and after, qua being countable, is the now."

Cf. b18-21: τοῦτο δὲ ὃ μέν ποτε ὂν τὸ αὐτό (ἢ στιγμὴ γὰρ ἢ λίθος ἤ τι ἄλλο τοιοῦτόν ἐστι), τῷ λόγῳ δὲ ἄλλο, ὥσπερ οἱ σοφισταὶ λαμβάνουσιν ἕτερον τὸ Κορίσκον ἐν Δυκείῳ εἶναι καὶ τὸ Κορίσκον ἐν ἀγορᾳ.

[&]quot;By virtue of what, on any occasion, it is, the moving thing is the same: point or stone etc. But in definition it is different in the way the sophists assume that being Koriscus-in-the Lyceum is different from Koriscus-in-the-Agora."

⁶⁶ On that phrase (*ho ... pote on esti to nyn*, b14-5) and the relevant passage see Brague 1982; Hussey 1983, 148 f.; Charles 2004, 153-5; Coope 2005,173 ff.; Bowin 2008. – My excessively expansive translation of this phrase is meant to emphasize both the free variable indicated by *pote* and the What-is? question indicated by *esti*.

^{67 219}b26-7 (see my footnote above).

may take. That's why, taken in the way indicated by Aristotle's *ho pote on*, the now involved in now-(a-in- φ) is "the same". But evidently, this does not prevent the now involved in now-(a-in- φ) from being different, as φ takes different values (such as φ 1 and φ 2).

3. If time follows change, and is defined in terms of change, change must not be described in terms of time. According to Russell (who thereby represents the canonical view of both modern and contemporery philosophy and natural science),

T 13: B. Russell (1901, 372) "motion consists merely in the fact that bodies are sometimes in one place and sometimes in another, and that there are intermediate places at intermediate times".

For Aristotle, this account of motion it is unavailable. Similarly, Galileo's account of what it is for motion to be uniform is unavailable for Aristotle. According to Galileo (*Discorsi* III), motion is uniform iff the proportion

s-t (with s = the distance in space traversed and t = the amount of time passed) holds – which, in Galileo, is just a prelude to the description of "naturally" accelerated motion" by means of the proportion v-t upon which, in turn, Galileo's derivation of the regularity of the pendulum is dependent. 68 The notion of distinct lapses of time being equal (in length) is thus presupposed by Galileo and cannot be derived from his account of uniform motion.

This being quite trivially so, more sophistication is required by the question as to if, and how, the vicious circle indicated is avoided by Aristotle. In view of that question, I propose to distinguish four levels in Aristotle's analysis of change: ontological, topological, cosmological, and metrical.

- **3.1.** On the **ontological** level, Aristotle presupposes distinctions
- (a) between things, properties of things (both essential and accidental), and collections of things,
- (b) between being actually (entelecheia: = energeia:) and being potentially (dynamei),
- (c) between being jointly or separately (i.e. disjointly) the case, with (b) and (c) respectively underlying
- the definition of change as "the activity of a potential as such",69 and

⁶⁸ I cannot go now into any details. The crucial step in the derivation is Prop. VIII in *Discorsi*, bk. 3. See also the relevant passage in die *Dialogue* (p. 470 f. tr. Strauss, Stuttgart 1982).

⁶⁹ Phys. III 1, 201a10-1: ή τοῦ δυνάμει ὄντος ἐντελέχεια, ἦ τοιοῦτον, κίνησίς ἐστιν – "activity": *entelecheia* (see Anagnostopoulos, 2010, 59 and passim). As an alternative, "(state of) complete activation" is worth considering.

• the principle of non-contradiction, viz. the principle that "for the same thing jointly (*hama*) to hold good (*hyparchein*) and not to hold good of the same thing and in the same respect is impossible ..."⁷⁰

Both the definition of change and the principle of non-contradiction are presupposed in Aristotle's account of time. Hence, it is important to see that no reference to time is presupposed in either. In Aristotle's statement of the principle of non-contradiction, *hama* was rendered in English as "simultaneously" by Kirwan and as "at the same time" by Ross (*ROT*). But the temporal connotation in either translation is superficial. The phrase 'at the same time' (in German: 'gleichzeitig', not just 'zugleich') may even occur in strictly tenseless discourse, such as in the statement "2 is even, and at the same time is a prime number ". For safety, I render *hama* in English by "jointly", so as to avoid the temporal connotation. Insofar as the principle of non-contradiction is employed to distinguish positions in time from each other, the key term is Aristotle's *kata to auto*.

In a sense, this is another case in which time follows change. But change, as it is described by the principle of non-contradiction, is just a collection of facts about one given object of which no two can without qualification jointly occur. Taken in this way, positions in time – i.e. Aristotle's "now" being "other and other" – are qualifications such that each fact is marked by the qualification with which it obtains,⁷¹ and the collection of all the facts thus marked may coexist. No direction or order are thus exhibited, neither in change nor in time.

By contrast, direction is exhibited by Aristotle's definition of change in terms of potentiality and activity. A straightforward way to distinguish a change from its converse is this. Consider some object a and a property F such that a admits of both being non-F and being F. By definition,

a changes from being *non-F* to being *F* iff *a* is not *F* but its potentiality to be *F* is active as such.

