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# Wavelet and fluctuation analysis of 20th-century monthly precipitation extremes in Germany

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## Abstract

Monthly precipitation extremes from gauging stations across Germany were studied using continuous wavelet tolls towards variance and frequency time localization and detection of dominant oscillations. In the late 19th and early 20th century, monthly precipitation could be characterized by high frequency oscillations whereas in the second half century, apart from the variance increase, low frequency oscillations with periods 11 or 14 years become noticeable. Wavelet analysis of single extreme events showed that the latter are likely caused by annual/seasonal oscillations rather than high frequency oscillations. Applying Detrended Fluctuation Analysis (DFA) and the Wavelet Transform Modulus Maxima method (WTMM) we look for correlation pattern in the precipitation time series through the computation of self-similarity parameter, namely, the Hurst coefficient, and teleconnections with large-scale hemisperical interdecadal weather pattern variations. The results indicate that the North Atlantic Oscillation (NAO) could be made responsible for the detected spectral and correlation characteristics.

## Methods

**Continuous wavelet transform** *[Simultaneous extraction of the time frequency information from a signal]*

Since the classical spectral analysis (Fourier transform) provides only the frequency content of a periodic signal and the Windowed Fourier transform (WFT) lacks due to under or over time-localization [1], we apply continuous wavelet transform tools [2] on time series of precipitation data towards variance and frequency time localization and detection of dominant oscillations.

**Detrended Fluctuation Analysis (DFA)**
**Wavelet Transform Modulus Maxima Method (WTMM)** *[Detection of long-range dependence scaling exponent]*

Scaling phenomena are manifested through a spectral power-law scale behavior of some scale dependant time series statistics. If this property applies to a whole range of scales, i.e. from a sampling unit to the time series length, the exponent in this power-low dependence is called a self-similarity exponent, that can be characterized by a fractal dimension D, or more suitably, by the Hurst coefficient H. In the case of self-similarity, the $q^{th}$ order moment of the time series wavelet coefficients scales as $s^{qH}$. Also, in the presence of long-range correlations, the decay of the autocorrelation function ACF(t) has a power-law character.

## Data

The wavelet tools [2], DFA [3] and the WTMM [4] method are applied to long-range precipitation records from the German Meteorological Service [ www.dwd.de] DWD gauging stations across Germany. We extract monthly precipitation extremes where the lowest scale becomes $s_0$=2 months. All wavelet-based analyses involved the Morlet wavelet with the nondimensional frequency equal to $\omega_0$=6. Normalization is done by substraction of the linear least square fit to define the fluctuation time series.

According to Hurell [5], a major source of interannual variability in the atmospheric circulation across Europe is the North Atlantic Oscillation (NAO). We used the NAO index from the Climatic Research Unit [http://www.cru.uea.ac.uk/ftpdata/nao.dat]. The Nino3 See Surface Temperature (N3 SST) data are from the University of Washington [http://tao.atmos.washington.edu/data_sets/globalsstenso/].

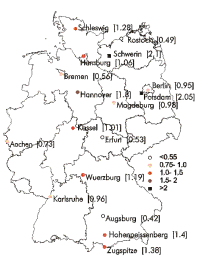


***Fig. 1:*** *7-14 yr. scale averaged global wavelet power for the selected DWD stations.*

## Results

**Wavelet based statistics**

The time series span showed to be a decisive factor for the quantification of the results. Only the annual oscillation shows interval-independent characteristics. It dominates the spectrum for all locations, except Zugspitze, the location with the highest altitude (2960 m), where a 14 year oscillation dominates. For a comparison of the frequency content and to test the hypothesis of significant changes of precipitation patterns in the second part of the century, the long time series were split into 30-50 years long continuous non-overlapping intervals and the wavelet tool was applied to each of them separately (see Figure 2).