Conversely,

a changes from being *F* to being *non-F* iff *a* is *F* but its potentiality to be *non-F* is active as such.

Note that in either case, the relevant initial and final states are distinguished in terms of the property *F*. But no intermediate states are exhibited. What is between the initial and final states is just the change, and is uniformly described as the relevant potentiality being active as such.

 $^{^{70}}$ Met. III 3, 1005b19-20: τὸ γὰρ αὐτὸ ἄμα ὑπάρχειν τε καὶ μὴ ὑπάρχειν ἀδύνατον τῷ αὐτῷ καὶ κατὰ τὸ αὐτό ... (Kirwan's tr., but with my "jointly" replacing his "simultaneously").

⁷¹ That is, given Kirwan's translation of *Met*. III 3, 1005b19-20 quoted above, by the "respect" in which it obtains..

3.2. Topological structures are described by Aristotle in terms of the before-after relation. Other topological concepts such as the concepts of betweenness, of a continuum (or interval), of beginning and end, of simultaneousness, of contact and of continuous connection are easily defined in terms of this before-after relation, together with the ontological concepts mentioned above. It is topological structure with respect to which Aristotle claims explicitly that change follows magnitude (i.e. the extension of its trajectory), and time follows change.⁷²

The direction involved in this structure derives from the distinction of initial and final states mentioned above and transmits itself to the intermediate states that correspond to the positions in the relevant trajectory. In the special case of locomotion – i.e. of change of place – the relevant structure of the trajectory may be described in terms of geometry. Yet, geometry is not sufficient. As geometry is indifferent with respect to our orientation in space, so the geometric structure of the trajectory is indifferent to the distinction between a change and its converse. Right and left, front and back, etc. depend upon our orientation in space, and are thereby, i.e. *pros hêmas kai thesei*, defined.⁷³ Similarly, the succession in the trajectory is given $t\hat{e}i$ thesei.⁷⁴ That is to say, it derives its direction from the distinction between initial and final states that corresponds to the direction of that change.

There is still another important respect in which it is not just geometry that serves Aristotle's purpose. Geometry, as it is known to Aristotle, has to do with regularly defined things such as straight lines, circles, and what can be constructed out of them. That's why he claims that only in special cases, such as optics, harmony and astronomy, does mathematics yield the physical account.⁷⁵ The general analysis of change and motion is no such case. Trajectories in space may be as regular as a stadium and as irregular as the road from Thebes to Athens.⁷⁶ The metrical structures exhibited in geometry cannot be straightforwardly transferred to the more irregular cases.⁷⁷ In addition, Aristotle insists that the existence of a trajectory is not

⁷² *Phys.* IV 11, 219a14-19, see above T 8 in section 2.2.

 $^{^{73}}$ Cf. *Phys.* III 5, 205b33-4. – Hussey's "conventionally" for *thesei* is as misleading here as his "by convention" is for $t\hat{e}i$ thesei at 219a16. A good discussion of the former occurrence is in Wagner's commentary (1979, 519 – on 205b34).

⁷⁴ *Phys.* IV 11, 219a16, see above 2.2.

 $^{^{75}}$ Cf. Phys. II 2, 194a7-12. My "mathematics yield(s) the physical account" corresponds to Aristotle's ή δ' ὀπτική μαθηματικήν μὲν γραμμήν (sc. σκοπεῖ, a10), ἀλλ' οὐχ ἧ μαθηματική ἀλλ' ἦ φυσική (a11-2).

⁷⁶ Phys. III 3, 202b13-4.

⁷⁷ As far as I can see, there is nothing in Aristotle that anticipates the modern definition length of a irregular line = least upper bound of the lengths of the corresponding polygon lines.

peculiar to locomotion, i.e. to the change of place.⁷⁸ In the other cases, the contraries involved in the change are claimed – accidentally, if not in itself – to exhibit a continuum of intermediate states.⁷⁹ But evidently, there is no straightforward transfer of metrical structures from geometry to that continuum.

It is, therefore, important to see that there is no such difficulty with the topological structures involved. Rather, the trajectories of all kinds of change – of place, in quality or in magnitude – are claimed by Aristotle to exhibit the same topological characteristics, viz. the characteristics of a linear continuum. Moreover, Aristotle claims that

- (i) for every change, the time the change takes and its trajectory are isomorphic with respect to order,⁸⁰ and
- (ii) time is the same for all changes.81

Taken together, the claims entail that all trajectories are isomorphic with respect to order. This is also the point in my claim that Aristotle's account of time in *Phys*. IV is no theory of time-measurement. Aristotle's analysis in *Phys*. IV – VI of the linear continua involved in change is offered in topolological rather than metrical terms. In particular, time and linear order, but not time-measurement, are presupposed in Aristotle's discussion of Zeno's paradoxes in *Phys*. VI. But, with the exception of a little note on Zeno's arrow,⁸² I will not go into any detail now.

3.3. Aristotle's **cosmology** is pluralistic. On the one hand, there are as many natures – i.e. irreducible regularities and, hence, principles of scientific explanation – as there are biological species and, more generally, natural kinds. On the other hand, things interact. Interaction requires contact according to Aristotle. Yet, whenever two or more kinds are involved, contact is not regularly established by the relevant natures but requires coincidences. Sensation is the exceptional case in which physical interaction takes place at a distance, and requires no contact.

In modern cosmologies, from Galileo to relativity and quantum theories, space-time is a universal frame of reference providing parameters for the description of all kinds of regularity. There is no such framework in Aristotle. Let a and a' be two objects which independent-

⁷⁸ This is tacitly presupposed at *Phys*. IV 11, 219a10 ff. (see above 2.2.) where *kinêsis* and *megethos* refer to any change and its trajectory. At *Phys*. V 3, 227a7 ff. trajectories are described in terms of contraries, of which distance in space is a special case.

⁷⁹ Phys. V 3, 226b23-5, cf. Phys. VI, passim – "accidentally ...": Phys. VI 4, 235a18, cf. a36 with the construction described in c. 5, 236b4-8.

⁸⁰ Phys. IV 11, 219a10 ff., Phys. VI, passim – "isomorphic": analogon (219a17 f.), tou autou logou (231b18), etc.

⁸¹ *Phys.* IV 14, 223a29-b12, see below T 14.

⁸² See below section 3.5.

ly of each other undergo changes K and K', respectively. In adition, let φ and φ' be intermediate states in the relevant trajectories M and M' such that a is in φ at t and a' is in φ' at t'. Aristotle's definition, described above, of t and t' is offered just in terms of K and K', respectively. No relation is thus defined that connects t with t'. Taken in this way, it is pointless to ask whether t = t', or which one of the two occurs before the other, since no meaning has yet been given to that.

The conclusion that every change has a time of its own is explicitly rejected by Aristotle as follows.

T 14: Phys. IV 14, 223b1-4, 4-10, 10-12

a) ἀλλ' ἔστι νῦν κεκινῆσθαι καὶ ἄλλο· ὧν ἑκατέρας τῆς κινήσεως εἴη ἂν ἀριθμός. ἕτερος οὖν χρόνος ἔστιν, καὶ ἄμα δύο ἴσοι χρόνοι ἂν εἶεν· ἢ οὔ; ὁ αὐτὸς γὰρ χρόνος καὶ εἷς ὁ ἴσος καὶ ἄμα· εἴδει δὲ καὶ οἱ μὴ ἄμα·

"There may be another change, and (sc. time) may be the number of either. Is there, then, another time and two equal times simultaneously, 83 or not? For time is the same when equal and simultaneous. And non-simultaneous times ought to be the same, too."84

b) εἰ γὰο εἶεν κύνες, οἱ δ' ἵπποι, ἑκάτεροι δ' ἑπτά, ὁ αὐτὸς ἀριθμός. οὕτω δὲ καὶ τῶν κινήσεων τῶν ἄμα περαινομένων ὁ αὐτὸς χρόνος, ἀλλ' ἡ μὲν ταχεῖα ἴσως ἡ δ' οὔ, καὶ ἡ μὲν φορὰ ἡ δ' ἀλλοίωσις· ὁ μέντοι χρόνος ὁ αὐτός, εἴπερ καὶ ὁ ἀριθμὸς ἴσος καὶ ἄμα,⁸⁵ τῆς τε ἀλλοιώσεως καὶ τῆς φορᾶς.

"Suppose there are dogs and horses, seven of each: the number will be the same. In the same way, of changes that are jointly accomplished the time is the same. It makes no difference whether their speed is the same or not, or one is locomotion and the other qualitative change. The time is nevertheless the same, if the number is also the same and is jointly exhibited of either, ⁸⁶ the qualitative change and the motion."

c) καὶ διὰ τοῦτο αἱ μὲν κινήσεις ἕτεραι καὶ χωρίς, ὁ δὲ χρόνος πανταχοῦ ὁ αὐτός, ὅτι καὶ ὁ ἀριθμὸς εἷς καὶ ὁ αὐτὸς πανταχοῦ ὁ τῶν ἴσων καὶ ἄμα.

⁸³ "simultaneously": *hama* (b3, cf. the two occurrences at b4). Later in T 14, when relations between changes rather than times are described by *hama*, I will prefer "jointly", as in the statement of the principle of non-contradiction (see above 3.1.). In T 14a, "simultaneously" does refer to time but should be taken in a colloquial rather than technical sense. The trouble is just with cases that count as simultaneous by the common standards of time-keeping: the question as to what it is to be simultaneous has not yet been answered. – But see also my note on *hama* at b9 below.

⁸⁴ The last claim is obvious in view of overlapping times.

⁸⁵ ὁ ἀριθμός (b9), deleted by Ross, is rightly retained by Wagner (1979, 588 – on 223b8-10).

⁸⁶ Differently from Wagner (ibid.), I understand *hama* (b9) as indirectly qualifying the number (*arithmos*, b9) involved: *hama* = *hama* (sc. *arithmoumenos*). Strictly speaking, "jointly" is yet as undefined for separate changes as "simultaneously" is for the relevant times.

"Though the changes are different and separate from each other, time is everywhere the same because the number too is everywhere one and the same of equal changes that are jointly accomplished."

Time is the number exhibited by recognition when events that occur before and after are marked off in the observation of change. Aristotle's point in the passages just quoted seems to be this.⁸⁷ It makes no difference whether one or more changes are involved; the same construction as in the former case applies to the joint observation of many changes.