Low frequency spectral characteristics of the analysed data sets proved to be time-dependant, while the high frequency, 1 yr. oscillation remains significant for all mean and low altitude locations. Namely, from the mid 20th century on, a distinct low frequency component with either a 11 or 14 yr. peak extends the lower tail of the precipitation frequency distribution. Because of the time-variability of the precipitation spectral characteristics we fixed our time window to the period from 1960 to 1999 for all time series.

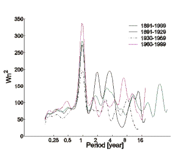


***Fig. 2:*** *Normalised global wavelet spectrum (Hamburg)*

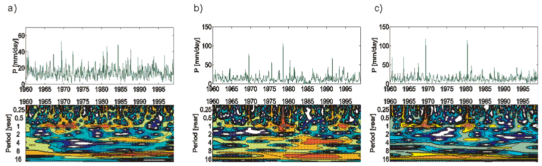


***Fig. 3:*** *Time series and the corresponding log wavelet power spectrum using Morlet wavelet with nondimensional frequency $w_0$=6 : a) Hannover, b) Potsdam, c) Schwerin. The y- axis of the wavelet spectra is the Fourier period. The thick solid line indicate the cone of influence.*

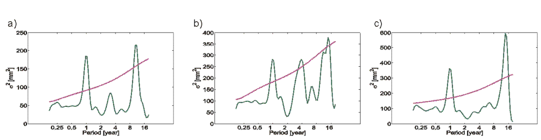


***Fig. 4:*** *Global wavelet spectrum and the 95% - confidence level: a) Hannover , b) Potsdam, c) Schwerin.*

**Long range correlations and memory effects**

The behavior of the monthly extrema as a function of observation scale s was analysed applying both the DFA and the WTMM method. These two approaches were tested in a numerical experiment using fractional Brownian motion (fBm) with H= 0.8 obtaining $H_{DFA}$ = 0.748 and $H_{WTMM}$ =0.776, respectively, therefore confirming that both methods estimate H suffiently close. The argument in favour of using the DFA method is the sensitivity of the WTMM method to the choice of the lower and the upper cutoff scale (scales over which power law is satisfied).

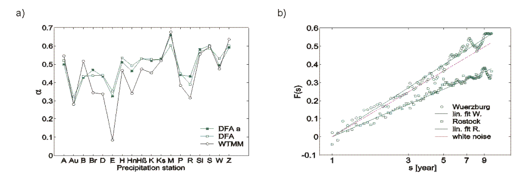


***Fig. 5:*** *a) Self-similarity (Hurst) parameter H for selected DWD gauging stations (alphabetically sorted- Table 1) by the DFA a method (the remaining part at the end of the time series was neglected), classical DFA and the WTMM method ; b) Fluctuation function as a function of scale for Rostock (square), Würzburg (circle) and white noise (H=0.5).*

**Inter-decadal teleconnections between the precipitation and the North Atlantic Oscillation (NAO)**

The North Atlantic Oscillation (NAO) atmospheric oscillation is a main determinant of the European climate and its variability [5]. The NAO index is defined as a standardised difference between the sea level pressure (SLP) average over the Azores and over Iceland. Analog to the wavelet spectrum of the precipitation time series, the global wavelet spectrum of the 20th century NAO is also multiscale but exhibits a pronounced dominance of low frequency components, unlike the El Nino SST spectrum (Fig.6). In Table 1 estimations of the correlation coefficient between winter precipitation time series and NAO, probability values for testing null hypothesis of no correlation and upper and lower bounds for the 95% confidence interval are listed. Locations with a strong low frequency pattern show also a significant correlation coefficient with the NAO. In particular, for Hannover, the cross-wavelet spectrum for winter precipitation and winter NAO indicate high power within the long period band 7-14 years (Fig. 7).

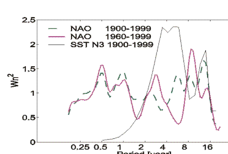


***Fig. 6:*** *Normalised NAO (the overall and the spectra for data intervals) and the N3 SST wavelet global spectra. Normalisation factor is $1/\sigma^2$.*

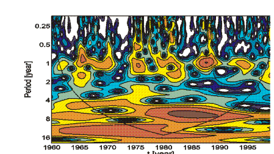


**Fig. 7:** *Cross-wavelet spectrum for winter precipitation data (Hannover) and the NAO index using Morlet with w0=6. The thick solid line is* the COI.

***Table 1:***

*Frequency content; correlation with NAO. The values in the second column are the significant oscillation periods assuming white or red noise background. Behind the slash, large peaks detected in the global spectrum, but which are statistically not significant, are indicated. H is the estimated Hurst-coefficient, $\sigma_\alpha$ the standard error of the estimated H, $R^2$ the coefficent of determination; r the estimated correlation coefficient between the NAO and DWD (Nov-Mar) monthly time series, and Prob the probability of the corresponding t-statistic.*

| Location | Sig. Per. | H | $\sigma_\alpha$ | $R^2$ | r | Prob |
|---|---|---|---|---|---|---|
| Aachen | 1/8 | 0.52 | 0.004 | 0.98 | 0.209 | 0.196 |
| Augsburg | 1 | 0.32 | 0.006 | 0.96 | 0.118 | 0.467 |
| Berlin | 1/14 | 0.43 | 0.005 | 0.98 | 0.048 | 0.770 |
| Bremen | 1/14 | 0.44 | 0.006 | 0.97 | 0.301 | 0.059 |
| Erfurt | 1 | 0.35 | 0.005 | 0.97 | 0.144 | 0.377 |
| Hamburg | 1/7 | 0.53 | 0.005 | 0.97 | 0.524 | 0.001 |
| Hannover | 1,11 | 0.49 | 0.004 | 0.99 | 0.434 | 0.005 |
| Hohenpeißenberg | 1/11 | 0.53 | 0.004 | 0.98 | 0.114 | 0.484 |
| Karlsruhe | 1/14 | 0.53 | 0.006 | 0.97 | 0.210 | 0.194 |
| Kassel | 1/14 | 0.52 | 0.011 | 0.94 | 0.267 | 0.095 |
| Magdeburg | 1/14 | 0.60 | 0.015 | 0.96 | -0.198 | 0.221 |
| Potsdam | 1,4,14 | 0.45 | 0.003 | 0.98 | 0.026 | 0.875 |
| Rostock | 1 | 0.39 | 0.006 | 0.97 | 0.288 | 0.071 |
| Schleswig | 1/11 | 0.57 | 0.007 | 0.98 | 0.248 | 0.123 |
| Schwerin | 1,14 | 0.59 | 0.007 | 0.97 | 0.448 | 0.004 |
| Würzburg | 1,7/14 | 0.53 | 0.007 | 0.98 | 0.148 | 0.363 |
| Zugspitze | 14 | 0.60 | 0.005 | 0.99 | 0.358 | 0.024 |

## Conclusions

Monthly precipitation extremes from 18 gauging stations across Germany were analysed using continuous wavelet tools towards variance and frequency time localisation and detection of dominant oscillations. In the late 19th and early 20th century, monthly precipitation in Hamburg could be characterised by high frequency oscillations with periods of 1, 2 and 4 years, while from the mid 20th century on, only the 1 year oscillation remains significant, and a smaller 21 year oscillation peak is recognizable. Similarly, a 6 year pattern is observable until the 1940's for Schwerin which almost disappears in the second half of the 20th century when a 14 year-period becomes significant.

Generally, for most of the analysed locations a low frequency component with periods of either 11 or 14 years is noticeable. The second half of the 20th century is also characterised by a slight variance increase, and with an increase of the peaks since late 1960's. Using DFA analysis the Hurst coefficient H was computed and found to be H ~0.4 in the eastern part of Germany, and H ~0.6 for most stations in the western part, i.e. which hints of anticorrelated and correlated precipitation pattern for these two hemisections of Germany, respectively. Attempts to put the long-period precipitation variations into a framework of possible intermediate-term climatic variations over Central Europe over the last half century provide evidence for some teleconnectivity with the North Atlantic Oscillation.

## References

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