Let K and K' be two changes – e.g., the sun moving along its daily path from east to west, and a wall being painted in my house – which can be jointly observed. In particular, if a is the sun and a' is the wall, and if φ and φ' are intermediate states on the relevant trajectories M and M', there may be a joint cognition such as

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now-(a-in-\varphi and a'-in-\varphi').
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Assume that in the ordering of M and M', respectively, ϕ_1 occurs before ϕ_2 , and ϕ_1 occurs before ϕ_2 . Of the nows involved in the relevant cognitions, viz.

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t^*_1 = _{\mathrm{def}} the now involved in now-(a-in-\varphi_1 and a'-in-\varphi_1) and
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t^*<sup>2</sup> = def the now involved in now-(a-in-\varphi<sup>2</sup> and a'-in-\varphi<sup>2</sup>),
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the former is exhibited in recognition as occurring before the latter. Hence, the lapse of time between t^{*_1} and t^{*_2} is defined in the same way as it was defined for a single movement. As the operation is iterated, time appears as the number which is common to the changes K and K'.88

Insofar as time is defined separately in terms of K or K', it is easily equated with the time defined jointly in terms of both K or K'. If

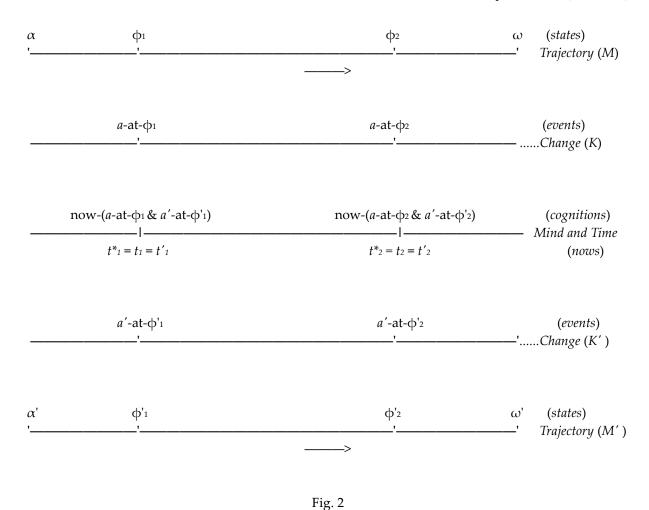
```
t_1 =def the now involved in now-(a-in-\varphi_1); t_2 =def the now involved in now-(a-in-\varphi_2) and
```

 $t'_1 =_{\text{def}}$ the now involved in now- $(a'-\text{in}-\varphi'_1)$; $t'_2 =_{\text{def}}$ the now involved in now- $(a'-\text{in}-\varphi'_2)$, it obvious that both t_1 and t'_1 may be equated with t^*_1 and both t_2 and t'_2 may be equated with t^*_2 , etc. In sum, the relevant times are the same.

⁸⁷ See Coope (2005, 123) on joint counting.

⁸⁷

^{**}S Loughlin (2011, 315 with reference to *Phys*. IV 14, 224a2-14) denies that the numbers involved are "truly identical". He seems to follow Hussey (1983, 160 f.) in conflating counted collections with counted numbers. At 224a3 and a14, *dekas* is just a collection of ten things (not, as Hussey translates, "the ten"). Hence, on the one hand, what it is to be a *dekas* formed by ten dogs or horses, respectively, is not the same. But what it is to be a *dekas* – and, hence, what the number ten is qua *arithmoumenos* (11, 219b7-8, see above T 6b; see also 12, 220b9) – is the same with respect to both horses, and dogs. Aristotle could not be more explicit in this than he is at 220b10-2: ἔστι δὲ ὁ ἀριθμὸς εἶς μὲν καὶ ὁ αὐτὸς ὁ τῶν ἑκατὸν ἵππων καὶ ὁ τῶν ἑκατὸν ἀνθρώπων, ὧν δ' ἀριθμός, ἕτερα, οἱ ἵπποι τῶν ἀνθρώπων.



There is, however, a gap which Aristotle is unable to fill.⁸⁹ The construction described above requires both that ϕ_1 occurs before ϕ_2 , and that ϕ_1 occurs before ϕ_2 . It may be adapted to the case in which one of the changes is interrupted by a state of rest (and hence, say, $\phi_1 = \phi_2$). But the construction is incoherent if in the ordering of M and M', respectively, ϕ_1 occurs before ϕ_2 , but ϕ_1 occurs after ϕ_2 .⁹⁰ In this case, t^*_1 and t^*_2 may be defined as above and, because of the incompatibilities of ϕ_1 with ϕ_2 , and of ϕ_1 with ϕ_2 , may still count as distinct. But neither may count as occurring before the other. Equating t^*_1 with both t_1 and t'_1 , and t^*_2 with both t_2 and t'_2 leads to inconsistency since t_1 occurs before t_2 but t'_1 occurs after t'_2 .

The difficulty does equally arise if one of the motions involved, say K, is a celestial rotation and, hence, has a claim to primacy in the cosmic order according to Aristotle. Still, the ordering of time depends upon the ordering of M (i.e. of relevant trajectory) which, in turn, depends upon the nature of a (i.e. of the thing that undergoes change). Let K be some celes-

⁸⁹ For a less technical description of the difficulty see Coope 2005, 79 f. According to Coope (ibid. 79), Aristotle does not even raise the relevant question.

⁹⁰ That is, in Fig. 2, the arrow on M is unchanged but the arrow on M' is directed to the left.

⁹¹ See my remarks on T 2 (i.e. *Phys.* IV 14, 223b18-20) above.

tial rotation, e.g. of the fixed stars or, as above, of the sun. It is important to see that the primacy of *K* assumed by Aristotle cannot be transferred to the relevant natures; Aristotle's pluralism would otherwise be denied. So Aristotle cannot rule out the possibility that

- (i) it is in the nature of a that in the ordering of M, ϕ_1 occurs before ϕ_2 ,
- (ii) it is in the nature of a' that in the ordering of M', ϕ'_1 occurs after ϕ'_2 , and
- (iii) there are cognitions now- $(a-in-\varphi_1)$ and $a'-in-\varphi_1$ and now- $(a-in-\varphi_2)$ and $a'-in-\varphi_2$.

With $t^*_{1,2}$, $t_{1,2}$, and $t'_{1,2}$ defined as above, the inconsistency of $t^*_{1} = t_1 = t'_{1}$ with $t^*_{2} = t_2 = t'_{2}$ may be avoided by claiming that, in view of the primacy attributed to K, universal time t^* must be equated with t rather than t'. But as this would leave no connection of t' with universal time, the unity of time would be denied thereby.

3.4. The fundamental **metrical** concept is that of something taking **as much time as** something else. Again, other metrical concepts are easily derived from this: A takes less time than B iff B has a proper part B' such that A takes as much time as B'. A takes half the time B takes iff B can be divided into equal parts B' and B'' such that A takes as much time as either B' and B'' take; etc.

The notorious trouble with this concept is this. In general, of the lapses of time thus compared, none is part of the other. Typical cases are: Breakfeast took longer time today than yesterday; I will spend less time in this country now than I did last year, etc. That's why metrical concepts cannot be reduced to such ontological or topological concepts as inclusion or the before-after relation. The difficulty is resolved by Aristotle in a similar way as it is in modern accounts: Given some uniform motion K such that both K and K and K are simultaneous, respectively, either covers as much ground as the other.

Again, change follows, and derives its structure from, its trajectory. And again, time follows change. So, if it may be taken for granted that equality in the relevant dimension is well defined, the crucial questions is what it is for a movement in that dimension to be uniform. As I said, this question is not even asked in *Phys*. IV.

It should be noted, however, that no metric is required by the division of lapses of time into parts. Given t_1 and t_2 such that t_1 occurs before t_2 , continuity requires that there is always some t such that t_1 occurs before t, and t occurs before t_2 . The time-interval $[t_1, t_2]$ is thus divided into the intervals $[t_1, t]$ and $[t_1, t_2]$, both shorter than $[t_1, t_2]$. But no length is defined of the parts such that one can be compared to the other.

Of course, the operation can be iterated indefinitely. I will not go into any details now since "the whole labyrinth about the composition of the continuum" would be thereby "unraveled" (quoting Leibniz, note of February 11, 1676; p. 159 in Loemker's edition). But two remarks are in order. (i) The division is straightforwardly transferred from one continuum to another

by any continuous motion (i.e., which has no turns and is not interrupted by rest). No uniformity of motion is required to secure the isomorphism in division and order. (ii) As the infinite division of a change is isomorphic with both the infinite divisions of the relevant trajectory and of the relevant time, such paradoxes as Zeno's dichotomy cannot even be restated within the framework described. Taken in this way, Aristotle's refutation of the paradox in *Phys.* VI 2 (233a23 ff.) is just to the point. No metric is required thereby – nor is any metric presupposed in the general argument of *Phys.* VI.

3.5. Appendix: Zeno's Arrow; analysis of *Phys.* **VI 1, 231b28-232a17.** The argument Aristotle presents in *Phys.* VI 9 as Zeno's *Arrow* is essentially this. ⁹²

Assuming that (i) "everything either is at rest or moves whenever it occupies a position equal to itself" and (ii) "the moving thing is always in the now", the flying arrow is (iii) "motionless" (239b5-7) and, therefore, (iv) "stands still" (ibid. b30).⁹³

This argument is based on the observation that instantaneous motion is a contradiction in terms and, hence, (v) "nothing moves in the now". 94 (ii) and (v) entail that (iii) the arrow is always "motionless" (and evidently occupies a space equal to itself). Taken together with (i), (iii) entails that (iv) the arrow "stands still".

Aristotle comments that (vi) "time is not composed of nows". His point is that, on the one hand, "always" in (ii) and hence in the whole argument (insofar as it is valid) only refers to "nows", i.e. indivisible positions in time. Hut since, on the other hand, "time is not composed of nows" nothing follows concerning the extended lapses of time required by motion and rest. In particular, instantaneous rest is as much as instantaneous motion a contradiction in terms. For instance, neither motion nor rest take place in the very moment when something has finished its movement, and will thereupon be at rest. Since at that moment the thing in question undeniably occupies a space equal to itself, (i) is false and, hence, Zeno's argument is fallacious.

⁹² See my reconstruction in Heinemann 2007, but see also below, remark (vii) on T 16. – "Arrow": oistos, 239b30.

 $^{^{93}}$ Modern interpreters usually follow Zeller (1876, 547n1) in deleting from (i) the clause "or moves" (\hat{e} *kineitai*, b6). But this clause makes perfectly sense, and is in the transmitted text.

⁹⁴ Phys. VI 3, 234a24; but see below.

⁹⁵ *Phys.* VI 9, 239b8, similarly b30-31.

⁹⁶ I prefer "positions" to the more common "points in time" since "point" (*stigmê*) is reserved by Aristotle as a terminus technicus for geometry.

⁹⁷ Phys. VI 3, 234a31-b9.

Aristotle's proof of (v) involves slower and faster motions taking more or less time, which they cannot when occurring "in the now". 98 This may suggest a metrical framework. But it is important to see that the relevant theorem is also prooved in *Phys*. VI 1 independently of any consideration of velocities. What follows is a preliminary version of my analysis and remarks on the relevant passage, *Phys*. VI 1, 231b28-232a17 (see T 16 below).

At the beginning of the relevant chapter, Aristotle has restated his definitions of continuous connection, contact, and succession (231a21-3), and has stated

Thm. 1. What is continuous cannot be composed of indivisibles.99

The proof of this (231a26-b15) is given in terms of point and line (introduced as examples at a24-6). It should be noted that in the statement of the theorem, continuity isn't just continuous connection. Rather, to be continuous is to be composed of parts which are continuously connected.¹⁰⁰ A terminological remark inserted in the proof is:

T 15: *Phys.* VI 1, 231b10-1. ἐξ ὧν ἐστιν ..., εἰς ταῦτα διαιρεῖται.

To be composed of ... means to be divided into

The proof of Thm. 1 is supplemented by

Corollary to Thm. 1. Whatever is continuous, is divisible into divisible parts, and so forth indefinitely.¹⁰¹

The next theorem is

Thm. 2. Consider some change K along the trajectory M taking the time T. Then either K, M, and T are all, and in isomorphic ways, composed of indivisible parts, or none is. 102

Aristotle's proof is split up into two sections in which it is argued that (1) the indivisible parts of a motion and of its trajectory correspond to each other, and (2) so do the divisions of a motion and of the time it takes .¹⁰³

As a digression, an argument is inserted between (1) and (2) of which the conclusion may be stated as follows.¹⁰⁴

⁹⁸ Phys. VI 3, 234a25-31.

 $^{^{99}}$ 231a24: ἀδύνατον ἐξ ἀδιαιρέτων εἶναί τι συνεχές. – *Phys.* VI is mostly composed like a modern mathematics text and will, in my analysis, be reported in that manner.

¹⁰⁰ Hence, to be continuous (*synechês*, *Phys.* VI 1, 231a24) is to be the unified whole formed of those parts (see *Phys.* V 3, 227a16: *houtô kai to holon estai hen*).

^{101 231}b16: πᾶν συνεχὲς διαιρετὸν εἰς αἰεὶ διαιρετά

 $^{^{102}}$ 231b18-20: τοῦ δ' αὐτοῦ λόγου μέγεθος καὶ χρόνον καὶ κίνησιν ἐξ ἀδιαιρέτων συγκεῖσθαι, καὶ διαιρεῖσθαι εἰς ἀδιαίρετα, ἢ μηθέν.

¹⁰³ 231b22-8 and 232a19-22, respectively.

Thm. 3. The trajectory of motion cannot be composed of indivisible parts.

Aristotle's proof of Thm. 3 is

T 16: Phys. VI 1, 231b28-232a17.

a) εἰ δὴ ἀνάγκη τὸ κινούμενον ποθέν ποι μὴ ἄμα κινεῖσθαι καὶ κεκινῆσθαι οὖ ἐκινεῖτο ὅτε ἐκινεῖτο (οἶον εἰ Θήβαζέ τι βαδίζει, ἀδύνατον ἄμα βαδίζειν Θήβαζε καὶ βεβαδικέναι Θήβαζε), (231a28-232a1)

Let a move from α to ω . a cannot at the same time travel and have completed travelling the trajectory it traversed. For instance, whatever walks to Thebes cannot at the same time walk to Thebes and have completed walking to Thebes.¹⁰⁵

b) (sc. εἰ) τὴν δὲ τὸ A τὴν ἀμερῆ ἐκινεῖτο τὸ Ω , ἦ ἡ τὸ Δ κίνησις $\pi\alpha$ ρῆν ιωστ'... (232a1-2) If a travelled the indivisible distance M_0 , and the corresponding motion was K_0 , then there are three cases:

c) εἰ μὲν ὕστερον διεληλύθει ἢ διήει, διαιρετὴ ἂν εἴη (ὅτε γὰρ διήει, οὔτε ἠρέμει οὔτε διεληλύθει, ἀλλὰ μεταξὺ ἦν), (232a2-4)

Case 1: a has completed traversing M_0 after it traversed M_0 . In this case, M_0 (?) is divided since, when a traversed M_0 , it was neither (still?) at rest, nor had it completed traversing M_0 , but was (sc. somewhere) between.

d) εἰ δ' ἄμα διέρχεται καὶ διελήλυθε, τὸ βαδίζον, ὅτε βαδίζει, βεβαδικὸς ἐκεῖ ἔσται καὶ κεκινημένον οὖ κινεῖται. (232a4-6)

Case 2: a has completed traversing M_0 at the same time when traversing M_0 . In this case, what walks will, while walking, be at the destination, having completed walking, and having completed travelling the distance it travels.

e) εἰ δὲ τὴν μὲν ὅλην τὴν ΑΒΓ κινεῖταί τι, καὶ ἡ κίνησις ἣν κινεῖται τὰ Δ Ε Z ἐστι, τὴν δ' ἀμερῆ τὴν A οὐθὲν κινεῖται ἀλλὰ κεκίνηται, εἴη ἂν ἡ κίνησις οὐκ ἐκ κινήσεων ἀλλ' ἐκ κινημάτων καὶ τῷ κεκινῆσθαί τι μὴ κινούμενον τὴν γὰρ A διελήλυθεν οὐ διεξιόν. ὥστε ἔσται τι βεβαδικέναι μηδέποτε βαδίζον ταύτην γὰρ βεβάδικεν οὐ βαδίζον ταύτην. (232a6-11)

Case 3: a travels along the whole trajectory $M_0M_1M_2$, the corresponding motion being $K_0K_1K_2$, yet never travels but only has completed traveling the indivisible part M_0 . (a6-8) In this case, the motion would be composed of jerks rather than motions, and would take place by virtue of having completed motion without moving. For a has completed traversing M_0 without traversing. Hence it is possible for something to have completed walk-

¹⁰⁴ In my analysis, I adopt the notation employed for *Phys*. IV (see above 2.2.). My *a* corresponds to Aristotle's Ω, my M_0 , M_1 , M_2 to Aristotle's A B Γ, my K_0 , K_1 , K_2 to Aristotle's Δ E Z.

¹⁰⁵ This is also the idea in Aristotle's tense test at *Met*. IX 6, 1048b18-35: When I see Thebes, and thus have a view at Thebes, I also have had that view in its completion.

ing without ever walking (which, of course, is absurd). For it has completed walking that distance without walking it. (a8-11)

f) εἰ οὖν ἀνάγκη ἢ ἠρεμεῖν ἢ κινεῖσθαι πᾶν, ἠρεμεῖ καθ' ἕκαστον τῶν Α Β Γ, ὥστ' ἔσται τι συνεχῶς ἠρεμοῦν ἄμα καὶ κινούμενον. τὴν γὰρ ΑΒΓ ὅλην ἐκινεῖτο καὶ ἠρέμει ότιοῦν μέρος, ὥστε καὶ πᾶσαν. καὶ εἰ μὲν τὰ ἀδιαίρετα τῆς ΔΕΖ κινήσεις, κινήσεως παρούσης ἐνδέχοιτ' ἂν μὴ κινεῖσθαι ἀλλ' ἠρεμεῖν εἰ δὲ μὴ κινήσεις, τὴν κίνησιν μὴ ἐκ κινήσεων εἶναι. (232a12-7)

Case 3 (cont.). If everything either is at rest or moves, a is at rest at M_0 , M_1 , M_2 each. Consequently, there will be something continuously at rest and moving at the same time. For a travelled along the whole trajectory $M_0M_1M_2$, and is at rest at each part and, hence, at the whole. (a12-5) And if the parts of the motion $K_0K_1K_2$ are motions, it would be possible in the presence of motion (i.e. while untergoing motion) not to move but to be at rest. If, by contrast, (sc. the parts are) not motions, the (sc. whole) motion would not be composed of motions. (a15-7)

Remarks: (i) Evidently, case 3 corresponds to Russell's account of motion according to which both trajectory and motion are composed of indivisible parts (viz. places and arrivals, respectively). Aristotle's point is that what motion is composed of must be motions, not just arrivals.

- (ii) Both cases 2 and 3 are precluded by the rules of tensed discourse. But Aristotle also attaches importance to the observations that in case 3, motion would not be composed of motions (a8, a17), and would be compatible with rest.
- (iii) Aristotle's argument concerning case 1 fails if the trajectory is a succession of indivisible parts such that each part M_n (i.e., where there is an arrival) has an immediate predeccessor M_{n-1} . For on that assumption, a may traverse M_n while having just traversed M_{n-1} , and before having traversed M_n (etc.) in the same way as I climb the stairs: climbing the 1st stair while still being on the floor, then climbing the 2nd stair while just having arrived at the 1st and, in general, climbing the nth stair while just having arrived at the (n-1)th.
- (iv) Similarly, if Aristotle insists that a is at M_n while traversing M_n (which he probably does): Assuming that M_n has an immediate successor M_{n+1} , a may be at M_n while traversing M_n and at M_{n+1} when having just traversed M_n in the same way as I climb the stairs: climbing the (n+1)th stair while just having arrived at the nth.
- (v) Evidently, Aristotle cannot allow the indivisible parts of the trajectory to form a discrete succession which interpretation of T 16 is, of course, far from orthodoxy. Nothing is presupposed in that passage concerning the topological structure of the trajectory except this. The trajectory exhibits some linear order such that in the corresponding order of time, for

each part of the trajectory, traversing that part takes place before having traversed it (T 16a).¹⁰⁶

(vi) In particular, no metric is presupposed in that passage. Rather, the requirements are just the same as in T 8-11.

(vii) I am still struggling with the *Arrow*. Arsenijevic et al. (2008) take its second premise, "the moving thing is always in the now", to be just one horn of a dilemma; the other horn being "the body is at least sometimes in a time interval" (op. cit. p. 32). Decads ago, I tried (unsuccessfully) to combine a similar construction with a version of Eleatic presentism. ¹⁰⁷ I also tried to exploit the idea that Aristotle's argument in T 16 was formed on a Zenonian model and may thus provide information concerning the *Arrow*. The latter item in my approach may be resumed in view of the interpretation of the *Arrow* by Arsenijevic et al. In particular, my cases 1 and 2 (T 16c-d) seem to form a dilemma similar to the dilemma they describe. – But going into any details is work for the future. ¹⁰⁸

References:

Anagnostopoulos, Andreas [2010] "Change in Aristotle's Physics 3", OSAP 39, p. 33-79

Arsenijevic, Milos, Scepanovic, Sandra and Massey, Gerald J. [2008]: "A New Reconstruction of Zeno's *Flying Arrow*", *Apeiron* 41.1, 1-43

Bowin, John [2008]: "Aristotle on Identity and Persistence", Apeiron 41.1, 63-88

Brague, Rémi [1982]: "Sur la formule aristotélicienne ho pote on", in his Du Temps chez Platon et Aristote, Paris: PUF, p. 99-144

Charles, D. [2004] "Simple Genesis and Prime Matter", in: *Aristotle: On Generation and Corruption, Book I.* Proc. XV Symposium Aristotelicum, ed. by J. Mansfeld and F. de Haas, OUP, p. 151-169.

Coope, Ursula [2005]: Time for Aristotle: Physics IV.10-14, OUP

Ford, Lewis S. [1984]: The Emergence of Whitehead's Metaphysics 1925-1929, Albany: SUNY Pr.

Frege, Gottlob [1892]: "Über Sinn und Bedeutung", in: G. Frege, Funktion, Begriff, Bedeutung. Fünf logische Studien, hg. von G. Patzig, 2. Aufl. Göttingen 1966, p. 40-65

¹⁰⁶ Aristotle's analysis in *Physics* VI of motion and the continuum is evidently designed to avoid the difficulties exhibited by Zeno's paradoxes. Diodorus Cronus, by contrast, is reported by Sextus Empiricus (*Adv. math.* 10,85 ff.) to have developed Zeno's arguments and explicitly endorsed the formula "never moves, but has moved". In Aristotle that formula serves to indicate the absurdity entailed by the assumption that time and magnitude are composed of indivisible parts.

¹⁰⁷ See pp. 105-141 in my *Zum Naturbegriff im archaischen und klassischen griechischen Denken*. Zwischenbericht, Teil 1. Kasseler Philosophische Schriften, preprint 1/91, Kassel 1991. – "Eleatic presentism": cf. Parmenides, DK 28 B 8.5-6: "... neither was, nor will it be, since it is now, all together, one, holding together"; Plato, *Parm*. 152e1-2: "... is always now, whenever it is"; Zeno in Aristotle, *Phys.* VI 9, 239b6-7: "... is always in the now".

¹⁰⁸ I am grateful to Niko Strobach for his helpful comments, and to P.H. Coetzee who did a great job polishing my English.

- Frege, Gottlob: *Schriften zur Logik und Sprachphilosophie*, ed. by G. Gabriel, 2nd. ed. Hamburg: Meiner 1978
- Heinemann, Gottfried [2007]: "Whitehead's Interpretation of Zeno", in: *A Handbook of Whiteheadian Process Thought*, ed. M. Weber, Frankfurt am Main: ontos
- Hussey, Edward [1983]: Aristotle's Physics. Books III and IV, tr. with notes, OUP
- Kirwan, Christopher [1993]: *Aristotle. Metaphysics. Books Gamma, Delta, and Epsilon*, tr. with notes, 2nd ed. OUP
- Leibniz, Gottfried Wilhelm: *Philosophical Papers and Letters. A selection*, tr. and ed. by L.E. Loemker, 2nd. ed. Dordrecht Boston: Reidel 1969, repr. 1976
- Loughlin, Tim [2011]: "Souls and the Location of Time in Physics IV 14", Apeiron 44, 307-325
- Ross, W.D. [1936]: Aristotle's Physics. A rev. text with introd. and comm. Oxford: Clarendon
- ROT = The Complete Works of Aristotle. The Revised Oxford Translation, ed. by J. Barnes, Princeton 1984
- Russell, B. [1901]: "Recent Work on the Principles of Mathematics", in: *The Collected Papers of Bertrand Russell*. Vol. 3. *Toward the "Principles of Mathematics": 1900-02*, ed. by Gregory H. Moore, London: Routledge 1993, p. 366-379 [reprinted as "Mathematics and the Metaphysicians", in Russell's *Mysticism and Logic* (1918), p. 74-94].
- Tugendhat, Ernst and Wolf, Ursula [1983], Logisch-semantische Propädeutik, Reclam
- Wagner, Hans [1979]: *Aristoteles. Physikvorlesung* (Aristoteles, Werke in deutscher Übersetzung, begr. von E. Grumach, hg. von H. Flashar, Bd. 11), 3. Aufl. Berlin: Akademie Verlag
- Zeller, Eduard [1876]: Die Philosophie der Griechen in ihrer geschichtlichen Entwicklung, 1. Teil: Allgemeine Einleitung. Vorsokratische Philosophie, 4. Aufl. Leipzig: Fues